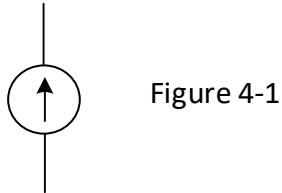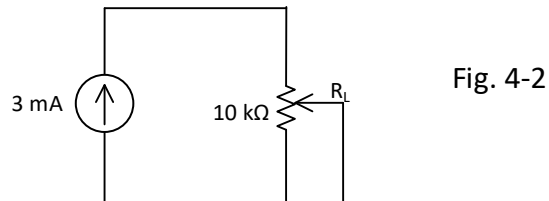4   Current and Power

The Ideal Current Source

An ideal current source is shown in Fig. 4-1 below:

Figure 4-1

It implies a current at a specified value that is unaffected by any value of resistance. The following Fig. 4-2 gives a 3 mA current source into a variable resistor that may vary from 0 to 10 kΩ.

Fig. 4-2

The current source is a guaranteed 3 mA no matter what the value of R_L. The wiper could be at the top and R_L = 0 or at the bottom and R_L = 10 kΩ. The current source doesn't change based on the load. This is referred to as an ideal current source. While most current sources are only ideal on paper, a current source may approach an ideal source depending on the circuit.

With the wiper at the top:

$$V = R_L I = 0 \times 0.003 = 0$$

With the wiper at the middle:

$$V = R_L I = 5{,}000 \times 0.003 = 15.0 \text{ V}$$

With the wiper at the bottom:

$$V = R_L I = 10{,}000 \times 0.003 = 30.0 \text{ V}$$

The V found is the voltage across the resistor which is also the voltage across the current supply. Therefore, the voltage must change across the current supply to almost any voltage to guarantee the current's value. Many times, the current source can only be built using electronic components such as a transistor.

Current sources, like voltage sources, can be either a dependent source or an independent source. To be dependent, the value of the current source may depend on a referenced current or voltage from another circuit. An example will show how a dependent source works.

The following circuit shows how to build a current source from a voltage supply. The current can only change a small amount from 100 µA regardless of the position of the wiper. All the way up on the wiper gives R_L = 0 and current I = 100 µA. With the wiper all the way down and R_L = 100 kΩ, I = 99 µA.

The calculation of I with the wiper at the top of the resistor gives:

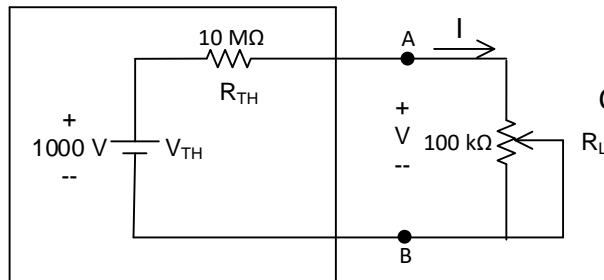$$I = \frac{V_{TH}}{R_{TH}} = \frac{1000}{10(10^6)} = 100 \ \mu A$$

Fig. 4-3
Approximate
Current Source

Load voltage is: V = R_LI = 0 x 100 µA = 0 V

The calculation of I with the wiper at the bottom of the resistor gives:

$$I = \frac{V_{TH}}{R_{TH} + R_L} = \frac{1000}{10.1(10^6)} = 99 \ \mu A$$

Load voltage is:

V = R_LI = 100(10³) x 99(10⁶) = 9.9V

For this circuit, the load voltage moves from 0 to 9.9 V while load current moves a very small amount from 100 µA to 99 µA. The circuit gives a good example of a circuit in which current changes very little over a range of variable resistance. We accomplish this relative constant current by placing a very large resistance at R_{TH} and a relative small resistance at R_L. This guarantees the current will not change much over a variable range of R_L.

The next two examples identify current sources that are independent and dependent. Fig. 4-4a has an independent current source that will not vary. Fig. 4-4b has a dependent current source that varies with the value of I in the left portion of the circuit. For Fig. 4-4a, the current source is constant at 2 mA. The value of current in the left part of the circuit has no effect on the 2 mA. In Fig. 4-4b, the current in the right portion of the circuit is 100 x I in the left portion of the circuit. The only question really to ask is what kind of circuit would give this result and the answer is a transistor. This circuit is a model of a bipolar junction transistor. We will not further discuss this circuit in this text but leave it for the study of bjt's.

The values of the voltage across R_L can be found for these two circuits. Those solutions are below:

Figure 4-4 Dependent and Independent Current Sources

(a)

(b)
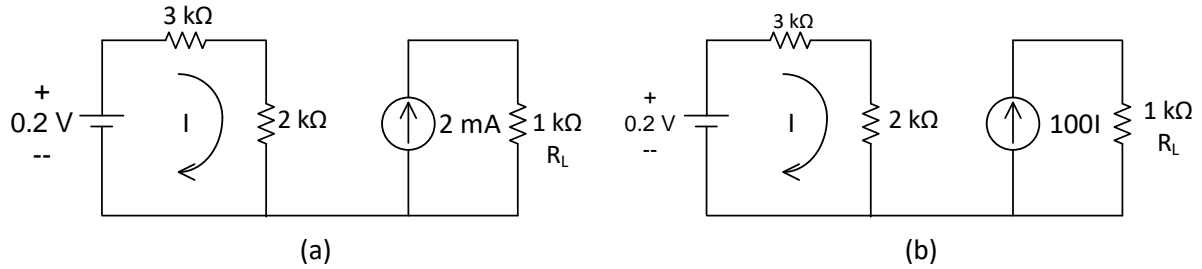
For Fig. 4-4a, the 2 mA of the circuit at the right produces the following voltage across R_L:

$$V_L = R_L I = 1000 \times 0.002 = 2 \text{ V}$$

For Fig. 4-4b, first find the current I from the left loop:

$$I = \frac{V}{R} = \frac{0.2}{5000} = 0.04 \text{ mA} = 40 \text{ μA}$$

Then, find the current in the right circuit:

$$100I = 100 \times 0.04 \text{ mA} = 4 \text{ mA}$$

Finally, find the voltage across R_L:

$$V_L = R_L \times 100I = 1000 \times 0.004 = 4 \text{ V}$$

How to Treat Current Sources in Thevenin, Norton and Superposition Problems

To find R_{TH}, we short all voltage supplies. We now need to discuss how we treat current supplies in this process. We reduce the current supply to 0 by opening the connection.
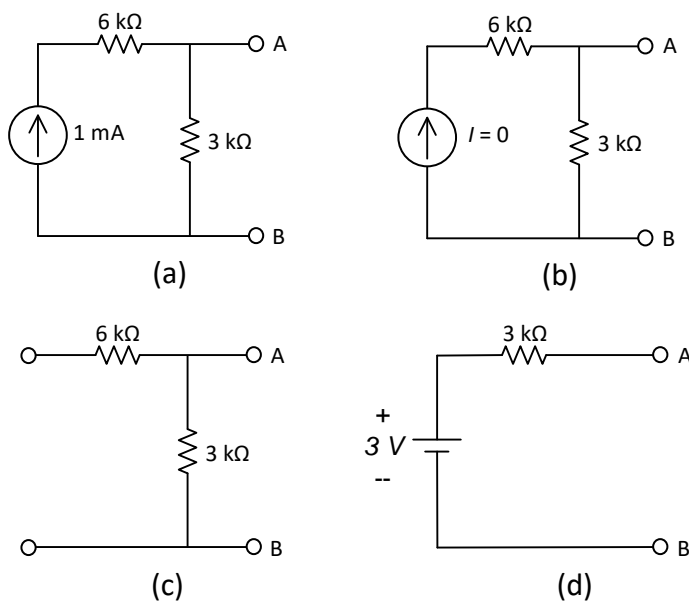

(a)

(b)

Figure 4-5

(c)

(d)

3

In Fig. 4-5 above, the Thevenin circuit is found by first finding the voltage from A to B. We observe that the current can only flow through the 3 kΩ resistor between A and B giving the voltage for V_{TH}.

$$V_{TH} = 3000 \times 0.001 = 3 \text{ V}$$

To find R_{TH}, walk the circuit starting at A and ending at B. Notice that in Fig. 4-5c, the current source is open not allowing the 6 kΩ resistor to be considered. Only the 3 kΩ resistance is used. The reconstructed Thevenin circuit is shown in Fig. 4-5d.

A second circuit with current supply and attached R_L yields similar results:
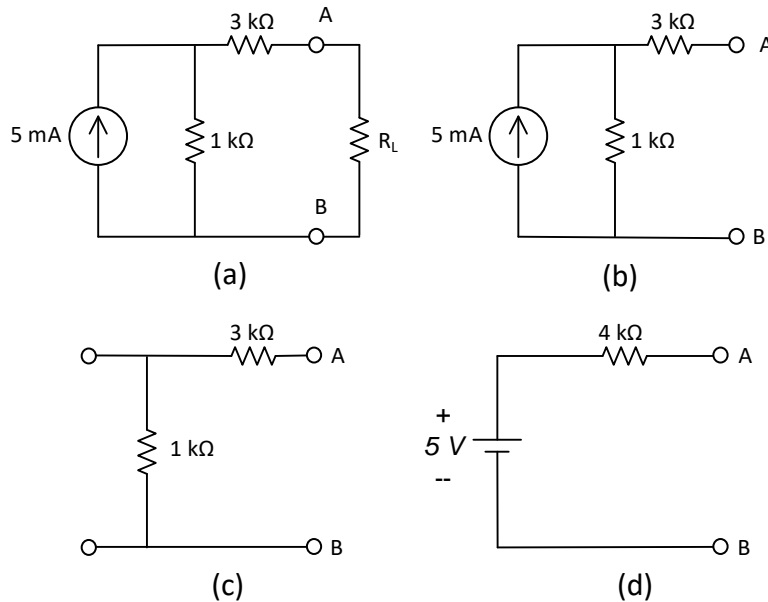

Figure 4-6

In Fig. 4-6 above, the Thevenin circuit is found by first finding the voltage from A to B. We observe that the current can only flow through the 1 kΩ resistor between A and B giving the voltage for V_{TH}.

$$V_{TH} = 1000 \times 0.005 = 5 \text{ V}$$

To find R_{TH}, walk the circuit starting at A and ending at B. Notice that in Fig. 4-6c, the current source is open and the 3 kΩ and 1 kΩ resistance are used in series. The reconstructed Thevenin circuit is shown in Fig. 4-6d.

The following example uses superposition to find the thevenin circuit. The first circuit is shown in Fig. 4-7b. Use voltage divider 3 kΩ to find V_{AB} (V_{TH}) for the voltage circuit.

$$V_1 = \frac{4000}{8000} 10 = 5 \text{ V}$$

For the current source, the 2 mA splits evenly through the two 4 kΩ resistors leaving 1 mA through 4 kΩ or

$$V_2 = 4000 \times 0.001 = 4 \text{ V}$$

To find V_{TH}, find the sum of V1 and V2:

$$V_{TH} = V_1 + V_2 = 5 + 4 = 9 \text{ V}$$

For R_{TH}, use Fig. 4-7d to get:

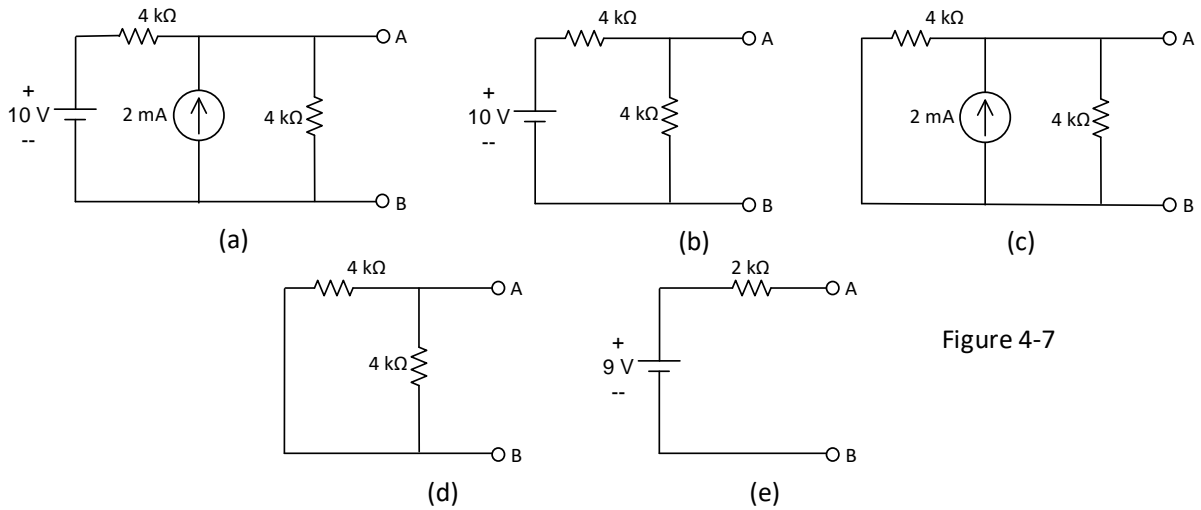$$R_{TH} = 4 \text{ k}\Omega \parallel 4 \text{ k}\Omega = 2 \text{ k}\Omega$$

Figure 4-7

Norton

Norton's Theorem is the current equivalent of Thevenin's Theorem. It is pictured in Fig. 4-8 below. With it we can find voltage and current across a load equal to the Thevenin Circuits of last chapter.
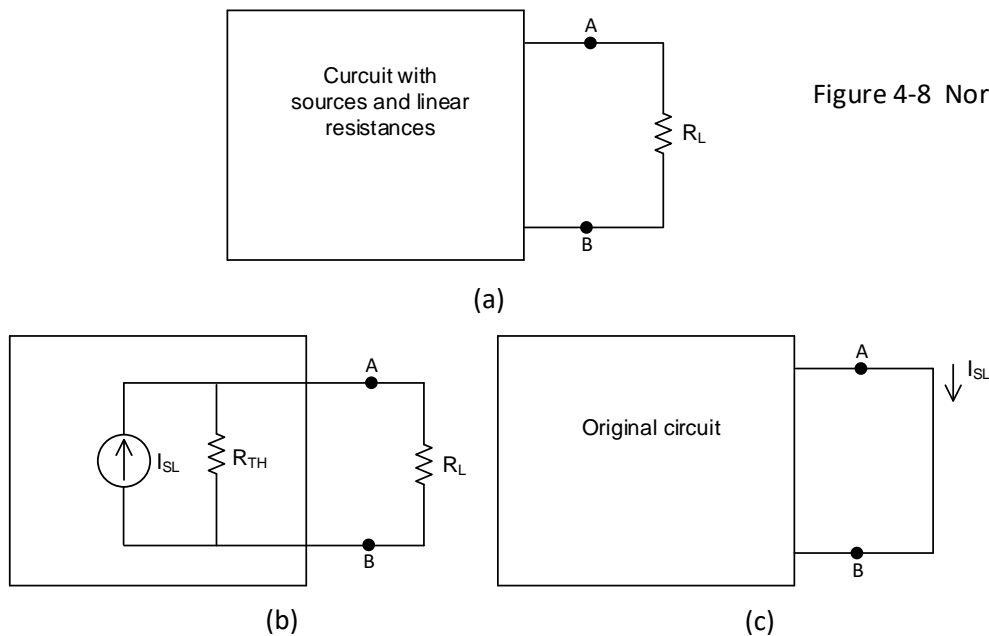

(a)

Figure 4-8 Norton's Theorem

(b)

(c)

Norton's Circuit moves the R_{TH} parallel to the current source and substitutes the voltage source V_{TH} with a current source, I_{SL}. The value of the current source is:

$$I_{SL} = \frac{V_{TH}}{R_{TH}}$$

In the example below, we find the Thevenin equivalent circuit and then find the Norton Equivalent circuit from which we can find the voltage and current across the A-B terminals similar to the Thevenin Circuits of last chapter.

Figure 4-9   Applying Norton's Theorem

(a) (b) (c) (d) (e)
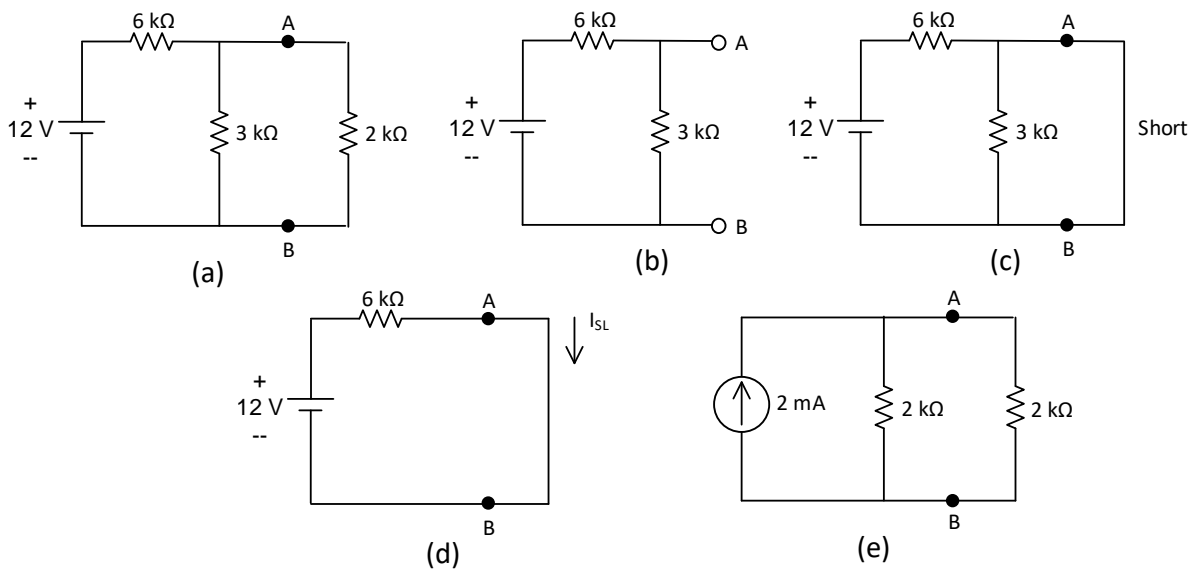
In Fig. 4-9a, we determine the A-B terminals and remove the 2-kΩ resistor right of A-B. Next, use the rules of Thevenin to find V_{TH} and R_{TH}. R_{TH} is 2 kΩ. V_{TH} is 4 V.

To find I_{SL}, we have two methods. One is the equation $V_{TH} = I_{SL} \cdot R_{TH}$. The other is to short the A-B terminals and determine I_{SL} by observation or the equation below:

$$I_{SL} = \frac{V}{R} = \frac{12}{6000} = 2 \text{ mA}$$

Fig. 4-9e shows the Norton Equivalent Circuit reattached ready to find the current through the 2-kΩ resistor.

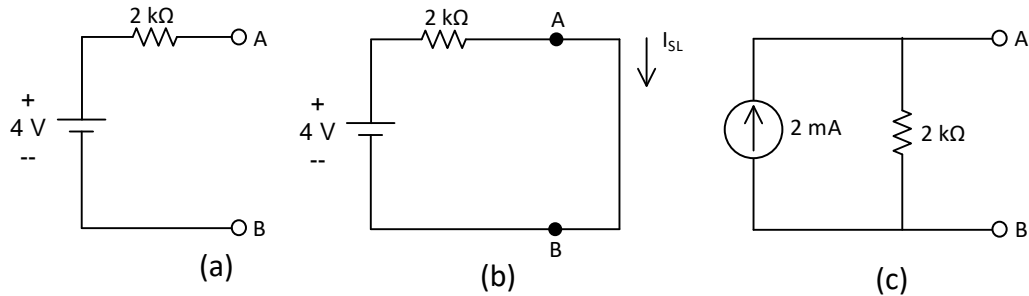The next Fig. 4-10 shows the same problem but worked first by finding V_{TH} and then I_{SL}.

$$V_{TH} = \frac{R_2}{R}V = \frac{3000}{6000}12 = 4 \text{ V}$$

and

$$R_{TH} = 6000 \parallel 3000 = 2 \text{ kΩ}$$

The following Fig. 4-10 with the results:

Figure 4-10 Converting Thevenin to Norton

(a) (b) (c)

Now apply Norton's theorem to Fig. 4-10a. With a short across the load terminals, Fig. 4-10b gives

$$I_{SL} = \frac{4}{2000} = 2 \text{ mA}$$

We can see from the above that Norton is to complement Thevenin. Since we usually have voltage, Thevenin is the preferred method. Sometimes, current is the lead and Norton may be used instead. Both work!

We now find a circuit that is a Norton circuit, Convert the Norton circuit to a Thevenin and find that either will produce the same result.

$$V_{TH} = 5000 \text{ x } 0.003 = 15 \text{ V}$$

Fig. 4-11a introduces a Norton circuit. Fig. 4-11b shows the Thevenin equivalent. Fig. 4-11c,d show the same circuits with a 5 kΩ resistor added at the A-B terminals. The current through the attached 5 kΩ is the same for both circuits. These circuits are the same! I = 1.5 mA in both and V_{AB} is 7.5 in both!
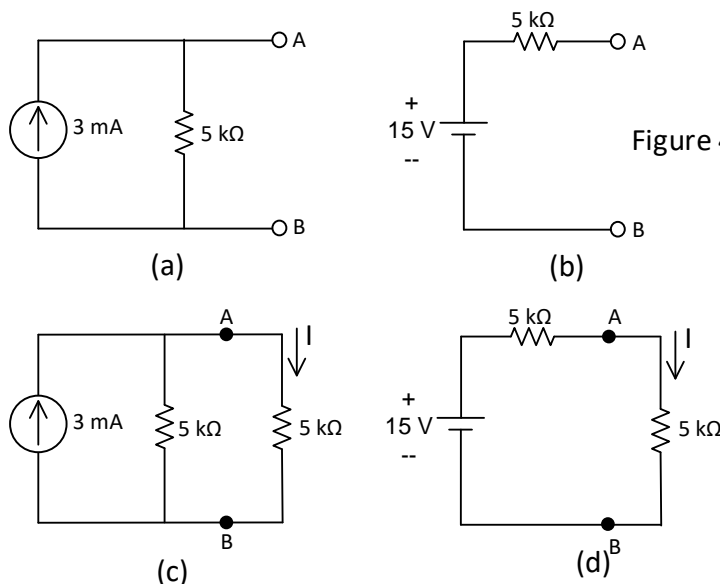

(a) (b)

Figure 4-11 Applying Thevenin's theorem

(c) (d)

7

Electrical Power

Power can be defined as work divided by time. The formula for Power is:

$$P = \frac{W}{t}$$

With   P = power
        W = work
        t   = time

In electrical circuits, work is defined in joules and time in seconds. If 5 J of energy are released in 2 s, then power is:

$$P = \frac{5\ J}{2\ s}$$

Power can also be related to voltage and current. In terms of V and I, power is given as:

$$P = V \cdot I$$

when   P = power
        V = voltage
        I = current

For V = 10 V and I = 5 A , power equals:

$$P = 10 \times 5 = 50\ W$$

Units of power are watts (W). Alternate methods of expressing the power across a resistance are:

$$P = V \cdot I = I^2 \cdot R = \frac{V^2}{R}$$

Matched-Load Power

We have used Thevenin circuits in a number of examples and now we ask how to get the most power from one. The answer is very useful in that we always want to find how to get the best output from a device whether electrical or otherwise. How do we maximize the thing! To do this, we use the matched-load power theorem which, believe it or not, says to match R_L with R_{TH} and we get the most bang for the buck. Figure 4-12a shows a black box driving an adjustable load resistance.

The circuit of Fig. 4-12a shows an unknown circuit with unknown V_{TH} and R_{TH}. First, find V_{TH} and R_{TH} and set $R_L = R_{TH}$ and the maximum power is realized from this circuit across the output terminals. It is given that we can't change the black box, but wish to realize the most power output from it. If we were able to change the parameters in the box, perhaps there would be more power that could be output. We do not consider this, only the parameter that R_{TH} and V_{TH} are fixed. The max value of power is realized when R_L is set to R_{TH} as seen in Fig. 4-12b.

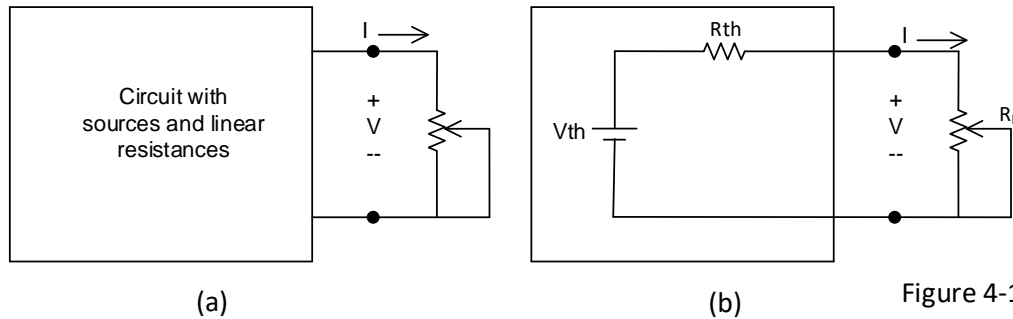(a)   (b)   Figure 4-12   Matched Load

The theorem will not be proven but rather shown by example. We start with the example of Fig. 4-13.

In the example of Fig. 4-13, if we match R_L to 4 kΩ, we get the maximum power. The power can be calculated by finding V_L which is 6 V. For this value of V, we can calculate I and finally P:
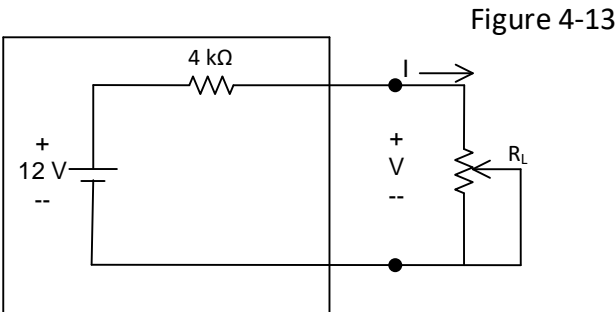
Figure 4-13

Load current:

$$I = \frac{12}{4000 + 4000} = 1.5 \text{ mA}$$

Max Power out (Matched-load power) =

$$P = V{\cdot}I = 6(0.0015) = 0.009 \text{ W} = 9 \text{ mW}$$

Now change R_L either a little up or down from 4 kΩ and observe the new value of I and P. It is a little less than 9 mW which we now are more likely to believe to be the maximum power out from this circuit.

Again, we are only looking at circuits with a fixed value of V_{TH} and R_{TH}. Fig. 4-14a shows a circuit that modifies R_{TH} and not R_L. This is not going to give maximum power out at R_L. The same can be said for Fig. 4-14b. This circuit does not fit the matched-load power output theorem either.
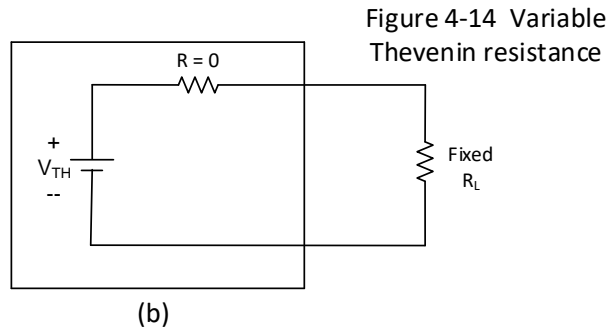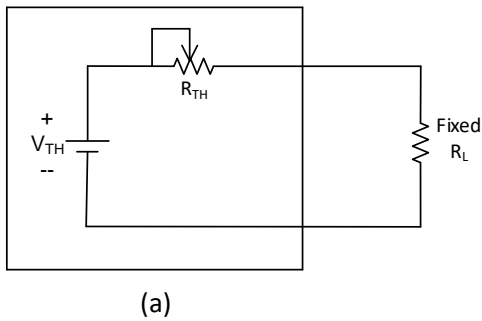
Figure 4-14 Variable
Thevenin resistance

Applications of Matched-Load Power Theorem

It is more important to match the output resistance to R_{TH} when the signal is weak as in a TV antenna. This is a good example of the need to match the antenna resistance closely.

The quality of a TV picture depends on the power to the TV receiver. In Fig. 4-15b the receiver has a resistance of R_L; this is the load resistance connected to the antenna.

Time-varying signals also apply to the maximum load power theorem. At t = 0, V_{TH} equals 1 V. We find R_{TH} equals 50 Ω. Find the maximum load power at t = 0 in Fig. 4-15 below:

(a)

(b)
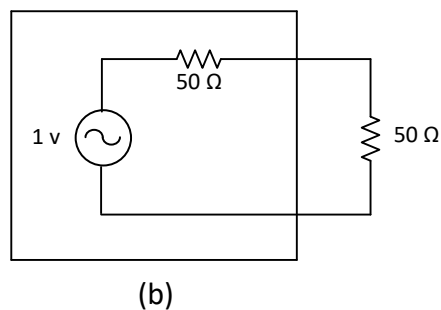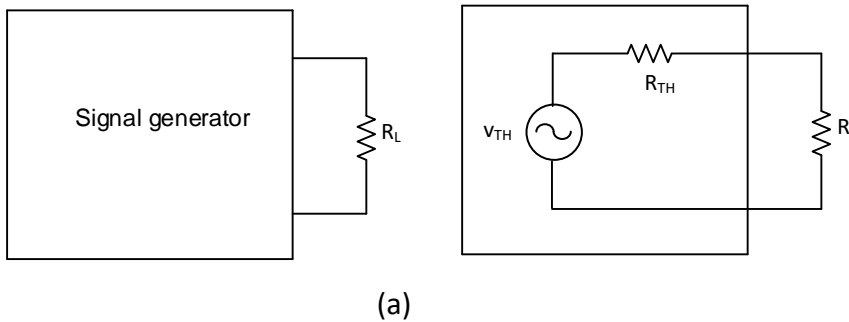
Figure 4-15 Thevenin circuit of signal generator

For max load power, set R_L = 50 Ω. For this resistance, V_L is:

$$V_L = \frac{V_{TH}}{2} = \frac{1}{2} = 0.5 \text{ V}$$

Load power equals:

$$P = \frac{V^2}{R_L} = \frac{0.5^2}{50} = 0.005 \text{ W} = 5 \text{ mW}$$

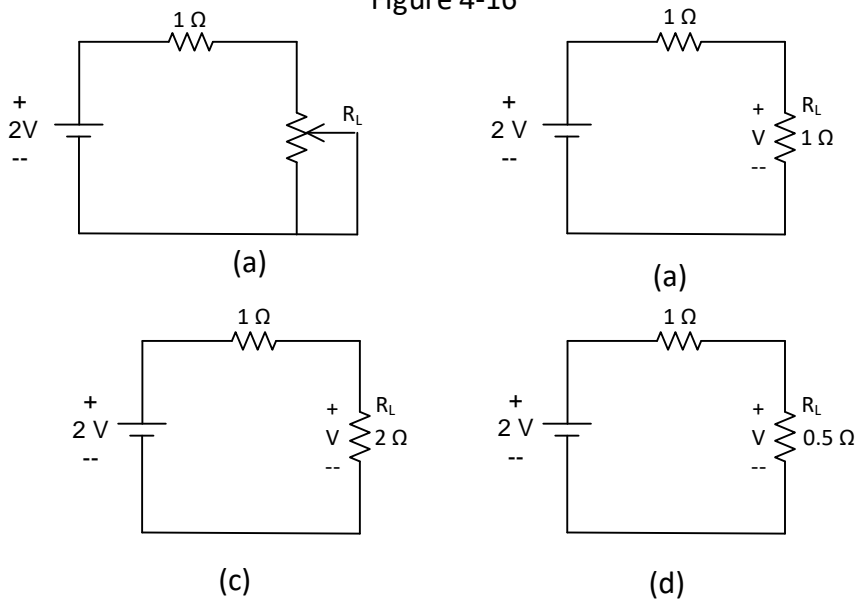The following example shows by example the maximum load-power theorem actually finds the max power out:

First, set $R_L = 1 \ \Omega$:

$$V_L = \frac{V_{TH}}{2} = \frac{2}{2} = 1 \text{ V}$$

Load current I =:

$$I_L = \frac{V_L}{R_L} = \frac{1}{1} = 1 \text{ A}$$

Figure 4-16

(a)

(a)

(c)

(d)

and:

P = V · I = 1 x 1 = 1 W

Moving the wiper to 2 Ω gives:

$$V_L = \frac{2}{3} 2 = 1.33 \text{ V}$$

Load current I =:

$$I_L = \frac{V_L}{R_L} = \frac{1.33}{2} = 0.667 \text{ A}$$

and:

P = V · I = 1.33 x 0.667 = 0.887 W

Moving the wiper to $R_L = 0.5\ \Omega$ gives:

$$V_L = \frac{0.5}{1.5}2 = 0.667\ \text{V}$$

Load current I_L =:

$$I_L = \frac{V_L}{R_L} = \frac{0.667}{0.5} = 1.33\ \text{A}$$

The load power is

$$P = V \cdot I = 0.667(1.33) = 0.887\ \text{W}$$

Each example moving away from 1 Ω gives a value of P less than 1 W, which we surmise is the maximum load power for the given circuit. Again we feel more comfortable that we have found the method for finding the maximum power output from a given Thevenin equivalent circuit.

Problems

4.1   Find the voltage of the 10-kΩ resistor of Fig. 4-17a.

(a)   (b)

Figure 4-17

(c)
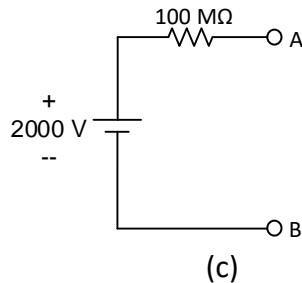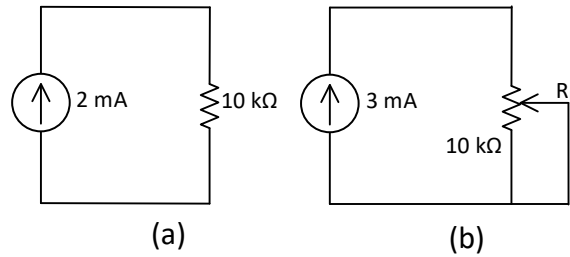
4.2   In Fig. 4-17b the resistor is a 10 kΩ pot. When the wiper is at the top, what is the voltage across R_L, when the wiper is at bottom? When the wiper is at the middle position?

4.3   If you were to have in your hand a number of resistors in the range of 10 Ω to 1 MΩ, what would be the max value across the A-B terminals of Fig. 4-17c if only one resistor was attached at a time? What would be the min value across the A-B terminals? Max current? Min current?

4.4   For Fig. 4-18a, find the Thevenin equivalent circuit.

4.5   Find the Thevenin equivalent circuit left of A-B in Fig. 4-18b.

4.6   Find the Thevenin equivalent circuit left of A-B in Fig. 4-18c.

4.7   Find the Thevenin equivalent circuit left of C-D in Fig. 4-18d. Then find the Thevenin equivalent circuit left of A-B in Fig. 4-18d.

4.8   Find the Thevenin equivalent circuit left of A-B in Fig. 4-18e when I_{CBO} = 1 μA.

4.9   Find the Norton equivalent circuit left of A-B in Fig. 4-19a.

4.10   Find the Norton equivalent circuit left of A-B in Fig. 4-19b.

4.11   Find the Norton equivalent circuit left of A-B in Fig. 4-19c. Then find the Thevenin equivalent circuit.

4.12   Find the Norton equivalent circuit left of A-B in Fig. 4-19d.

4.12   A boat battery dissipates 24,000 J in a 10 s time period. Find the kilowatts.

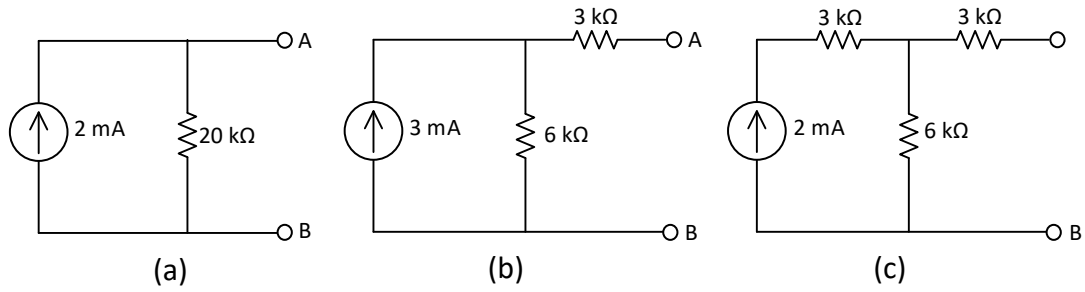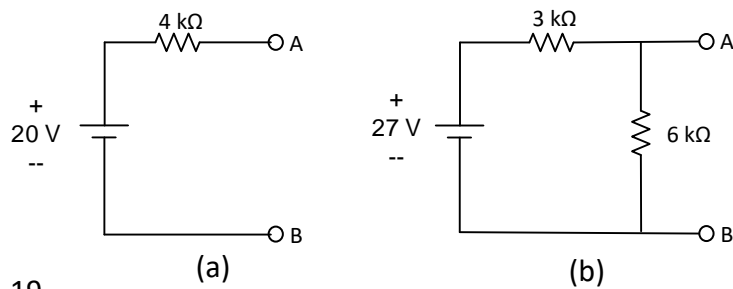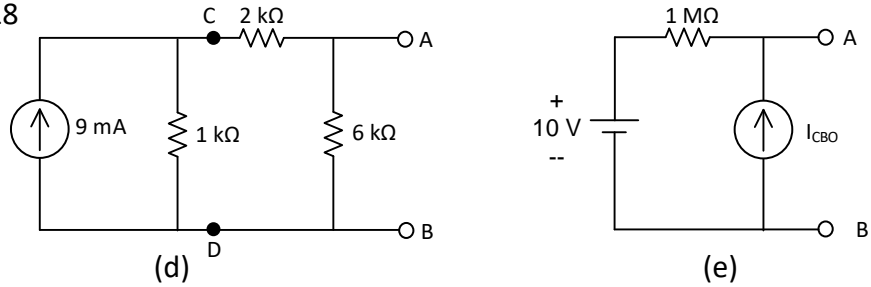4.13   A battery outputs 9 V. If the measured current equals 10 mA, what is the power dissipated?

(a)　　　　　(b)　　　　　(c)
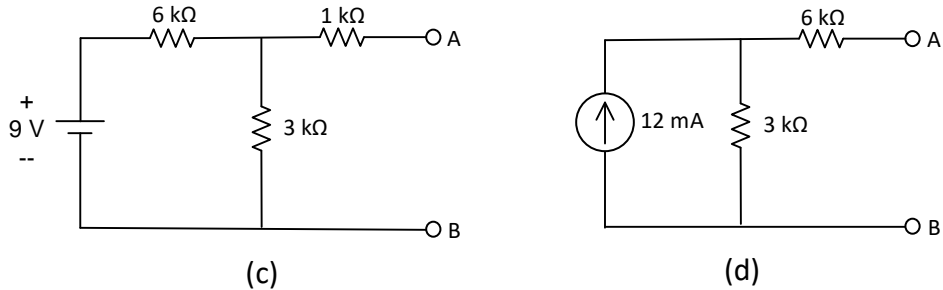
Figure 4-18
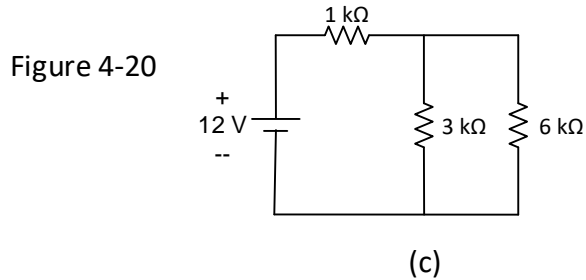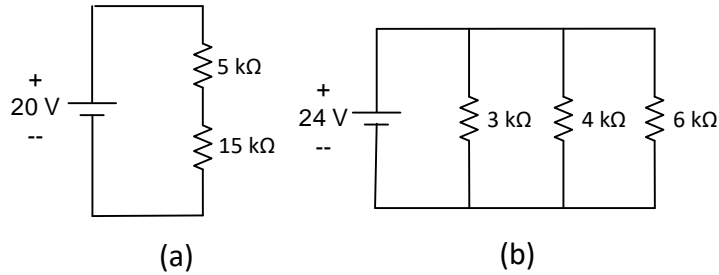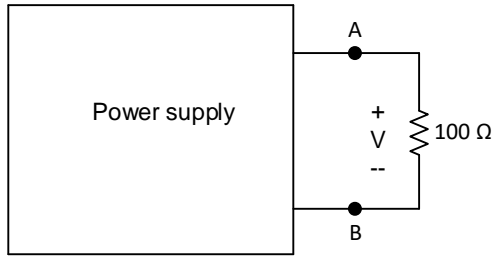
(d)　　　　　(e)

(a)　　　　　(b)

Figure 4-19

(c)　　　　　(d)

4.14 Find the power dissipated in the 5-kΩ resistor of Fig. 4-20a. In the 15-kΩ resistor. In the total R_L.

4.15 Find the power dissipated in each resistor in Fig. 4-20b. In the total R_L.

4.16 Find the power dissipated in each resistor in Fig. 4-20c. In the total R_L.
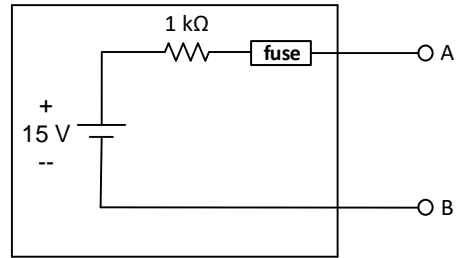
(a)   (b)

Figure 4-20

(c)

4.17   With the power supply of Fig. 4-21a, assume R_{TH} = 0. Find the power dissipated if the voltage is set to 10 V, 30 V.

4.18   Rate the fuse in Fig. 4-21b when the following occur:
   a.   There is a short between A and B
   b.   There is a 2- Ω resistor between A and B
   c.   There is a 75- Ω resistor between A and B

4.19   For Fig. 4-21c, find the max load power.

4.20   For an antenna V_{TH} = 100 µV and R_{TH} = 50 Ω. Find the max load power.

4.21   In Fig. 4-21d, adjust R_{TH} to get the max load power to the 10-kΩ resistor. Find the max load power at this value.

4.22   Next, adjust R_{TH} to 10-k Ω resistor in Fig. 4-21d. Find the max load power.

4.23   A black box has V_{TH} = 10 V and R_{TH} = 600 Ω. Find the max load power.

4.24   Fig. 4-21e gives the black box having V_{TH} = 10 V and R_{TH} = 50 Ω. Show through calculations at the following values of R_L that max power occurs at R_L = 50 Ω. Graph max load power across the range (except e).

   a. R_L = 0
   b. R_L = 25 Ω
   c. R_L = 50 Ω
   d. R_L = 100 Ω
   e. R_L = open circuit

Figure 4-21

(a)

(b)

(c)

(d)

(e)