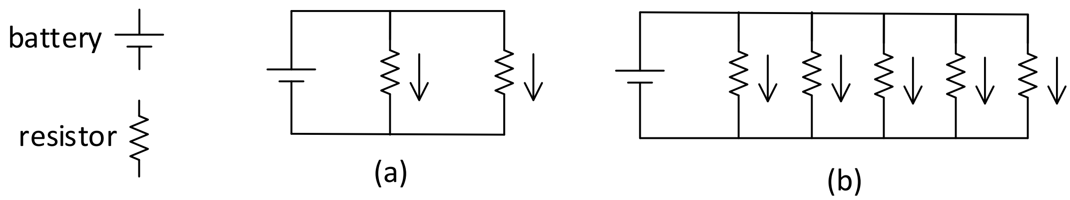
2 The Series, Parallel and Series-Parallel Circuit:

Figure 2-1 below show a battery and multiple resistors arranged in parallel. Each resistor receives a portion of the current from the battery based on its resistance. The split is proportional to the value of the individual resistors. Higher resistance would mean less current. Lower resistance would mean more current. The sum of these splits flows from the battery to the resistors and then the same total current would again flow back into the negative of the battery.

Fig. 2-1



The Kirchoff Current Law or KCL

KCL states that the sum of the currents into a point equals the sum of the currents out of the point. A point is defined as a common tie point where wires are actually touching or connected together. In the figure below (Fig. 2-2), current flows into a point ‘a’ and then out. In Fig. 2-2a, the entire current (I_1) flows out into I_2. We can write KCL for Fig. 2-2a as follows:

$$I_1 = I_2$$

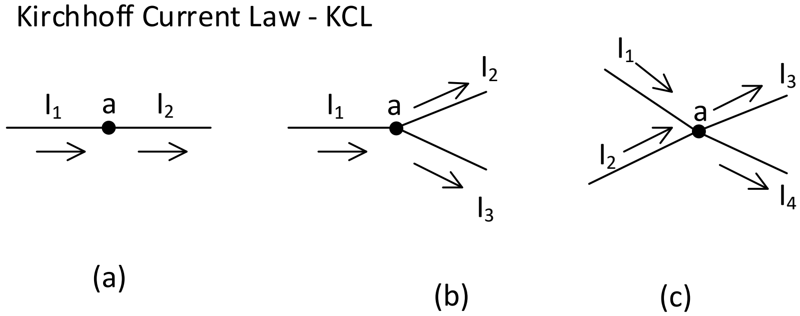
In Fig. 2-2b, the current I_1 splits with some traveling down one wire and some traveling down the other wire. We can write KCL for Fig. 2-2b as follows:

$$I_1 = I_2 + I_3$$

In Fig. 2-2c, the current comes to point ‘a’ from two sources, I_1 and I_2. The current traveling out from point ‘a’ in two wires as I_3 and I_4. We can write KCL for Fig. 2-2c as follows:

$$I_1 + I_2 = I_3 + I_4$$

Fig. 2-2 Kirchhoff Current Law - KCL



You can summarize KCL with the following equation:

$$\sum \text{currents into point} = \sum \text{currents out of point}$$

KCL is a crucial law in circuit analysis. It is simple in concept and problems are easy. KCL is intuitive in that it is easy to see electrons flowing into a point ‘a’. Where do they go? Do they accumulate at a point or do they flow out immediately. Obviously the electrons must continue to flow out of the point as they enter. They do not accumulate at a point!

An example of KCL:

Find the value of the unknown I in Fig. 2-3(a) below. Applying KCL at point 'a', write:

$$7 = 3 + I$$

Therefore, $I = 4A$

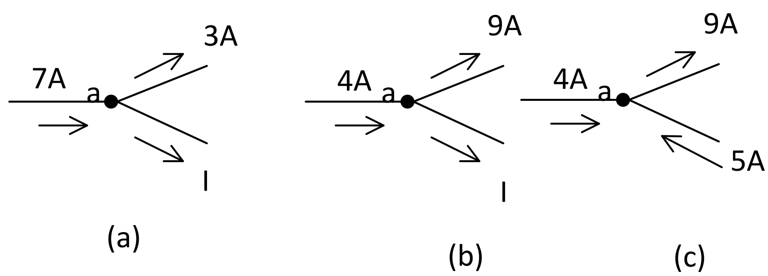
In Fig. 2-3(b), apply KCL at point 'a' to write:

$$4 = 9 + I \qquad -> \qquad I = -5A$$

In the equation, we notice that a negative sign occurs. The current that flows in this branch is opposite the assumed direction. The circuit is redrawn in Fig. 2-3c showing the actual direction of the current.

The direction of current will be determined by writing the KCL equation and noticing the sign of any unknown current. A minus sign shows that current is actually flowing in the opposite direction. KCL automatically determines current direction. This was shown in the example in Fig. 2-3(b) and (c).

Fig. 2-3 Examples of KCL
(Kirchhoff's Current Law)



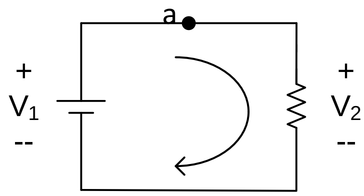
Remember that KCL is true because electrons do not disappear or accumulate at a point but rather continue to travel through the circuit.

The Kirchhoff Voltage Law of KVL

With KCL, all unknowns were currents. With the Kirchhoff Voltage Low, unknowns are voltages. The law states that the sum of voltages around a closed path equals zero. This law says to start at any point in a circuit, walk in a clockwise direction around the circuit and write voltages of each voltage source or load in an equation with the sum equal zero. A number of examples will give us a better understanding of this law. Another question is why is this law so important? It is important in that it leads to a number of equations with unknown voltages in any circuit. It is also useful to apply the voltage divider formula – a formula we will discuss in a few pages.

A first simple example of KVL:

Fig. 2-4



A single source (battery) and single load (resistor):

Start walking at point 'a' above in Fig. 2-4. The first voltage encountered is V_2 (a resistor). Since the sign is +, write (+) in the equation. At V_2, write:

$$+ V_2$$

Keep walking clockwise. At V_1 write $- V_1$ since the battery was entered from the negative side:
:

$$+ V_2 - V_1$$

End the walk back at the point 'a'. There write:

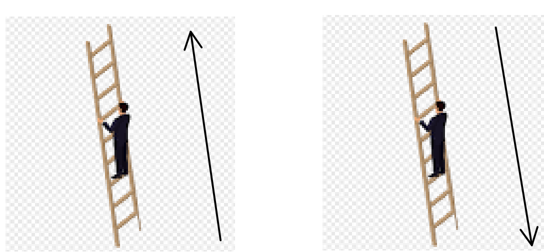
$$+ V_2 - V_1 = 0$$

From this, one gets:

$$V_2 = V_1$$

Climbing a ladder shows an equivalent analogy. When we climb up a number of steps, to get back to the origin (or ground), we must climb down the equal number of steps.

Fig. 2-5 Going Up Ladder – then Down



The source at the left is the man going up the ladder a number of steps. The resistance or load shows the same number of steps down the ladder back to zero volts or ground. The number of steps up must equal the number back down.

One source, two loads:

Fig. 2-6 (a) shows another example of KVL. Starting at point 'a' and walking clockwise gives the following three voltages:

$$+V_2 + V_3 - V_1$$

from which we find:

$$+V_2 + V_3 - V_1 = 0 \quad \text{or} \quad V_1 = V_2 + V_3$$

From these equations, it can be shown that the source voltage equals V_1 equals the sum of the load voltages V_2 and V_3.

Fig. 2-6

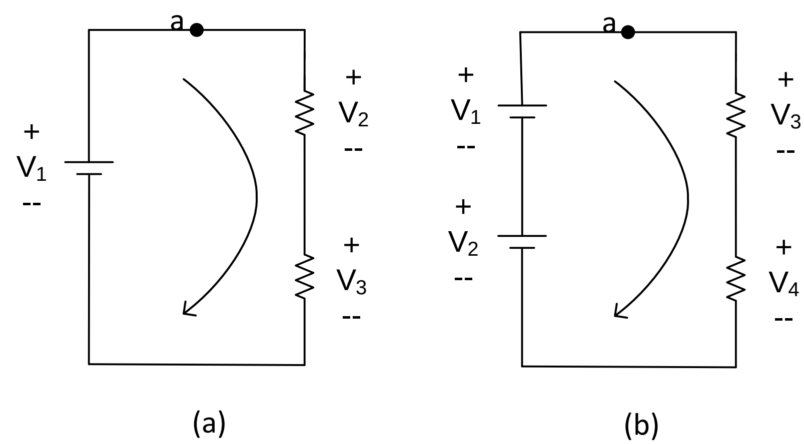
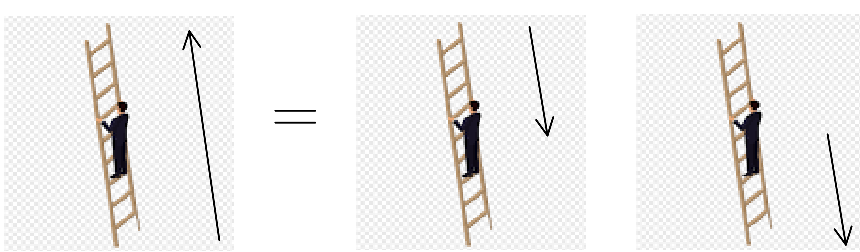


Fig. 2-7 Going Up Ladder – then Down



The same energy used to go up the ladder now is divided between the two efforts going down the ladder.

Two sources, two loads

Fig. 2-6 (b) shows another example of KVL. Starting at point 'a' and walking clockwise gives the following four voltages:

$$+V_3 + V_4 - V_2 - V_1 = 0 \quad \text{or} \quad V_1 + V_2 = V_3 + V_4$$

From these equations, one can see that the source voltages V_1 and V_2 equal the sum of the load voltages V_3 and V_4.

The equation for the Kirchhoff Voltage Law (KVL) states:

$$\sum \text{Voltages around closed loop} = 0$$

How to use KVL and KCL will be the subject of problems throughout this chapter and the next.

Find the unknown value of V in Fig. 2-8a:

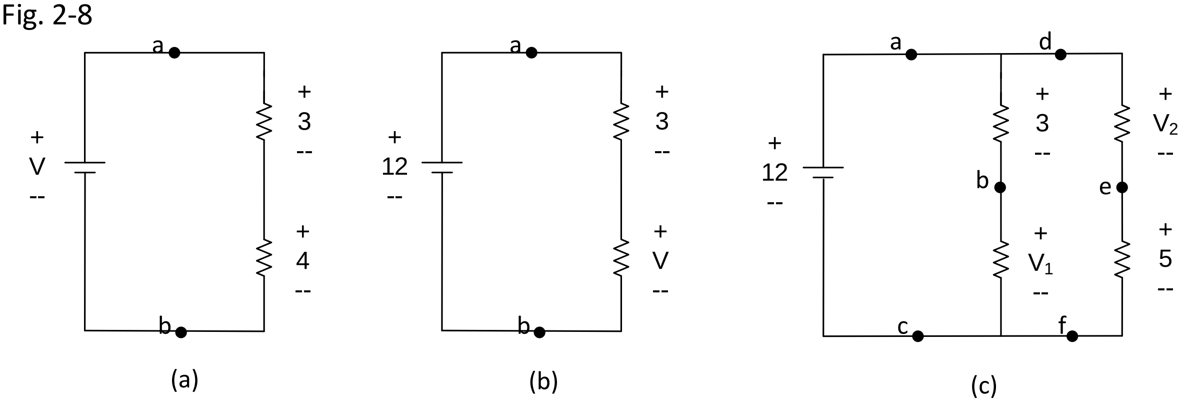
Starting at 'a' and moving around the loop clockwise gives the following:

$$3 + 4 - V = 0$$

Solving for the unknown V,

$$V = 7\ V$$

Fig. 2-8



The example gives each voltage except one giving a method for finding an unknown voltage. This method (KVL) can be used for any loop. The next example shows the same principle in use:

Find the unknown value of V in Fig. 2-8b:

Again, using KVL:

$$3 + V - 12 = 0$$

giving:

$$V = 9V$$

Another example:

Find the unknown values of V_1 and V_2 in Fig. 2-8c:

It is noticed that each loop with resistors have an unknown voltage. The first voltage to find is V_1 and can be found by choosing loop a-b-c-a which gives:

$$3 + V_1 - 12 = 0$$

Solving for V_1 we find:

$$V_1 = 9V$$

The second voltage to find is V_2. To find it, use path d-e-f-c-a-d or:

$$V_2 + 5 - 12 = 0$$

Solving for V_2 we find:

$$V_2 = 7V$$

Remember to be flexible with which loop to choose. We could have found V_1 and then use that branch to find V2 instead of the original path chosen (d-e-f-c-a-d). Either path works.

Analysis of Series Circuits (One Path)

Remember with a series circuit that the current is the same at any point in the circuit. Use this fact when solving problems that are series ones. This can be summarized in general as:

$$I_1 = I_2$$

This is an application of KCL (current in equals current out of any point).

Also an output of the series circuit is that the voltage ratio equals the resistance ratio.

Fig. 2-9b, is a series circuit with the R_1 and R_2. The equations of voltages are:

$$V_1 = R_1 \cdot I$$
$$V_2 = R_2 \cdot I$$

A ratio of voltages can be written:

$$\frac{V_1}{V_2} = \frac{R_1 \cdot I}{R_2 \cdot I}$$

or

$$\frac{V_1}{V_2} = \frac{R_1}{R_2}$$

Fig. 2-9

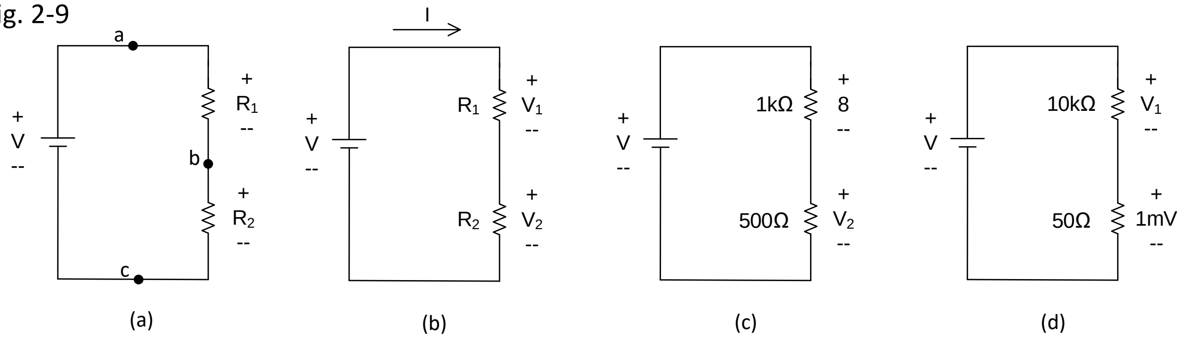


Fig. 2-9c adds values for R_1, R_2 and V_1 to allow us to write:

$$\frac{8}{V_2} = \frac{1000}{500}$$

or $\quad V_2 = 4V$

Fig. 2-9d adds values for R_1, R_2 and V_2 to allow us to write:

$$\frac{V_1}{0.001} = \frac{10{,}000}{50} = 200$$

or $\quad V_1 = 0.2\ V = 200\ mV$

Equivalent resistance

The equivalent resistance of two or more resistors in a series circuit equals the sum of the resistances in the circuit. Fig. 2-10a shows two resistors in series between a and b. The equivalent resistance is shown in Fig. 2-10b and equals:

$$R = 4000 + 2000 = 6000\ \Omega$$

The equivalent resistance is usually referred as R_{eq} or R equivalent. R_{eq} has the same resistance as the two resistors together in series.

Voltage across the two resistors equal the voltage across the equivalent R_{eq}.

$$V_1 + V_2 - V = 0$$

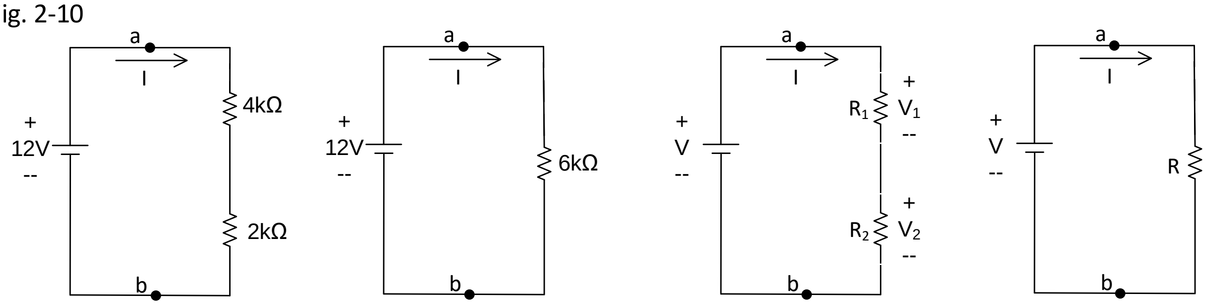
$$V = V_1 + V_2$$

Also $V_1 = R_1 I$

and $V_2 = R_2 I$

The current of the equivalent R_{eq} is equal to the series circuit.

Fig. 2-10



(a) (b) (c) (d)

For Fig. 2-10d, the equation for I is:

$$I = \frac{V}{R}$$

for the series circuit: $R_{eq} = R_1 + R_2$

The current in the equivalent circuit of Fig. 2-10d equals the current in Fig. 2-10c. This is an example of an equivalent resistance giving the same results as the two separate resistances of Fig. 2-10c.

An equation for R_{eq} for series circuits with more than two resistors is capable of being developed. This equation is:

$$R_{eq} = R_1 + R_2 + R_3 + \ldots + R_n$$

This is many times referred to as R-total or R_T.

The circuit below in Fig. 2-11 has values of resistors: $R_1 = 20\ \Omega$, $R_2 = 50\ \Omega$, and $R_3 = 10\ \Omega$. The equivalent resistance is 80 Ω. For a voltage source of 8 V, the current can be calculated as:

$$I = \frac{V}{R} = \frac{8}{80} = 0.1\ A = 100\ mA$$

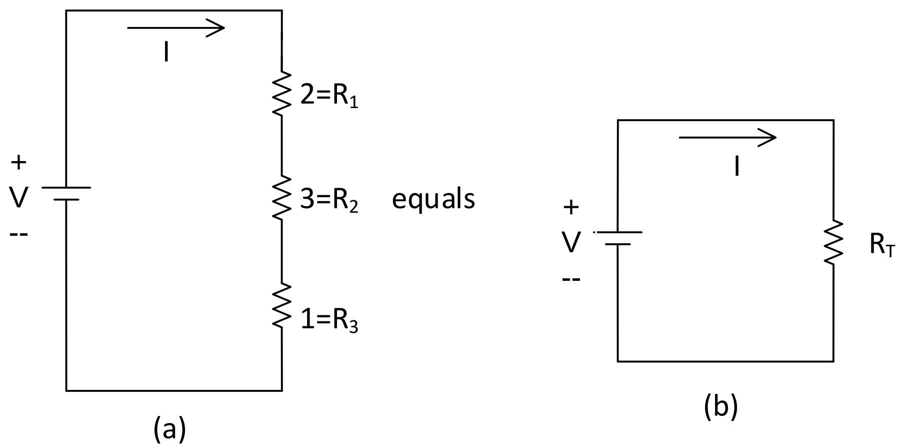


Fig. 2-11

Find the current I in Fig. 3-12a:

The first step is to find the equivalent resistance R_T:

$$R_T = R_1 + R_2 + R_3 = 2 + 3 + 1 = 6\ \Omega$$

After finding R_T, use the following equation to find I:

$$I = \frac{V}{R_T} = \frac{12}{6} = 2\ \text{A}$$

The resultant current is 2 A. This is also the current in the original circuit of Fig. 2-12a.

We can use 2 A to add the voltage through each resistor in Fig. 2-12a and add the three voltages to find the sum or source voltage.

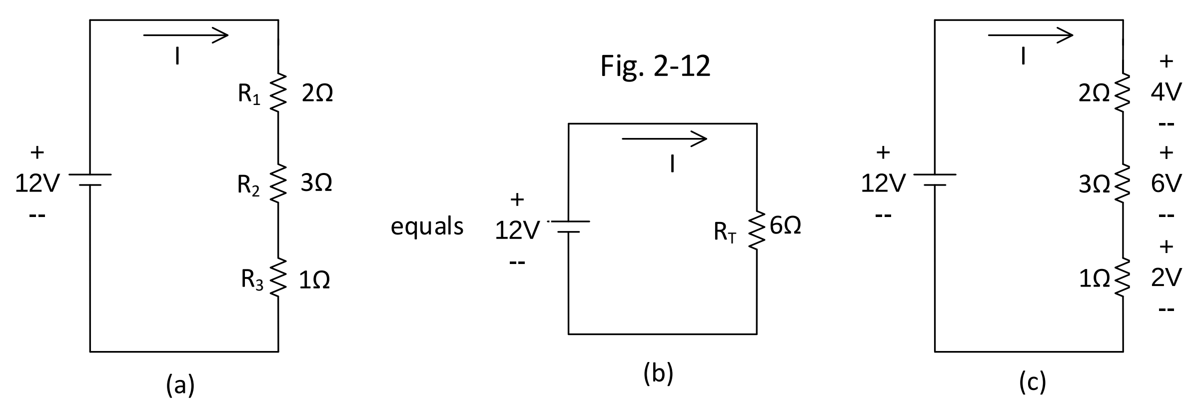
$$V_1 = R_1 I = 2 \times 2 = 4\ \text{V}$$

$$V_2 = R_2 I = 3 \times 2 = 6\ \text{V}$$

$$V_3 = R_3 I = 1 \times 2 = 2\ \text{V}$$

The voltages sum to 12 V (the value of the source). This gives proof that the current is indeed 2 A.

Fig. 2-12



An example of finding R_T from Series Resistors:

Find the total or equivalent resistance R_T in Fig. 2-13(a) and (c). Just add the two resistances to combine into a single resistor.

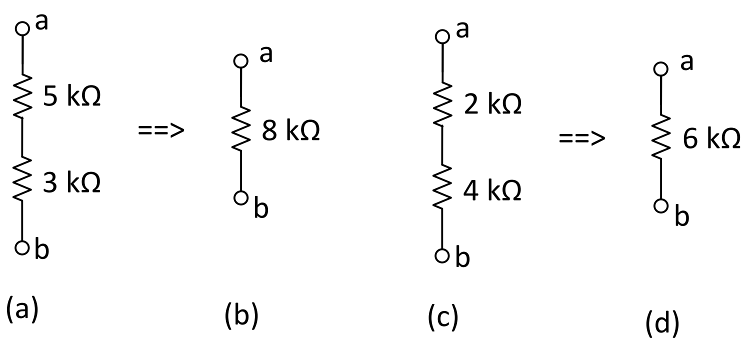
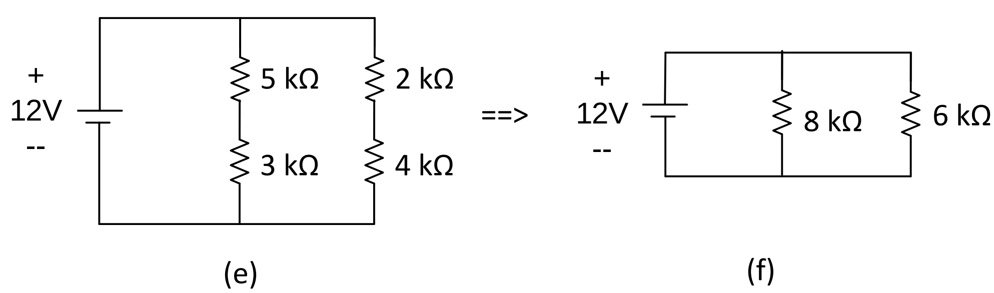


Fig. 2-13

Another example of finding R_T from Series Resistors, this time with multiple paths:

What if the series resistances are inside a larger circuit? These resistances are to be combined first as shown in Fig. 2-13(e) and (f). If the new circuit is parallel, then the parallel rules will apply to that portion of the circuit, rules that are to come.

Fig. 2-13 cont



Another example from the Holidays:

Using the older style incandescent Christmas tree bulbs in series, form a series circuit. Assume 12 lights at 100 Ω each and find R_T by addition. A voltage of 120 V is across the entire string. Find the current through the string.

$$R_T = 12 \times 100 = 1.2\ \text{k}\Omega$$

Current equals:

$$I = \frac{V}{R} = \frac{120}{1200} = 0.1\ \text{A} = 100\ \text{mA}$$

Analysis of Parallel Circuits (Many Paths)

For series circuits, current is the same and voltage splits. For parallel circuits, voltage is constant and current splits. Voltage is equal across parallel circuits per KVL, which states that the voltage around any closed loop must sum to zero. Therefore, any closed loop between parallel branches must have the same voltage. This can be shown in Fig. 2-14a below as:

$$V = V_1 = V_2$$

or

$$V_1 - V = 0$$

and

$$V = V_1$$

Around the right loop of Fig. 2-14a gives similar results:

$$V_2 - V_1 = 0$$

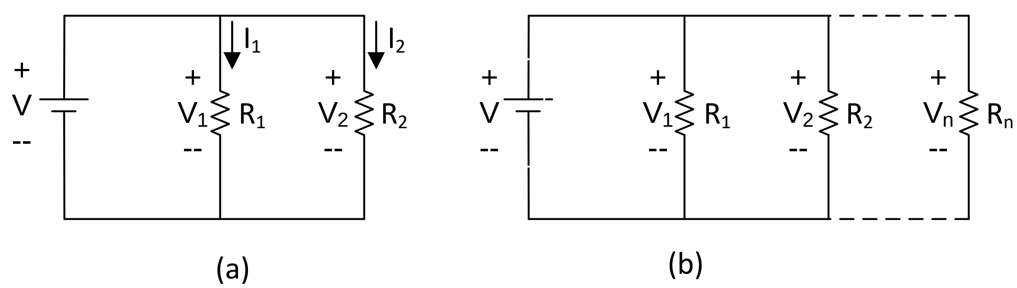
or

$$V_1 = V_2$$

and

$$V = V_1 = V_2$$

Fig. 2-14



Currents in Parallel Circuits

It follows from the fact that the voltage across each resistor is equal that we can write V as an unknown and find the currents in each branch of the parallel circuit of Fig. 3-14b as:

$$I_1 = \frac{V}{R_1}$$

$$I_2 = \frac{V}{R_2}$$

$$\vdots$$

$$I_n = \frac{V}{R_n}$$

By applying KCL (Kirchhoff Current Law), we can see the sum of the currents from each branch is I_T or $I_{(Total)}$:

$$I_T = I_1 + I_2 + \cdot\ \cdot\ \cdot + I_n$$

One can see from the following parallel equations that current is the inverse of resistance. If a resistance is larger, the current is smaller. If we have double the resistance, current is half, etc.

In general:

$$I_1 = \frac{V}{R_1}$$

$$I_2 = \frac{V}{R_2}$$

We can show the ratio of I_1/I_2 as:

$$\frac{I_1}{I_2} = \frac{V/R_1}{V/R_2}$$

or

$$\frac{I_1}{I_2} = \frac{R_2}{R_1}$$

We use this formula to find currents in parallel circuit in Fig. 2-15a below. If we know the current through R_1, we can find I_2. In Fig. 2-15a, I through R_1 equals 1 mA:

$$\frac{I_1}{I_2} = \frac{R_2}{R_1}$$

$$\frac{0.001}{I_2} = \frac{500}{1000}$$

or $I_2 = 0.002$ A = 2 mA

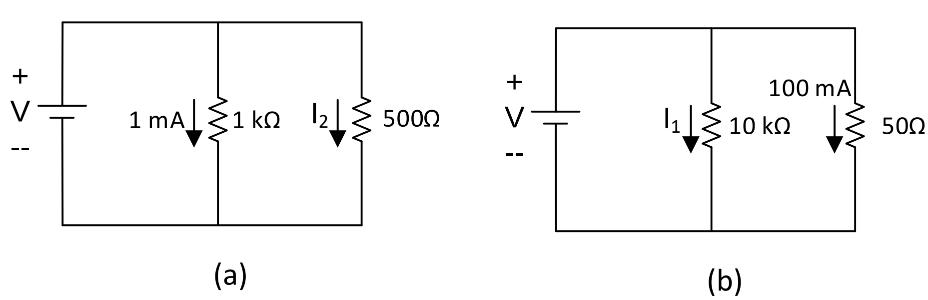


Fig. 2-15

Fig. 2-15b is a similar example but with I_1 unknown. The same equation gives I_1:

$$\frac{I_1}{0.1} = \frac{50}{10{,}000} = \frac{1}{200}$$

$$\text{or} \quad I_1 = 0.5\ \text{mA}$$

The ratio of current is 1:200 based on the ratio of resistance.

Equivalent Resistance in Parallel Circuits

For parallel-circuit analysis, there is a choice between formulas to use. First is the reciprocal rule, the more general rule. The development of this rule follows:

$$I = I_1 + I_2$$

Note

$$I_1 = \frac{V}{R_1}$$

and

$$I_2 = \frac{V}{R_2}$$

By substitution:

$$I = \frac{V}{R_1} + \frac{V}{R_2}$$

and:

$$I = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)V$$

For the simplified circuit,

$$I = \frac{V}{R_{eq}}$$

The equivalent resistance for a parallel path noted as R_{eq} is:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

(reciprocal rule)

Algebraic combination of equation's right side:

$$\frac{1}{R_{eq}} = \frac{R_1 + R_2}{R_1 R_2}$$

or

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

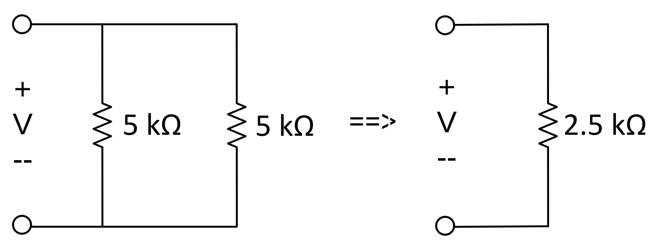
This is the product over sum rule and applies to two resistors, unlike the reciprocal rule (which applies to any number of resistors).

Examples Using Only Two Resistors:

Fig. 2-16a has two 5-kΩ resistances arranged in parallel. We calculate R_{eq} using the product/sum rule:

$$R_{eq} = \frac{5000x5000}{5000 + 5000} = \frac{25{,}000{,}000}{10{,}000} = 2{,}500\ \Omega$$

Fig. 2-16



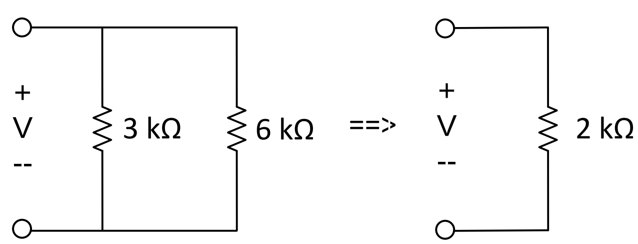
(a)

It can be noted that if the two resistors in parallel are equal, the combination of the two in parallel drops the resistance by half.

The two resistors in Fig. 2-16b are not equal. Again, we use the product/sum rule to calculate Req:

$$R_{eq} = \frac{3000x6000}{3000 + 6000} = \frac{18{,}000{,}000}{9{,}000} = 2{,}000\ \Omega$$

Fig. 2-16 cont

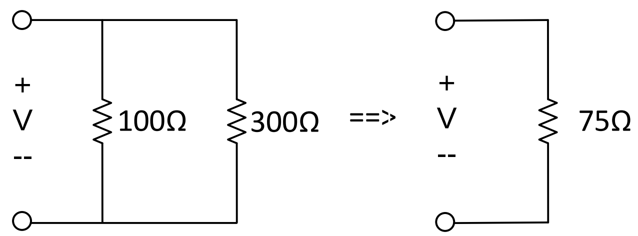


(b)

Again with Fig. 2-16c:

$$\mathrm{R_{eq}} = \frac{100x300}{100+300} = \frac{30{,}000}{400} = 75\ \Omega$$

Fig. 2-16 cont



(c)

The reciprocal or product over sum rule may be used for two resistors but we usually use the product over sum rule since it is usually viewed as easier.

Several Parallel Resistances (More than Two)

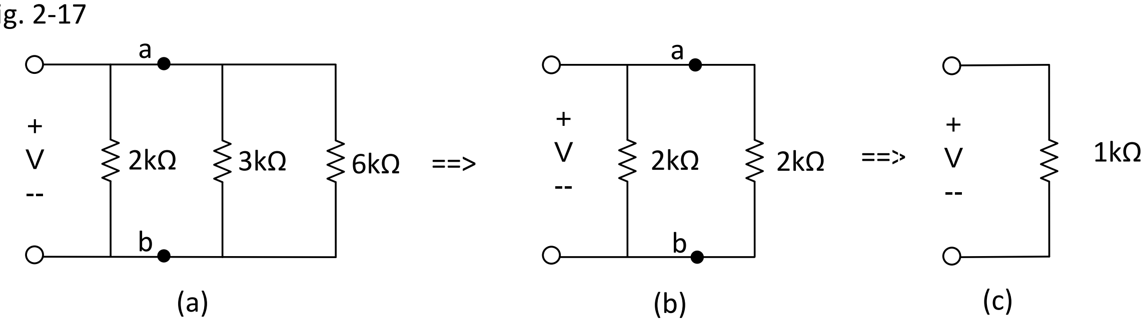
When several resistors (more than two) are in parallel, two methods are available to find the equivalent resistance. One may solve the resistances two at a time. The second method will solve for all resistances in a single equation. This method is the reciprocal rule:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdot\cdot\cdot + \frac{1}{R_n}$$

The product over sum rule may be used, even with a large number of resistors in parallel if we combine two resistors at a time. The following example shows how this method is used with three resistors in parallel:

We first combine the 3-kΩ and 6-kΩ resistors into an equivalent 2-kΩ resistor. We redraw the circuit in 2-17b and find the equivalent resistance R_T in 2-17c. The value is 1-kΩ. See Fig. 2-17c.

Fig. 2-17



The one-step approach uses the reciprocal rule to find R_{eq} in Fig. 2-17. The reciprocal equation is:

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}}$$

Substituting into this equation, we find R_{eq}:

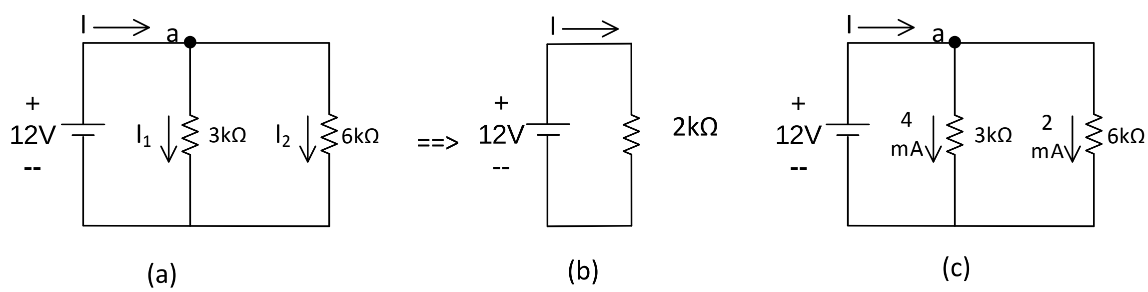
$$R_{eq} = \frac{1}{\frac{1}{2K\Omega} + \frac{1}{3K\Omega} + \frac{1}{6K\Omega}} = 1k\Omega$$

Either method gives the same answer. Sometimes the reciprocal rule is quicker and to be used. Other times, the product over sum rule is better. You will need to choose on a case-by-case basis.

Finding the Current I in Parallel Circuits:

Calculate the current I in Fig. 2-18a:

Fig. 2-18



In Fig. 2-18a find I_1 and I_2. Next, use KCL to add the two currents:

$$I_1 = \frac{12}{3000} = 4 \text{ mA}$$

and

$$I_2 = \frac{12}{6000} = 2 \text{ mA}$$

The value of I_T in Fig. 2-18b is the same in Fig. 2-18a and c. It is I_T = 4 mA + 2 mA = 6 mA.

Checking the ratio of currents as inverse of the resistance:

$$\frac{I_1}{I_2} = \frac{R_2}{R_1}$$

From Fig. 2-18:

$$\frac{I_1}{I_2} = \frac{4\text{ mA}}{2\text{ mA}} = 2 = \frac{R_2}{R_1} = \frac{6\text{k}\Omega}{3\text{k}\Omega}$$

Again, from the Holidays, another Christmas Tree Light Problem:

This time 20 bulbs are connected in parallel. Each light has a resistance of 10 kΩ. Find the entire string's equivalent resistance? Apply 100 V across the string of lights and find $_{IT}$ (total current). Next, find the current in each bulb.

Find R_{eq} using:

$$R_{eq} = \frac{1}{\frac{1}{10{,}000} + \frac{1}{10{,}000} + \frac{1}{10{,}000} + \frac{1}{10{,}000} + (\text{total of } 20)} = \frac{1}{20 \text{ x } 0.0001} = \frac{1}{0.002} = 500\Omega$$

Total current =

$$I = \frac{V}{R} = \frac{100}{500} = 200\text{ mA}$$

The current in each bulb is calculated as 200 mA/20 = 10 mA.

Looking at Fig. 2-19, find R_{eq}:

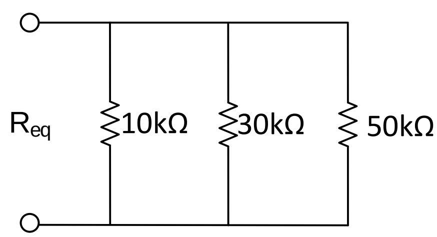


Fig. 2-19

Usually, the one-step reciprocal rule is best. Use it to find R_{eq}:

$$R_{eq} = \frac{1}{\frac{1}{10\text{k}\Omega} + \frac{1}{30\text{k}\Omega} + \frac{1}{50\text{k}\Omega}} = 6{,}520\ \Omega = 6.52\text{ k}\Omega$$

To check the validity of the answer, the answer cannot be greater than the smallest resistance (10 kΩ). It can also not be less than 1/3 the smallest resistance (3.3 kΩ). The answer definitely fits between the limit of this max and min limit and is a valid answer.

Combination Circuits – Let the Real Fun Begin

Often, circuits may be combined into series/parallel combinations. These are referred to as combination circuits. By astute application of the series and parallel rules, an equivalent resistance may be found and various currents and voltages found throughout the circuit.

The following Figure 2-03 will demonstrate finding R_{eq} from a series-parallel combination circuit:

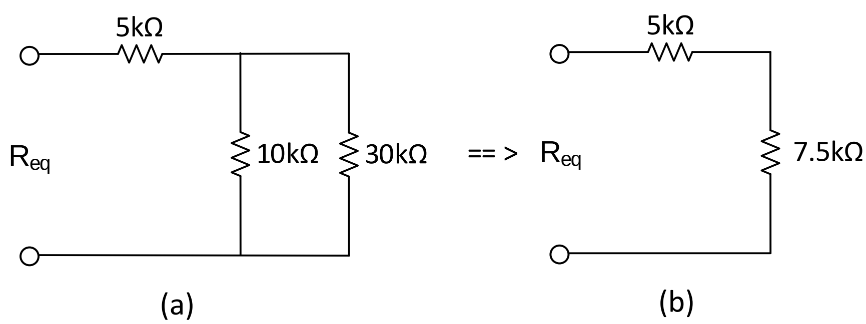


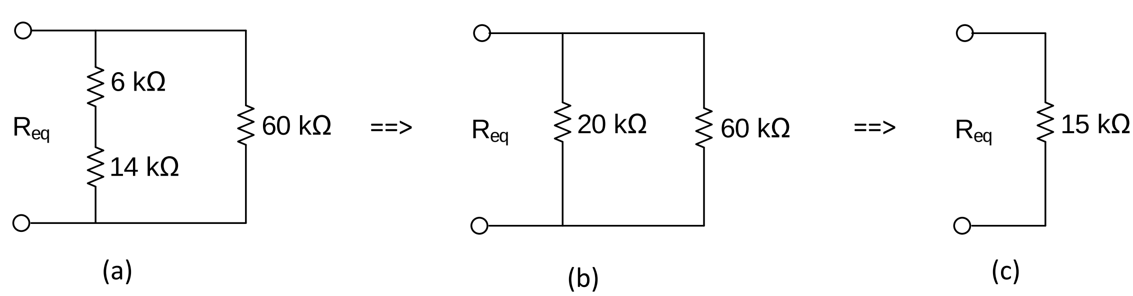
Fig. 2-20

As can be seen, the 10 kΩ and 30 kΩ resistors are in parallel and can be combined using the product over sum rule. From the product-over-sum rule, we get 7.5 kΩ for the resistance of this portion of the circuit. We observe also that the 5 kΩ is in series with the 7.5 kΩ resistance. To find R_{eq}, we add series resistors giving a total R_{eq} of 12.5 kΩ.

Another example:

In Fig. 2-21a below, there is a series-parallel combination that is reduced in (b) and (c). First, observe the 6 kΩ and 14 kΩ combined into a 20 kΩ resistor still in parallel with the 60 kΩ resistor. The product-over-sum rule or reciprocal rule may be used to find the final R_{eq} of 15 kΩ

Fig. 2-21



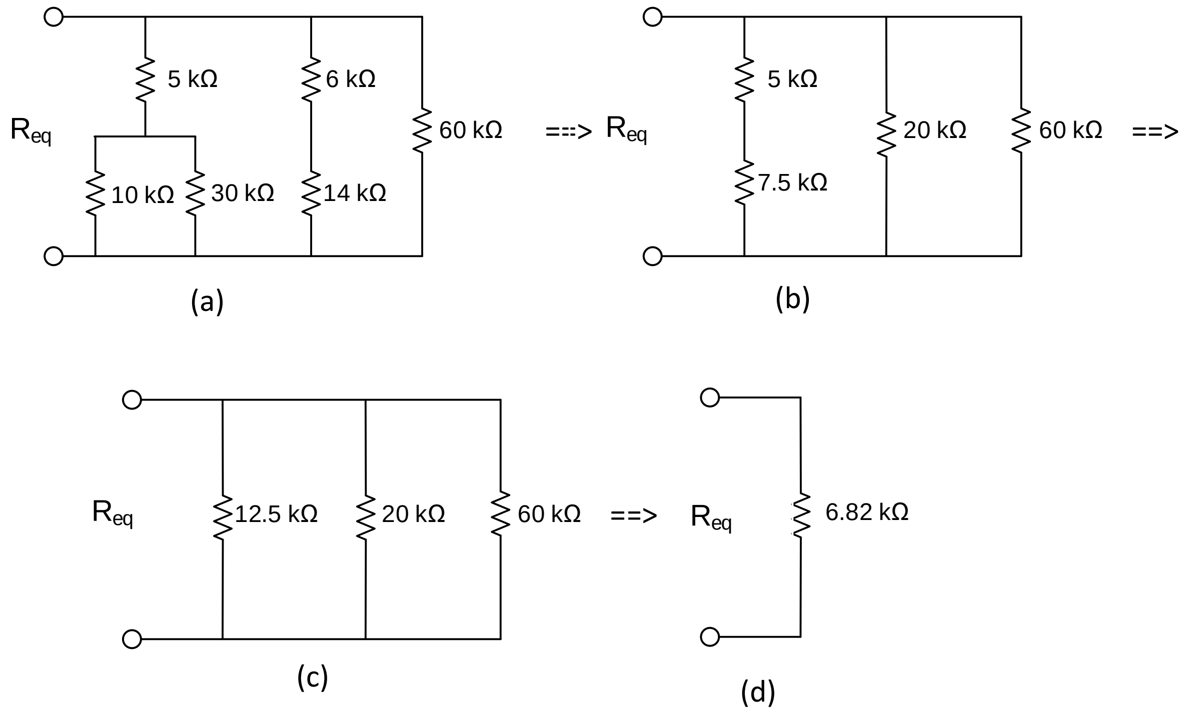
Some general rules for reducing these series-parallel combinations are:

1. Look at the circuit to identifying series and parallel combinations.
2. Replace series and parallel combinations until one equivalent resistance remains.

More examples:

From Fig. 2-22a, find the equivalent resistance of the circuit between the terminals:

Fig. 2-22



First, observe that the 10 kΩ and 30 kΩ resistors are in parallel and the 6 kΩ and 14 kΩ resistors are in series. Simplify both leaving results shown in (b). We are now presented again with a series combination of 5 kΩ and 7.5 kΩ resistors in series which we combine to 12.5 kΩ. Now we are presented with a choice. Either combine two resistors in parallel and then repeat the process or use the reciprocal rule once. I always try to do one operation instead of two so I choose the reciprocal rule which gives 6.82 kΩ for R_{eq}.

Remember:

Series Combination Rule: $R_{eq} = R_1 + R_2 + R_3 \ldots$

Reciprocal Parallel Rule:

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}}$$

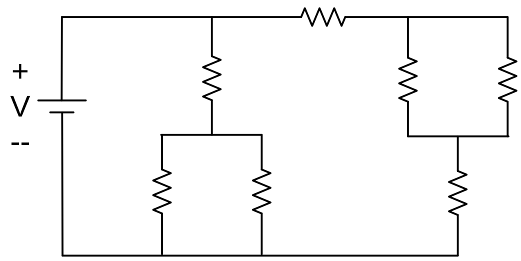
Product over Sum Parallel Rule (two resistors only):

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

Problems:

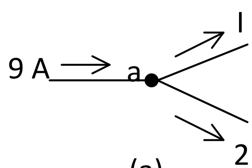
2.1 Find the number of equipotential common tie-points in Fig. 2-23:

Fig. 2-23

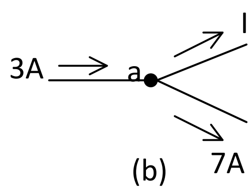


2.2 Calculate current and current direction in Fig. 2-24(a) and (b):

Fig. 2-24



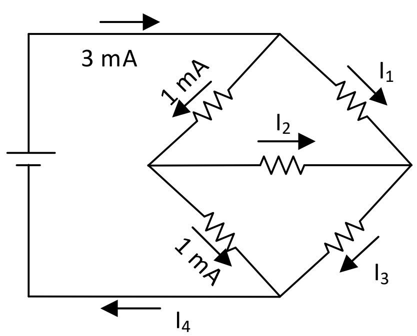
(a)



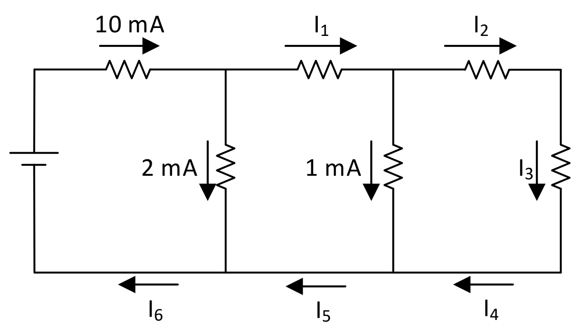
(b)

2.3 For Fig. 2-25(a) and (b), find the unknown currents:

Fig. 2-25



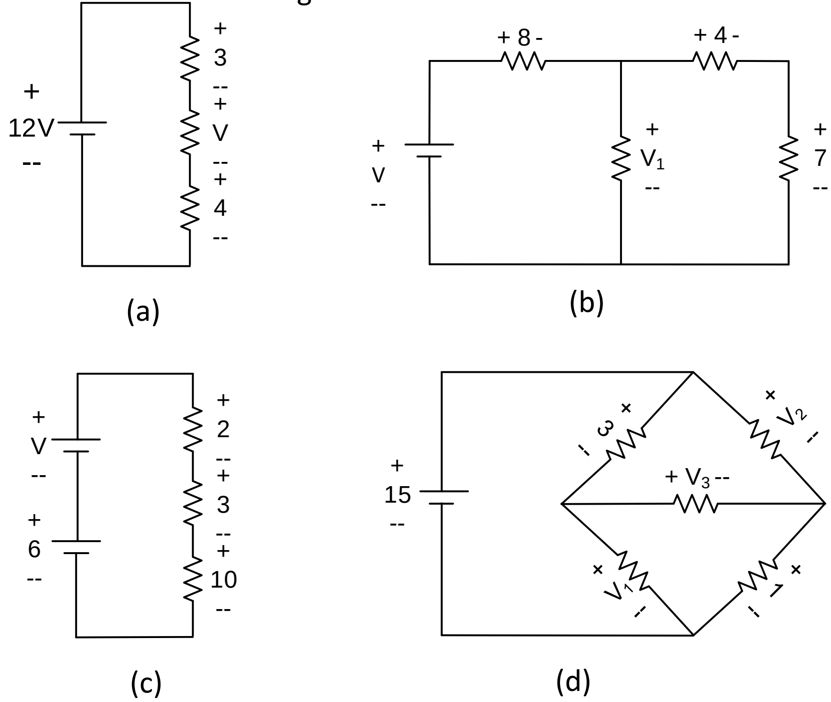
(a)



(b)

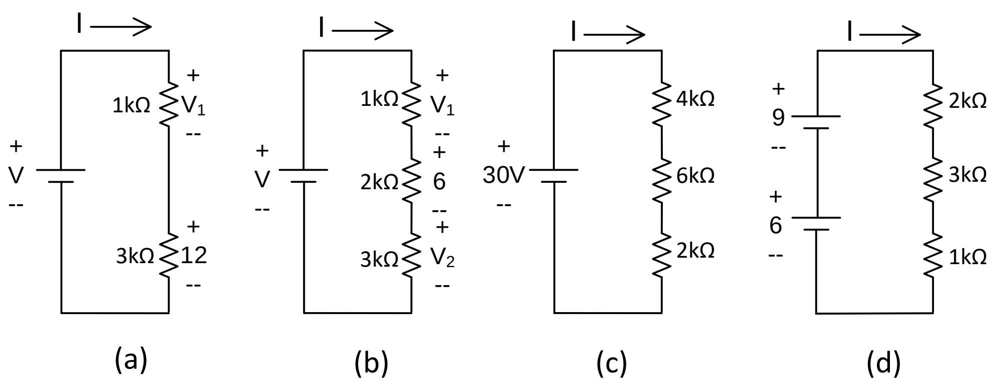
2.4 For Fig. 2-26a-d, find the unknown voltages:

Fig. 2-26



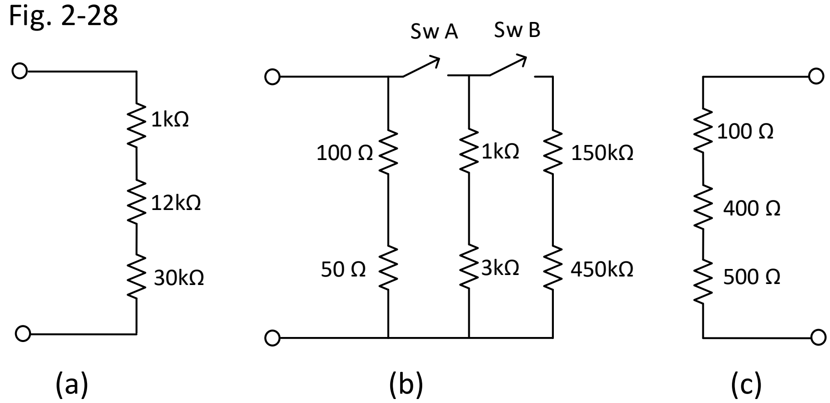
2.5 For Fig. 2-27a-d find the unknown voltages and currents:

Fig. 2-27



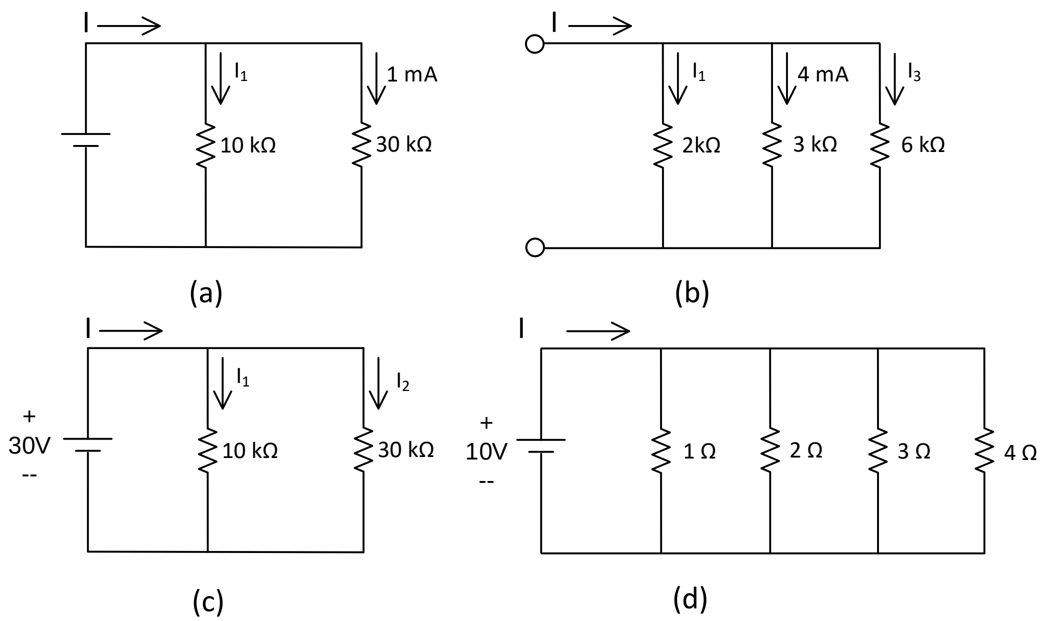
2.6 For Fig. 2-28a-c, find R_{eq} at the terminals:

Fig. 2-28



2.7 For Fig. 2-29a-d find all unknown currents (I):

Fig. 2-29



2.8 For Fig. 2-30 a-d, find all equivalent resistances (Req):

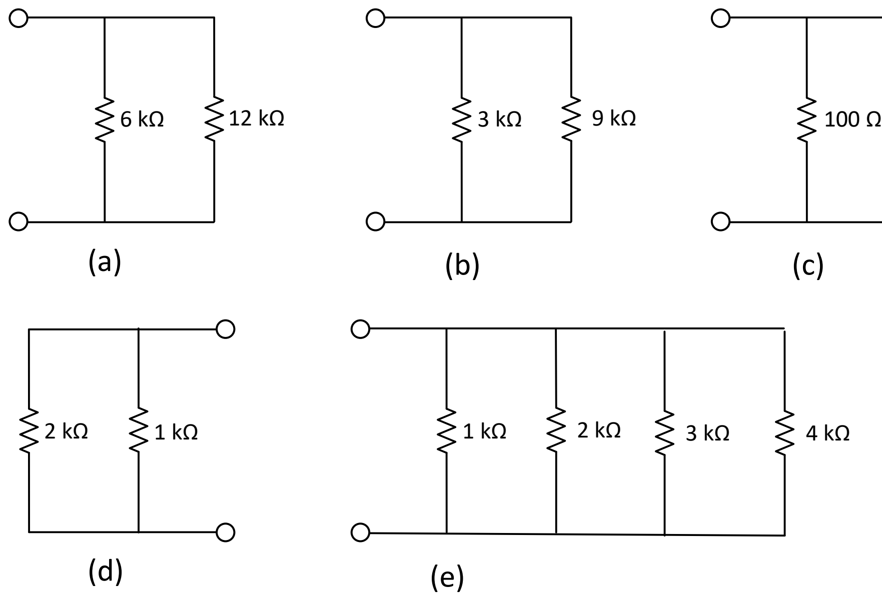
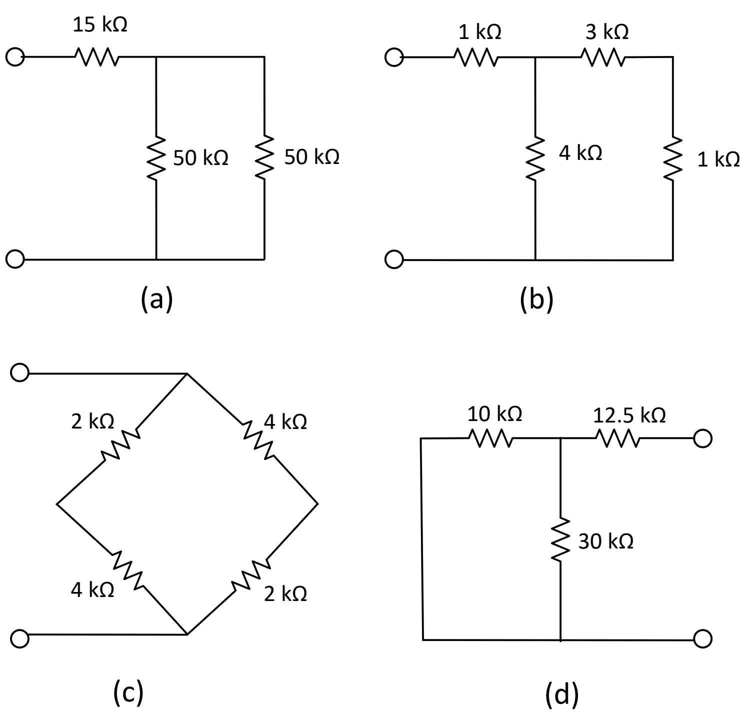


Fig. 2-30

2.9 For Fig. 2-31 a-d, find all equivalent resistances (Req):

Fig. 2-31



2.10 Attach a 10 V load to the terminals and find the voltage and current across each resistor in the above circuits of Fig. 2-31.