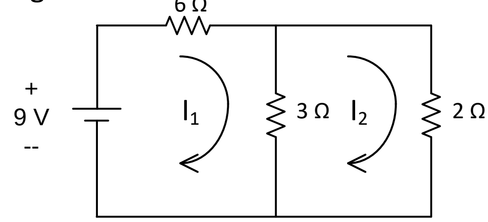
5 Mesh or Loop and Node Equations

Writing mesh or loop equations

To solve the Fig. 5-1 question of finding all unknown currents, we draw loop currents as found in the figure. I_1 is found in the first loop and I_2 in the second. KVL states that the sum of voltages around a closed loop adds to zero and we use this to write the equations for these two loops.

If we approach a voltage supply from the negative side, we write the minus sign. If we approach from the plus, we write the plus sign. If we write a voltage across a resistor, it is with a plus sign. If we write a voltage across a resistor with both currents flowing through it, we write the current of the loop we are in with a plus and the other current with a minus. These rules will become apparent as we use them in the various examples below:

Figure 5-1



First, we write the loop equation for the I_1 loop. We approach the 9V supply from the negative side so we write -9V to start the equation. We then approach the 6-Ω resistor and notice that only I_1 flows through it so we write $6 \cdot I_1$. Next we approach the 3-Ω I_1resistor and notice that both I_1 and I_2 flow through it. In the I_1 loop, we write $3 \cdot (I_1 - I_2)$.

We can now write the entire KVL equation for the left or I_1 loop:

$$6 \cdot I_1 + 3 \cdot (I_1 - I_2) - 9 = 0$$

Walking around the I_2 loop gives:

$$2 \cdot I_2 + 3 \cdot (I_2 - I_1) = 0$$

We rearrange the loop equations to the following in order to solve these equations. This arrangement is referred to as standard form:

$$9 \cdot I_1 -- 3 \cdot I_2 = 9$$
$$-3 \cdot I_1 + 5 \cdot I_2 = 0$$

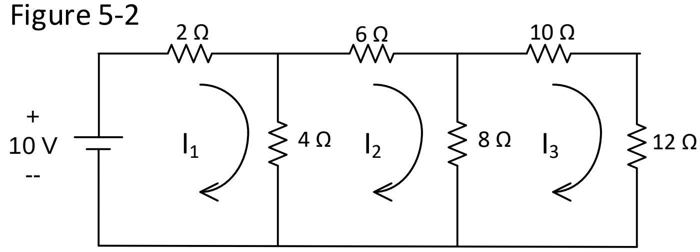
Finding the values of I_1 and I_2 now involves linear algebra. Your instructor may or may not decide to get involved with this. However, the values of I_1 and I_2 are:

$$I_1 = 1.25 \text{ A}$$
$$I_2 = 0.75 \text{ A}$$

We will discuss a number of examples in the following but the rules we laid out above apply to all.

Write loop or mesh equations for Fig. 5-2:

Figure 5-2



The I_1 loop:

$$2 \cdot I_1 + 4 \cdot (I_1 - I_2) - 10 = 0$$

or

$$6 \cdot I_1 - 4 \cdot I_2 = 10$$

The I_2 loop:

$$6 \cdot I_2 + 8 \cdot (I_2 - I_3) + 4 \cdot (I_2 - I_1) = 0$$

or

$$- 4 \cdot I_1 + 18 \cdot I_2 -- 8 \cdot I_3 = 0$$

The I_3 loop:

$$10 \cdot I_3 + 12 \cdot I_3 + 8 \cdot (I_3 - I_2) = 0$$

or

$$- 8 \cdot I_2 + 30 \cdot I_3 = 0$$

Written in standard form, we have:

$$\begin{aligned} 6 \cdot I_1 - 4 \cdot I_2 \quad\quad\quad &= 10 \\ - 4 \cdot I_1 + 18 \cdot I_2 -- 8 \cdot I_3 &= 0 \\ - 8 \cdot I_2 + 30 \cdot I_3 &= 0 \end{aligned}$$

Next, write the loop equations for Fig. 5-3 in standard form:

In the first loop we have:

$$3 \cdot (I_1 - I_2) + 6 \cdot (I_1 - I_3) -- 9 = 0$$

or

$$9 \cdot I_1 + 3 \cdot I_2 -- 6 \cdot I_3 = 9$$

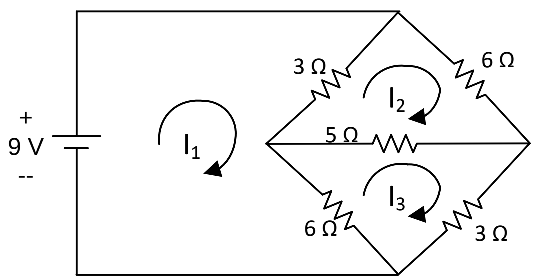


Figure 5-3

In the I_2 loop:

$$3 \cdot (I_2 - I_1) + 6 \cdot I_2 + 5 \cdot (I_2 - I_3) = 0$$

or

$$-3 \cdot I_1 + 14 \cdot I_2 - 5 \cdot I_3 = 0$$

In the I_3 loop:

$$3 \cdot I_3 + 6 \cdot (I_3 - I_1) + 5 \cdot (I_3 - I_2) = 0$$

or

$$-6 \cdot I_1 - 5 \cdot I_2 + 14 \cdot I_3 = 0$$

Standard form for these equations:

$$9 \cdot I_1 - 3 \cdot I_2 -- 6 \cdot I_3 = 9$$
$$-3 \cdot I_1 + 14 \cdot I_2 - 5 \cdot I_3 = 0$$
$$-6 \cdot I_1 - 5 \cdot I_2 + 14 \cdot I_3 = 0$$

Next, write the two equations in standard for Fig. 5-4.

In the I_1 loop:

$$6 \cdot I_1 + 3 \cdot (I_1 - I_2) + 6 - 9 = 0$$

or

$$9 \cdot I_1 - 3 \cdot I_2 = 3$$

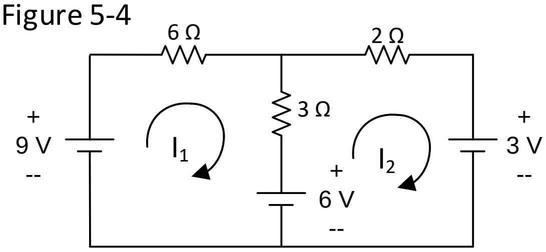
In the I_2 loop:

$$2 \cdot I_2 + 3 - 6 + 3 \cdot (I_2 - I_1) = 0$$

or

$$-3 \cdot I_1 + 5 \cdot I_2 = 3$$

Figure 5-4

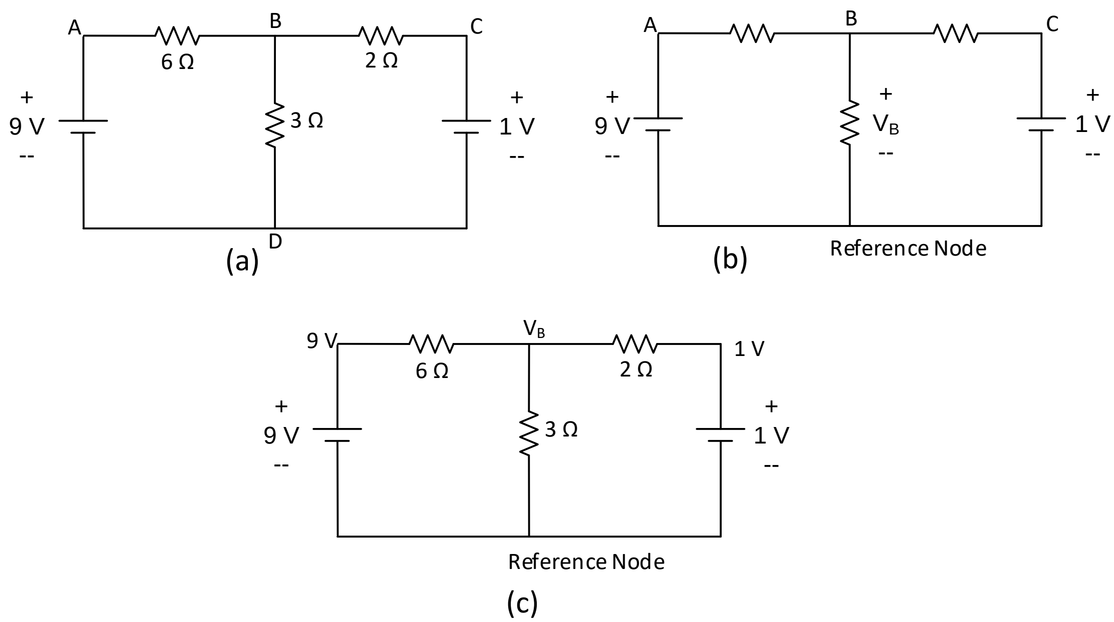


Node Equations

We used KVL and current to solve our loop or mesh equations. It is logical that we use KCL and voltage to solve our node equations. We picture each node as a point at which currents flow either into or away from (KCL). Next we find these nodes and write the current equations but not with I's. We write our currents with V/R. This method yields one less equation than the loop or mesh method usually and might seem simple. However, because we are always dealing with fractions, the method may seem more cumbersome. You decide.

In looking at Fig. 5-5a, we see the three nodes up top and one at the bottom. We ignore D because the D node equals 0 volts. However, we still have A, B, and C nodes. We notice that A and C have the same current flowing in as flowing out and we can ignore them. However, at B, we can write a KCL and get an unknown voltage V_B. If we find V_B, we can find the currents and voltages elsewhere easily.

Figure 5-5



Notice that in Fig. 5-6 we show the three currents:

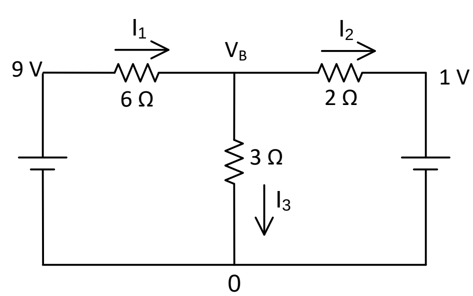


Figure 5-6

We write KCL at V_B in terms of current:

$$I_1 = I_2 + I_3$$

Now write the above KCL as:

$$\frac{9 - V_B}{6} = \frac{V_B - 1}{2} + \frac{V_B}{3}$$

The equation has V_B as its unknown. Solving for V_B gives:

$$V_B = 2 \text{ V}$$

With V_B known, we can find the currents. They are:

$$I_1 = \frac{9 - 2}{6} = 1.167 \text{ A}$$

$$I_2 = \frac{2 - 1}{2} = 0.5 \text{ A}$$

$$I_3 = \frac{2}{3} = 0.667 \text{ A}$$

We can observe in Fig. 5-7 that multiple resistors can be combined into one in the same branch:

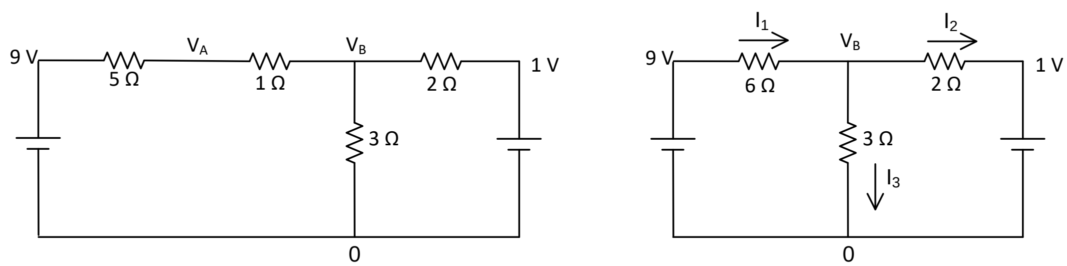


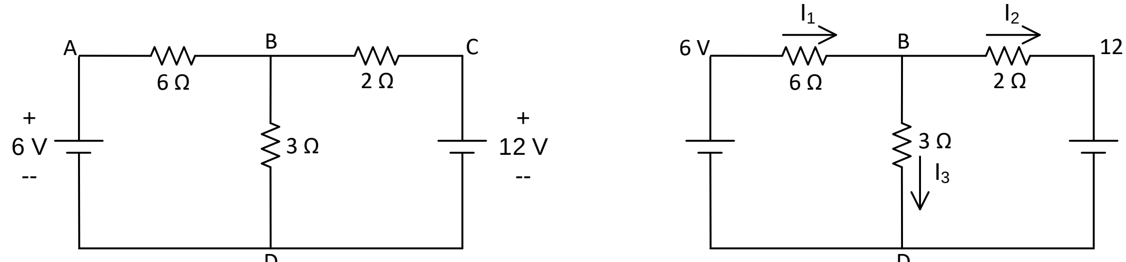
Figure 5-7

The following example again uses KCL to find current at the unknown node. In Fig. 5-8b:

$$I_1 = I_2 + I_3$$

or

$$\frac{6 - V_B}{6} = \frac{V_B - 12}{2} + \frac{V_B}{3}$$



(a) (b) Figure 5-8

Solving for V_B:

$$6 - V_B = 3 \cdot V_B - 36 + 2 \cdot V_B$$

and:

$$V_B = 7\ \text{V}$$

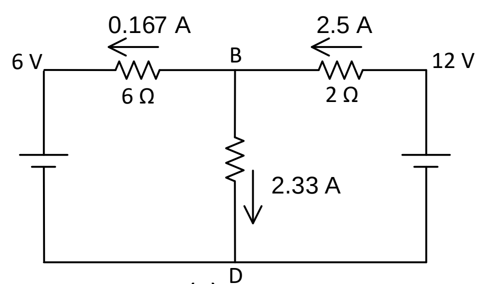
Solving for currents give:

$$I_1 = \frac{6-7}{6} = -0.167\ \text{A}$$

$$I_2 = \frac{7 = 12}{2} = -2.5\ \text{A}$$

$$I_3 = \frac{7}{3} = 2.33\ \text{A}$$

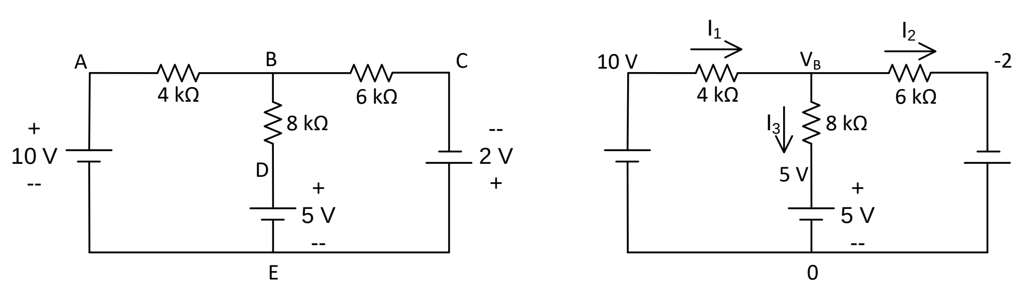
Actual current flow is shown in Fig. 5-8c:



(c)

Figure 5-8

Next, we add voltage supplies in all the loops with the following results:



(a) (b)

Figure 5-9

We write:

$$I_1 = I_2 + I_3$$

and

$$\frac{10 - V_B}{4000} = \frac{V_B + 2}{6000} + \frac{V_B - 5}{8000}$$

Now we look at a circuit with multiple unknown voltage nodes. We label the nodes as V_B and V_C:

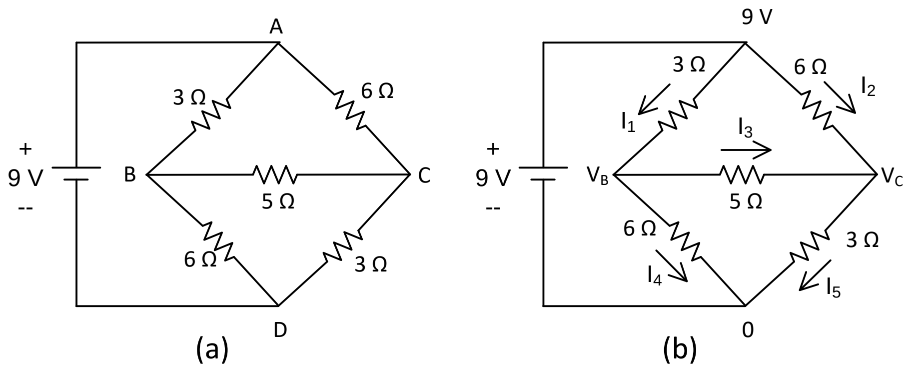


Figure 5-10

For currents at node B, we write:

$$I_1 = I_3 + I_4$$

or

$$\frac{9 - V_B}{3} = \frac{V_B - V_C}{5} + \frac{V_B}{6}$$

For currents at node C, we write:

$$I_2 + I_3 = I_5$$

or

$$\frac{9 - V_C}{3} + \frac{V_B - V_C}{5} = \frac{V_C}{3}$$

In standard form the unknown voltages are:

$$21 \cdot V_B - 6 \cdot V_C = 90$$
$$6 \cdot V_B - 21 \cdot V_C = -45$$

Problems

5-1. Write loop and node equations for each of the following: