6 Time-Varying Signals

In this chapter, we will begin the study of time-varying signals. These signals may be either current or voltage signals and may have a number of different shapes. First, we will discuss the basic idea of time-varying signals, learning to visualize it with a timing diagram. Then we will work a variety of problems using what we learned previously to calculate the time-varying signal's output from a circuit.

Time is measured in seconds. It is a fundamental quantity of nature which is defined as:

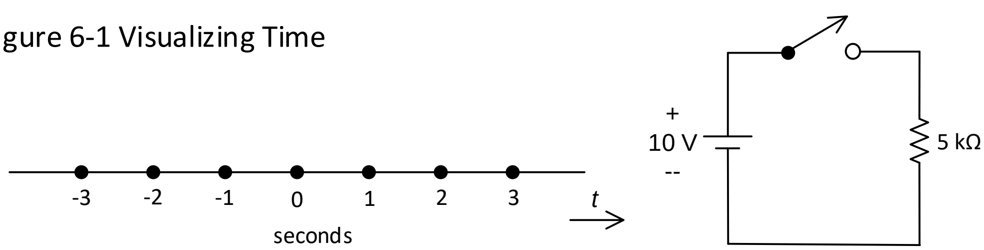
$$1\ second = \frac{Average\ solar\ day}{86{,}400}$$

We visualize time with a chart similar to Fig. 6-1a. The origin is defined as the point t = 0. Time increases to the right with earlier time referenced from the left.

We can choose where t = 0, our starting point. It is usually picked to coincide with an important part of the signal, where the signal starts to do something that we believe to be important or the starting point of a repeatable waveform. Likewise, we reference t = 1 as what happens 1 sec. after t = 0. Likewise, t = 2 references what happens 2 seconds later. . . .

When we pick a particular point in time, we freeze and take a picture at that instant. The problem at that point becomes just like a chapter 2-5 problem. Then we move on to another point and do the same. We move through the various points in the waveform doing the freeze until we have successfully built the output waveform required.

Figure 6-1 Visualizing Time



In the waveform of Fig. 6-2a, we see the following:

v = 0 V at	t = 0 s
v = 5 V at	t = 1 s
v = 10 Vat	t = 2 s
v = 15 Vat	t = 3 s

This waveform is a ramp waveform.

Figure 6-2b gives us the shape of the triangular waveform.

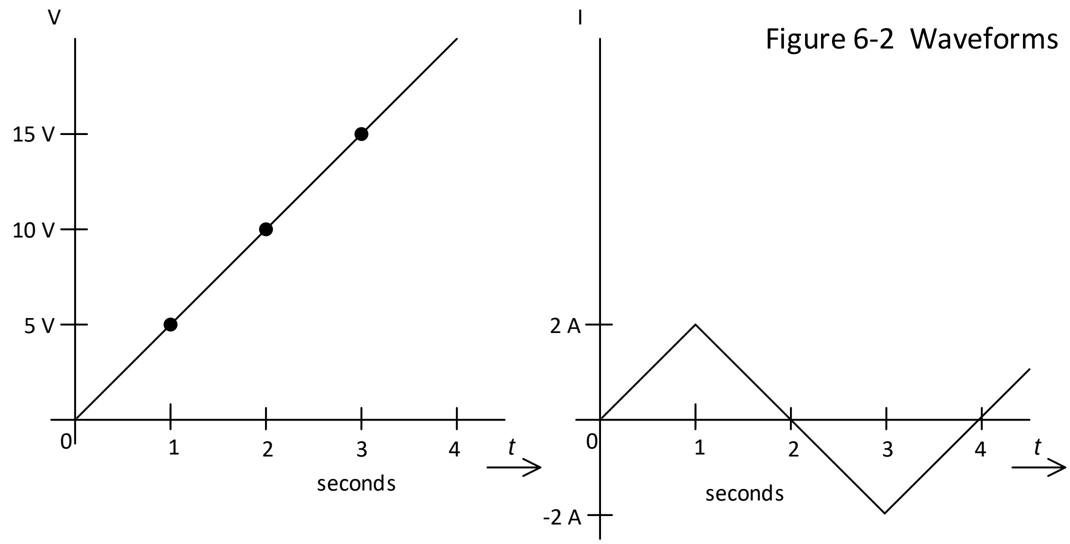


Figure 6-2 Waveforms

To build the waveform of Fig. 6-2a, we place points on the graph for each time interval. These points form a sequence that we can plot. The result is the plot of Fig. 6-2a.

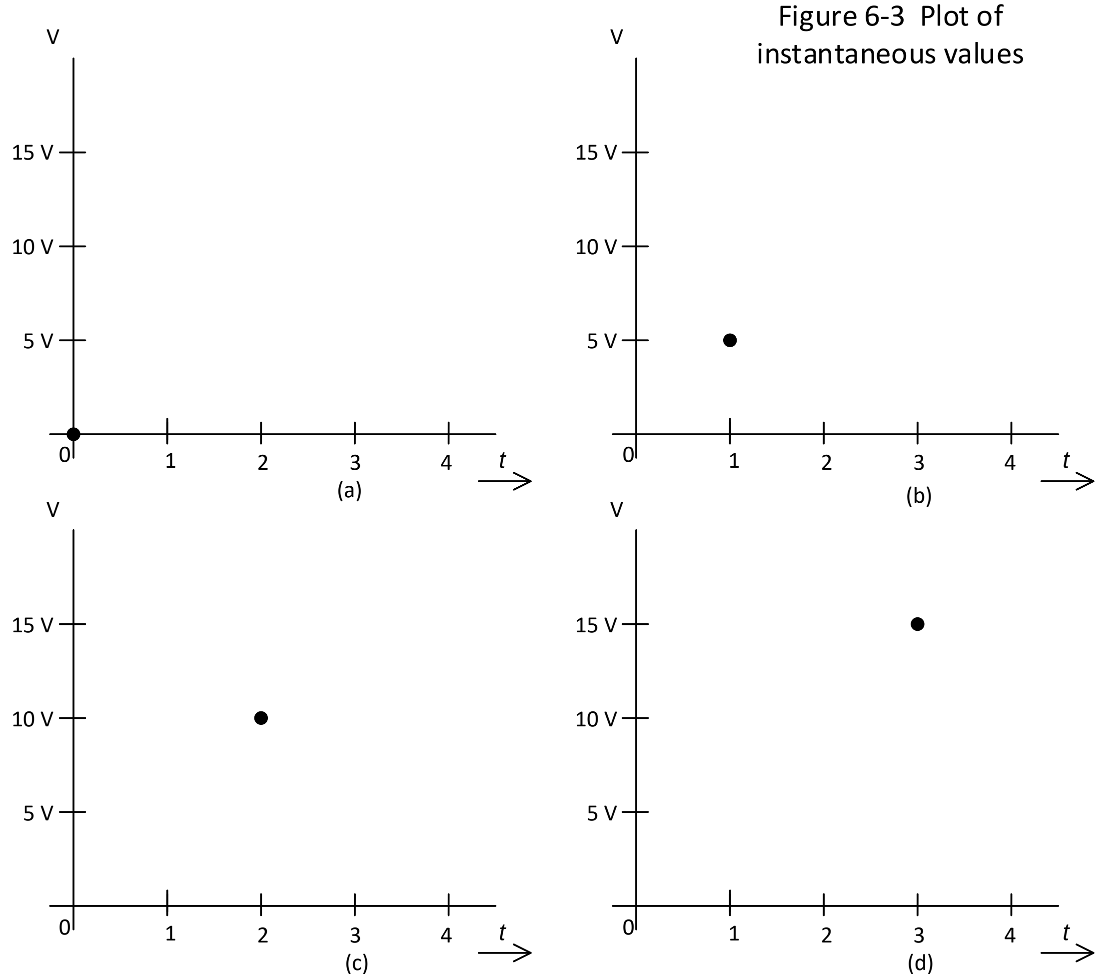
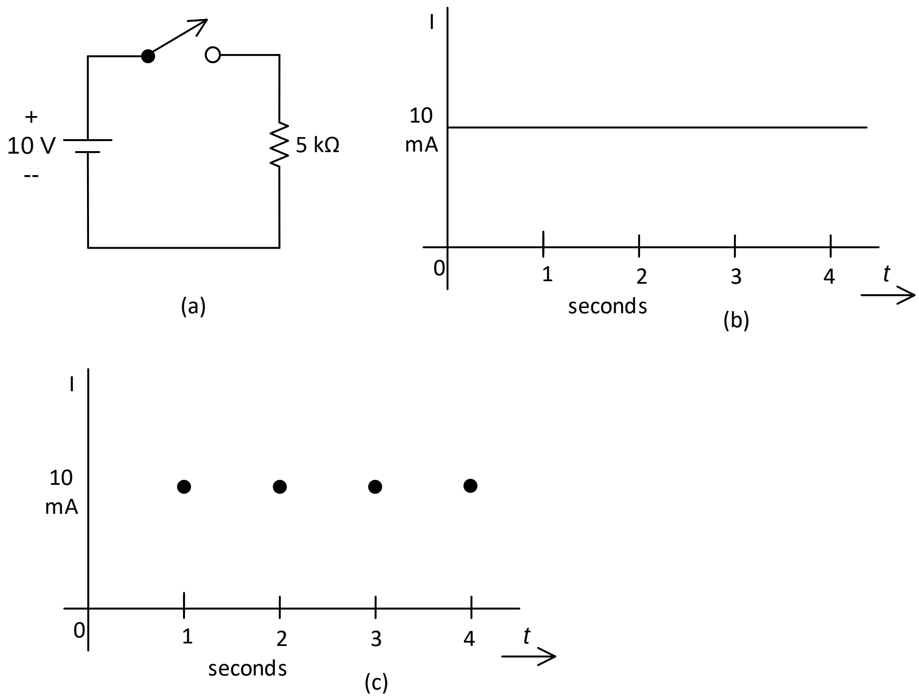


Figure 6-3 Plot of instantaneous values

In the following example of Fig. 6-4, we close the switch and plot the results. Before t = 0, the voltage across the 5 kΩ resistor is 0 V. After the switch closes, 10 V is found across the resistor. The current can be plotted in the timing diagram if Fig. 6-4b.

Figure 6-4 Current in Time Varying Circuit



The oscilloscope shows a time-varying waveform on the screen of the oscilloscope. This signal is a voltage waveform that displays a time-varying signal. It is very useful in showing time-varying waveforms and responses to these waveforms.

In Fig. 6-5a, we see a figure with three switches labelled A, B, and C. Switch A closes at t = 0 sec., B at t = 2 sec. and C at t = 4 sec. We are to sketch the current waveform i.

At time before t = 0, no switches are closed and current *i* = 0. Then, switch A closes and:

$$i = \frac{9}{6000} = 1.5 \text{ mA}$$

At t = 2, B closes and the parallel connection of the two 6-kΩ resistances form to an R_{EQ} of 3 kΩ and:

$$i = \frac{9}{3000} = 3 \text{ mA}$$

At t = 4, switch C closes and the circuit of Fig. 6-5d is seen. R_{EQ} = 2 kΩ, and:

$$i = \frac{9}{2000} = 4.5 \text{ mA}$$

The resultant step waveform is shown in Fig. 6-5e.

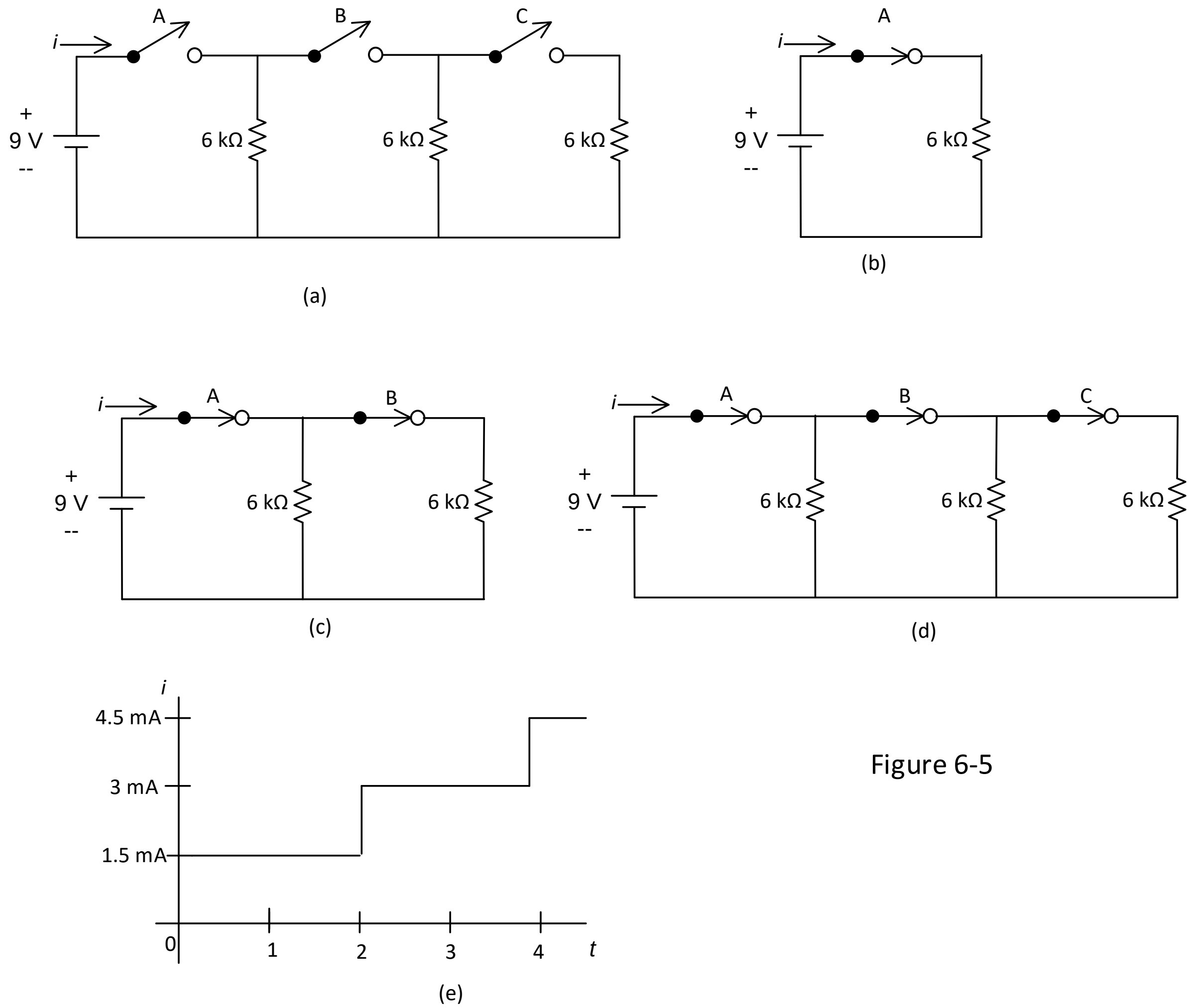


Figure 6-5

We can see for time-varying circuits that all previous theorems apply, just at an instant in time.

We again show the calculations for current at various times in the graph from Fig. 6-6. We apply the same equations again and again with the result of a number of points for current at the various times. We show the time-varying source differently than the previous battery source and define the source with a graph. We then calculate the output current at various times and plot on a graph similar to the original voltage graph. This graph of current becomes our resultant graph of current through the 10-kΩ resistor.

At t = 0, v = 0 and

$$i = \frac{v}{R} = \frac{0}{10{,}000} = 0$$

At t = 1, v = 5 and

$$i = \frac{v}{R} = \frac{5}{10{,}000} = 0.5\ mA$$

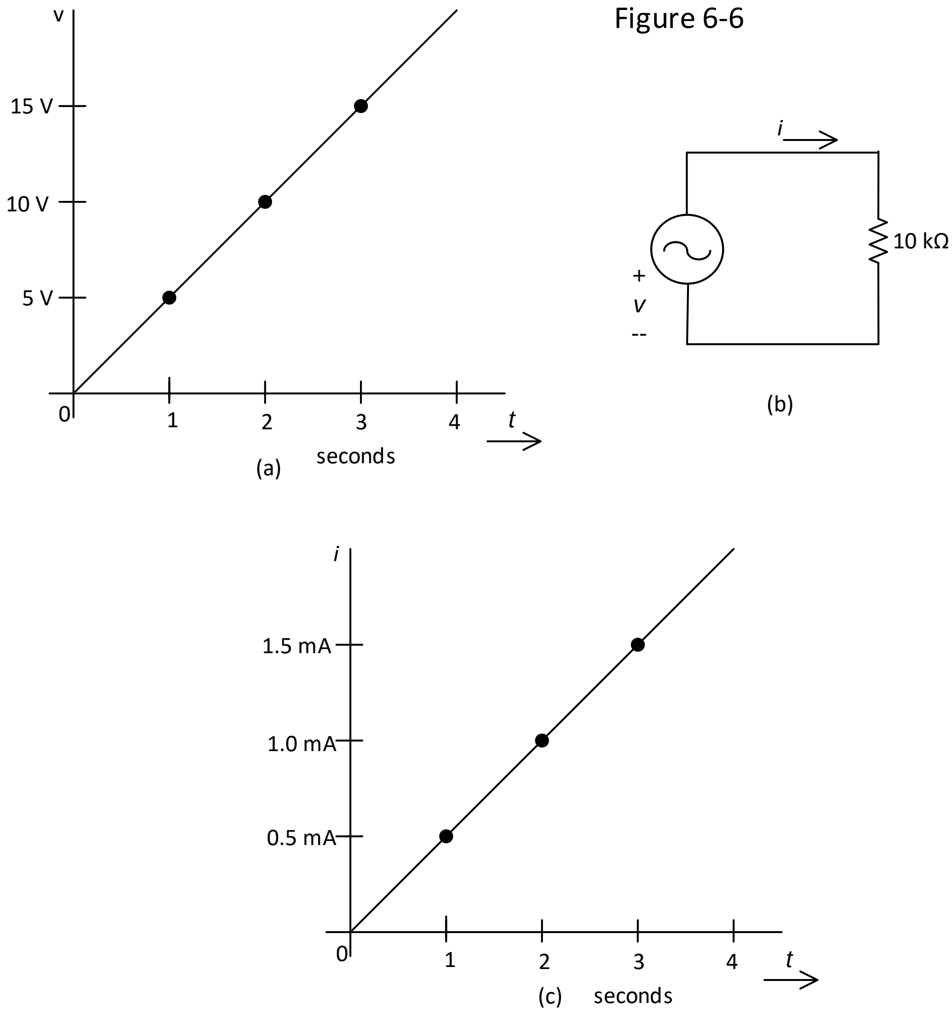
At t = 2, v = 10 and

$$i = \frac{v}{R} = \frac{10}{10{,}000} = 1\ mA$$

At t = 3, v = 15 and

$$i = \frac{v}{R} = \frac{15}{10{,}000} = 1.5\ mA$$

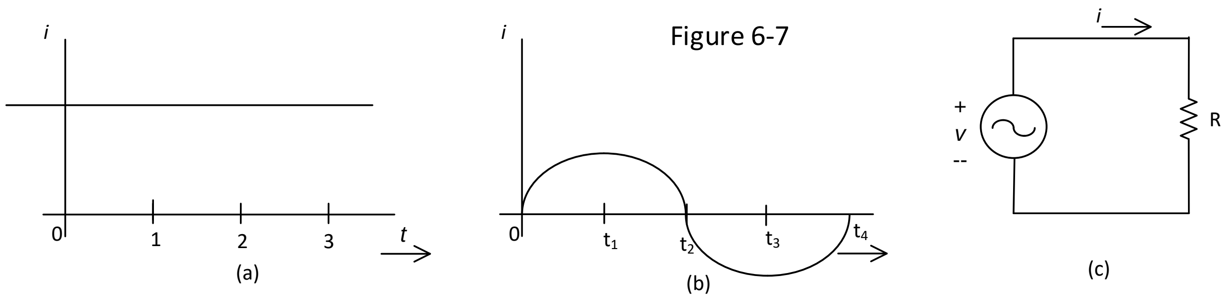
Figure 6-6



Time Varying Current Shapes

Fig. 6-7 a describes the direct current or non-varying waveform for current. Fig. 6-7b describes a sin waveform for current. The sin wave also is used to describe alternating current (or voltage). We will give more time (much more time) to alternating current and voltage waveforms in future chapters. For now, it is good enough to say it wiggles.

Figure 6-7



We will use capital letters for dc (direct current) and lower case letters for ac (alternating current or wiggling current). All the laws learned previously still apply, just for lower case values.

While for DC, we would have had:

$$\mathrm{I} = \frac{\mathrm{V}}{\mathrm{R}}$$

Now, we have:

$$i = \frac{v}{R}$$

DC Theorems such as the following do not apply for time-varying values. These are:

$$I = \frac{Q}{t}$$

and

$$P = \frac{W}{t}$$

We will discuss power using:

$$i = \frac{v}{R}$$

and

$$p = v \cdot i$$

Now we begin to use the formulas of previous chapters to draw waveforms similar to those of dc circuits. In Fig. 6-8a, find the current through R_L, the 2-kΩ resistor.

We begin by thevenizing the circuit left of A-B. With the 2-kΩ resistor removed, we use voltage-divider to find:

$$v_{TH} = \frac{R_2}{R}v = \frac{3000}{9000}v = \frac{v}{3}$$

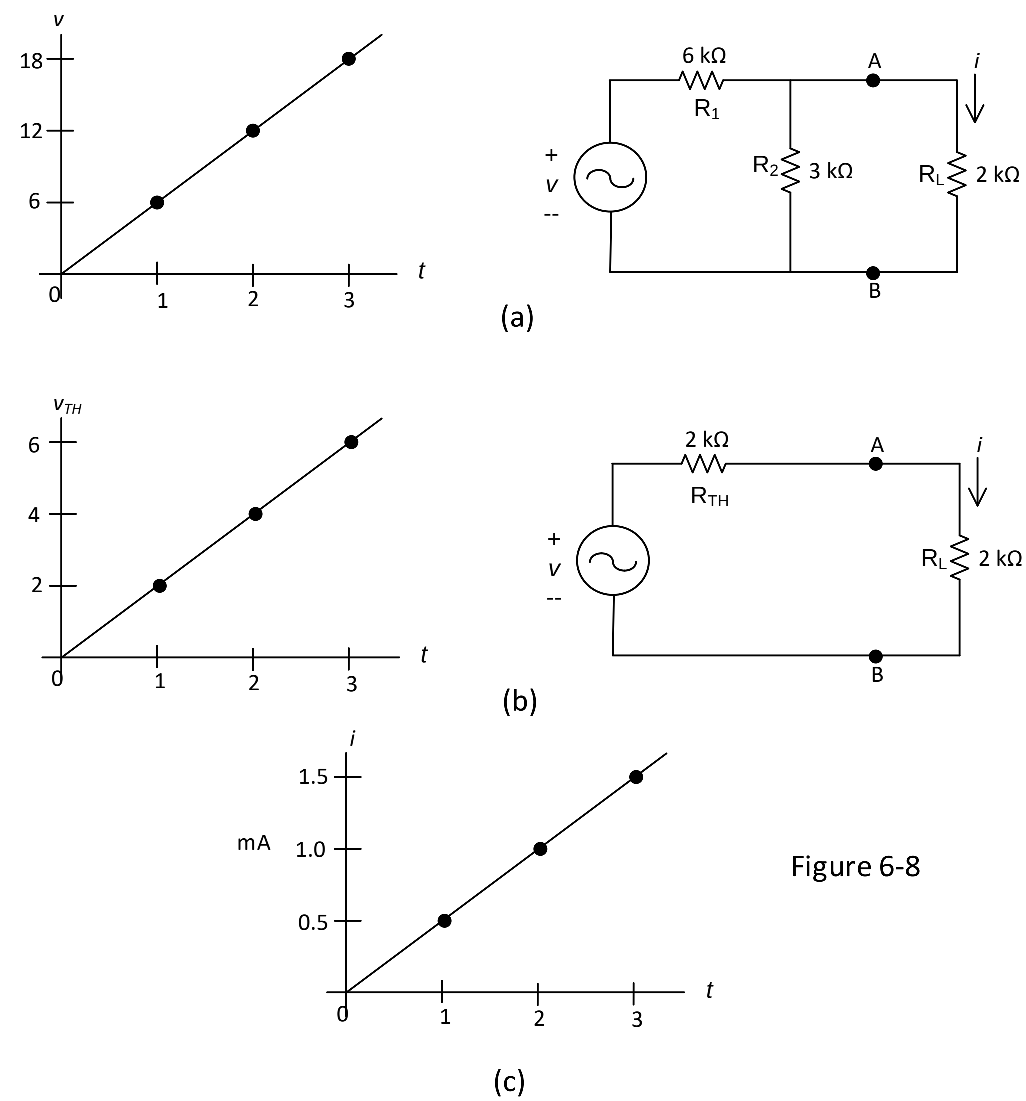


Figure 6-8

R_{TH} is found:

$$R_{TH} = R_1 \parallel R_2 = 6000 \parallel 3000 = 2k\Omega$$

We have the value of v_{TH} for various values of the input voltage. We use this relationship (1/3) to find vTH at various points in time. These calculations follow:

At t = 1, v_{TH} = 2 and

$$i = \frac{v_{TH}}{R_{TH} + R_L} = \frac{2}{4000} = 0.5\ mA$$

At t = 2, v_{TH} = 4 and

$$i = \frac{4}{4000} = 1\ mA$$

At t = 3, v_{TH} = 6 and

$$i = \frac{6}{4000} = 1.5\ mA$$

We construct the waveform for current in Fig. 6-8c. We can see by observation that a ramp input yields a ramp output. What a concept!

Next, we want to find the current through the 10 kΩ resistor for the waveform of Fig. 6-9a. We remember our Thevenin rules and begin by removing the 10-kΩ resistor. The 3-kΩ resistor becomes a dangling resistor and we calculate v_{TH} as follows:

$$v_{TH} = v\ \frac{R_2}{R} v = \frac{4000}{8000} v = \frac{v}{2}$$

R_{TH} equals:

$$R_{TH} = R_3 + R_1 \parallel R_2 = 3000 + 6000 \parallel 3000 = 5k\Omega$$

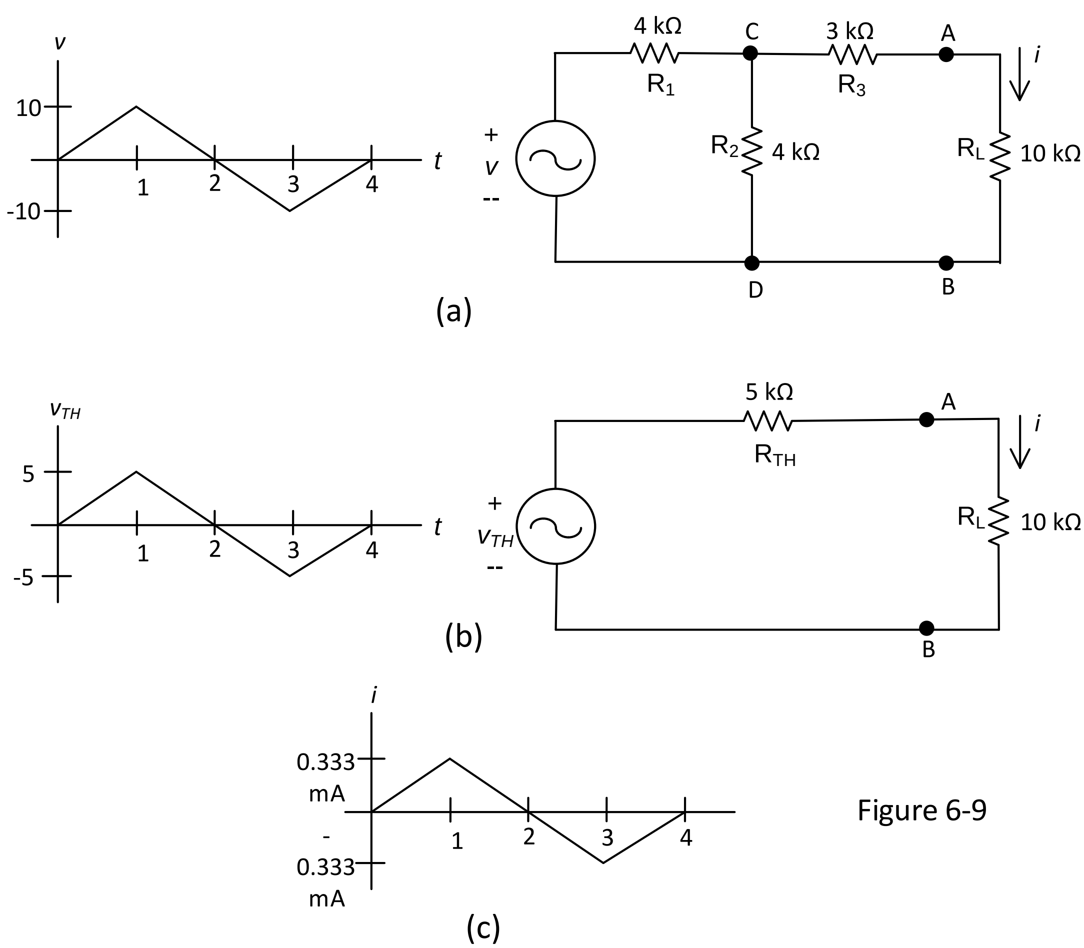


Figure 6-9

Next, we recombine the circuit with R_L and find R_{Total}:

$$R_{TH} + R_L = 5000 + 10{,}000 = 15k\Omega$$

At t = 1, v_{TH} = 5 and

$$i = \frac{v_{TH}}{R_{TH} + R_L} = \frac{5}{15{,}000} = 0.333\ mA$$

For Fig. 6-10 below, we observe a current source as a square wave. We are asked to graph *v* across the 20 kΩ resistor.

We observe that between t = 0 and t = 1, the current is a dc wave of 1 mA. The voltage in this time is calculated:

$$v = R \cdot i = 20{,}000(0.001)$$
$$= 20v$$

with Polarity positive.

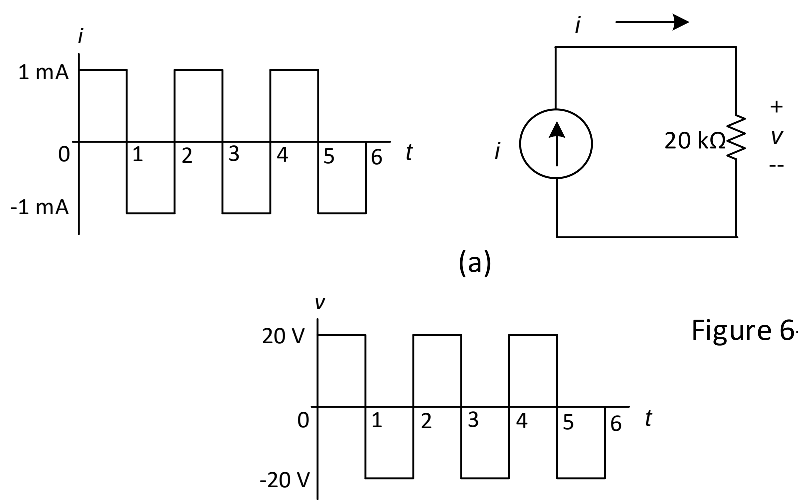


Figure 6-10

At t = 1, the current changes and equals -1 mA. The value is the same but of opposite direction. We calculate voltage:

$$v = R \cdot i = 20{,}000(-0.001)$$
$$= -20v$$

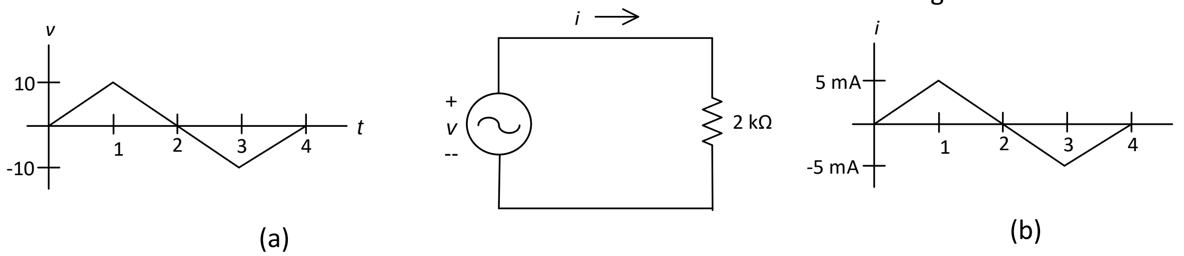
Again, we graph the output voltage waveform and find the same waveform as the current input waveform.

We observe the similarity theorem in all these examples. The similarity theorem simply states that the output waveform has the same shape as the input waveform. Again, we are bound by the linearity rules of resistors and we are also bound to a single source although we can add waveforms if superposition is to be used.

We see in Fig. 6-11a the same waveform as in Fig. 6-11b. At the positive peak for voltage, we calculate: (t = 1, v = 10)

$$i = \frac{v}{R} = \frac{10}{2000} = 5\ mA$$

Figure 6-11



We can see the same principle at the negative peak at t = 3 and v = -10. There, i = -5 mA.

Another example from a school of sharks (sawtooth waveform):

We see in Fig. 6-12a a sawtooth waveform. The current and voltages for the output are calculated and displayed in Fig. 6-12b, c, and d.

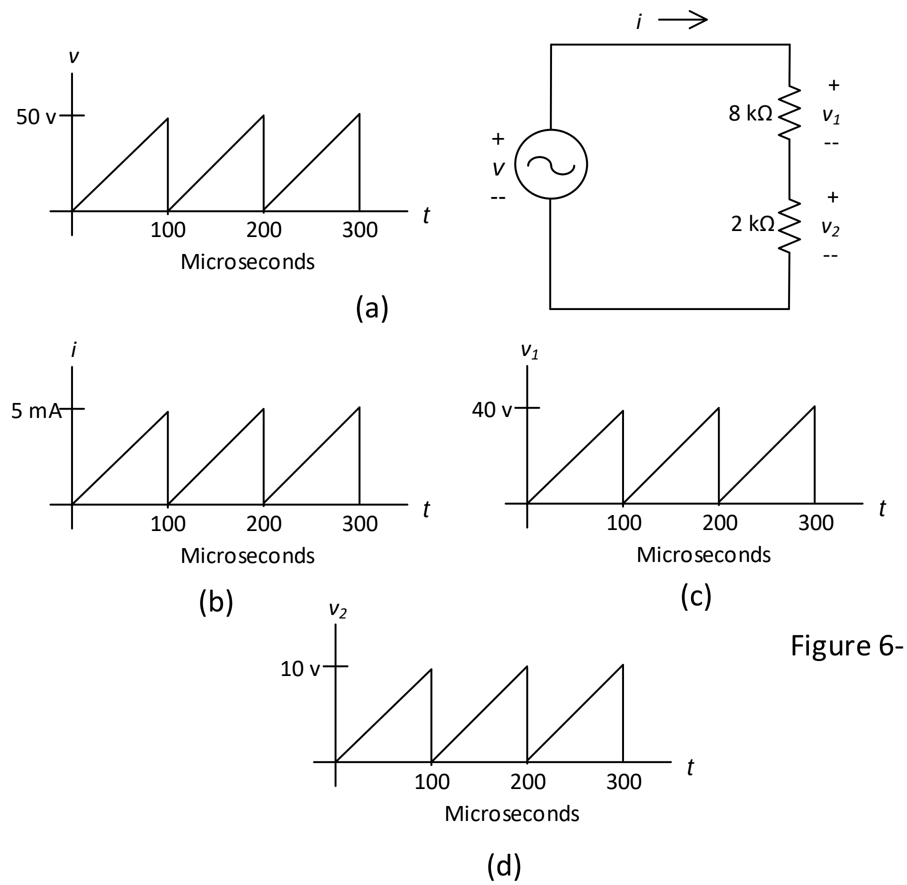


Figure 6-12

At t = 100 μs, we observe that v = 50 V and:

$$i = \frac{50}{10{,}000} = 5\ mA$$

(this is the peak current)

Likewise, v_1 and v_2 are sawtooths as well with peak:

$$v_1 = R_1 i = 8000(0.005) = 40\ V$$

and

$$v_2 = R_2 i = 2000(0.005) = 10\ V$$

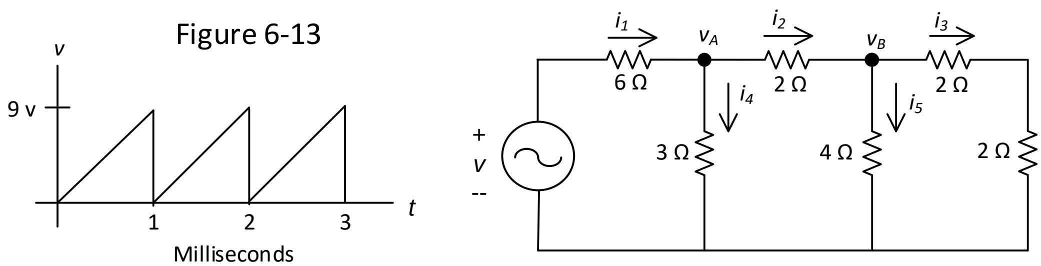


Figure 6-13

The example of Fig. 6-13 gives more practice with voltage sources and split currents. At t = 1 ms, t = 2 ms, t = 3 ms we find currents for v = 9 as follows:

$$i_1 = 1.17\ A$$

$$i_2 = 0.5\ A$$

$$i_3 = 0.25\ A$$

$$i_4 = 0.667\ A$$

$$i_5 = 0.25\ A$$

$$v_A = 2\ V$$

$$v_B = 1\ V$$

A Look at the Sine Wave

The sine wave is the wave generated by the power company and is what is expected when we plug a device into a household outlet. We will discuss such quantities as the period and frequency as well as the amplitude of a sine wave.

While we begin our study of electricity with dc circuits, most circuits actually are ac or alternating voltage and current circuits. With dc circuits, the input is from a battery or dc power supply. With ac, the input is from a wall socket or from the incoming wiring to the facility, either a house or factory. Three phase circuits are the most efficient with three-phase motors used for most factory applications.

An AC generator is used to generate power for the power company. They work by spinning a magnetic field which induces a current in a coil of wire. The rotation of the magnet will generate a sine wave in the wire of the circuit below. One complete revolution will supply one complete ac cycle of voltage.

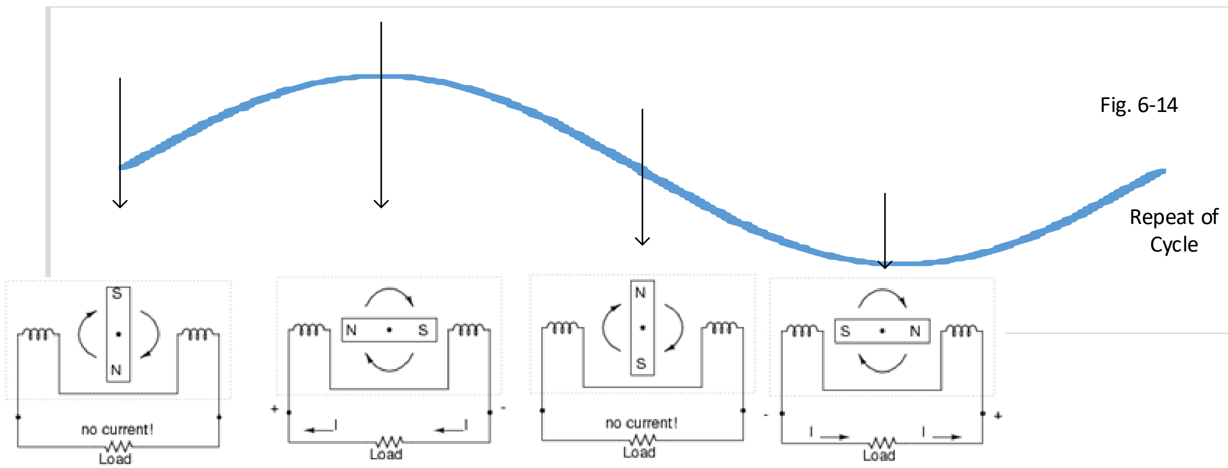


Fig. 6-14

Motors work in a similar manner. The operation is reversed with the field voltage producing a magnetic field that spins the armature of the motor.

When the generator rotates, the waveform generated looks like the following waveform. This is a sinewave form. We picture the waveform as an oscillating waveform.

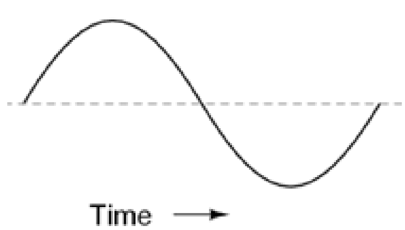


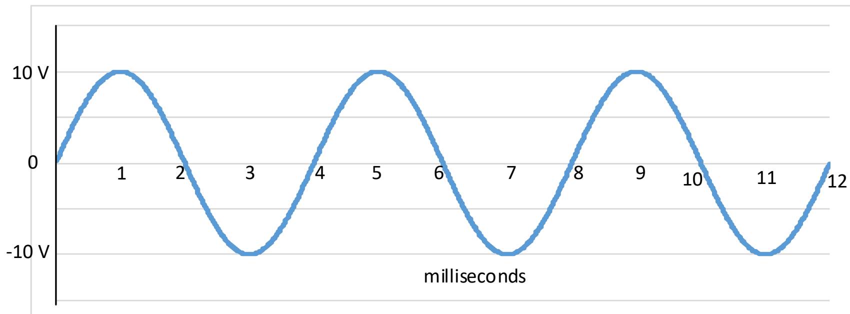
Fig. 6-15

Period and frequency are mathematical reciprocals of one another. That is to say, if a wave has a period of 10 seconds, its frequency will be 0.1 Hz, or 1/10 of a cycle per second:

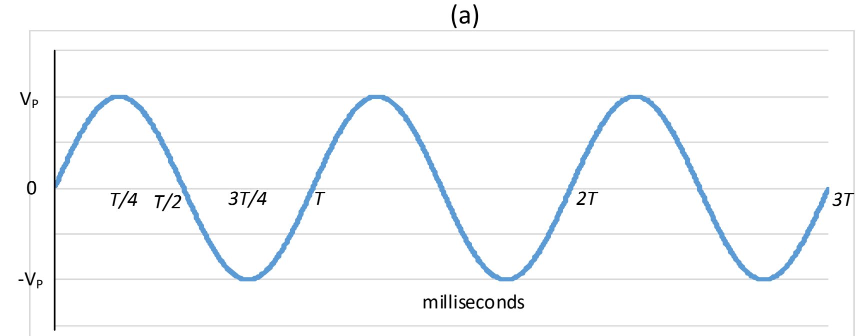
Period

We see in Fig. 6-16a and b a sine waveform. The waveform first climbs and then goes negative and finally returns back to the original value. When looking at frequency, we first look at the period. The period is determined as the point at which a waveform goes through a particular point to when it does the same again. At t = 0, the sin wave starts at 0 and is moving in the positive direction. We see that the waveform does the same at t = 4. Therefore the waveform's period is 4 msec. On the way, at t = 1 ms, v = 10 V, the positive peak. At t = 2 ms, v = 0, the crossover point. At t = 3 ms, v = -10 V, the negative peak and finally at t = 4 ms, v = 0, the point where the waveform repeats. This represents one period or cycle of the waveform. The waveform's period is defined as T. This is found in Fig.g 6-16b where the portion of the period is labelled.

Figure 6-16



(a)



(b)

We calculate the portions of the period as follows:

$$t = \frac{T}{4} = \frac{10\mu s}{4} = 2.5\ \mu s$$

(positive peak)

$$t = \frac{T}{2} = \frac{10\mu s}{2} = 5\ \mu s$$

(crossover)

$$t = \frac{3T}{4} = \frac{30\mu s}{4} = 7.5\ \mu s$$

(negative peak)

$$t = T = 10\ \mu s$$

(end of cycle, T)

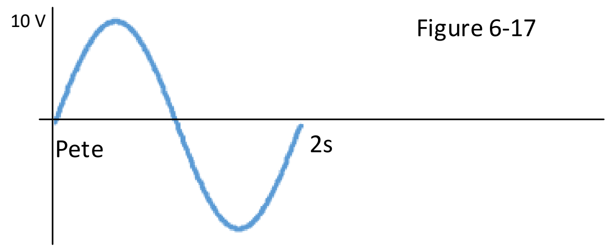
$$t = 2T = 20\ \mu s$$

(end of cycle 2, 2T)

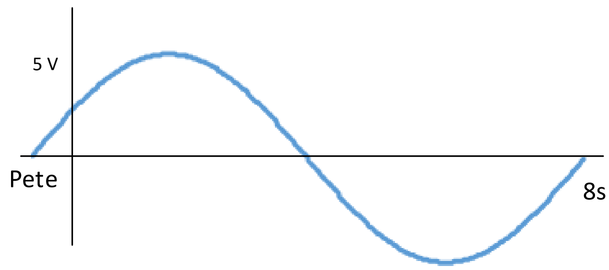
$$t = 3T = 30\ \mu s$$

(end of cycle 3, 3T)

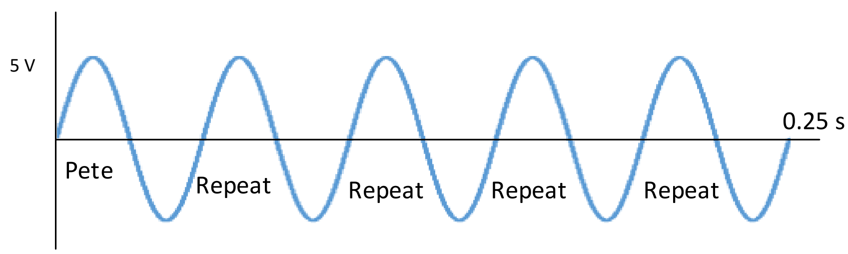
Pete and Repeat were in a boat. Pete fell out, who was left?



(a)



(b)



(c)

The above three sin wave forms show when the repeating of the waveform occurs. This is an important concept along with finding V_P. We see in Fig. 6-17a:

> V_P = 10 V, T = 2 sd
> From this we can also see:
> negative peak = -10 V
> positive peak @ t = 0.5 s
> crossover at t = 1 s
> negative peak @ t = 1.5 s

Fig. 6-17b gives similar results. As we can see, with Fig. 6-15b and c, the period changes which changes the frequency of these waveforms.

Frequency is defined as:

$$f = \frac{n}{T}$$

with f = frequency
n = cycles (number of)
T = time

Usually this can be simplified if we consider only one cycle to:

$$f = \frac{1}{T}$$

If we observe 12 cycles in 4 seconds, we calculate frequency as:

$$f = \frac{n}{t} = \frac{12\ c}{4\ s} = 3\ hertz$$

From Fig. 6-17c, we observe 5 cycles in 0.25 s. We can calculate frequency as:

$$f = \frac{n}{t} = \frac{5\ c}{0.25\ s} = 20\ hertz$$

We can reference cycles per second as hertz which is the accepted unit of frequency.

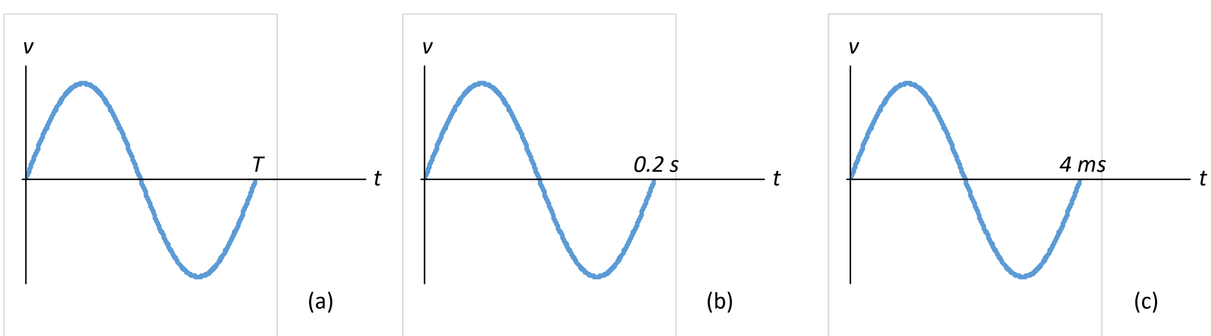
1 hertz = 1 cycle per second

or 1 Hz = 1 c/s

Also, the frequency is the inverse of the period T:

$$f = \frac{1}{T}$$

Figure 6-18



We use this equation to find the frequency of various waveforms. In Fig. 6-18a, we observe the period as T. The frequency is therefore 1/T. In the (b) and (c), we get numeric results. These results are:

Fig. 6-18b period = 0.2 s;

$$f = \frac{1}{T} = \frac{1}{0.2} = 5\ Hz$$

Fig. 6-18c period = 4 ms;

$$f = \frac{1}{T} = \frac{1}{0.004} = 250\ Hz$$

Figure 6-19a period = 10 μs;

$$f = \frac{1}{10(10^{-6})} = 100\ kHz$$

If the current through a resistor is from a voltage with sin wave, the frequency of the current is the same as that of the voltage. If R = 1000:

$$I_P = \frac{V_P}{R} = \frac{5}{1000} = 5\ mA$$

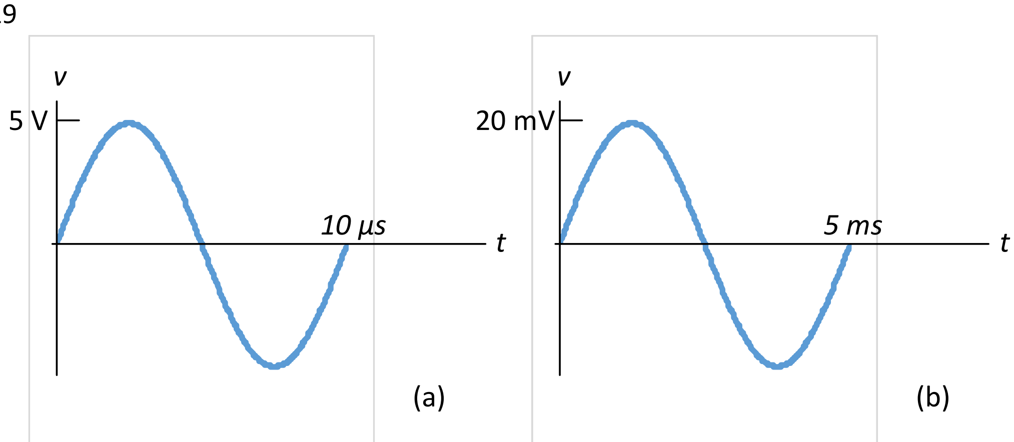
For Fig. 6-19b the voltage is applied across a 6-kΩ R. We observe again the frequency is the same and the current is:

$$I_P = \frac{V_P}{R} = \frac{0.02}{6000} = 2.5\ \mu A$$

Frequency:

$$f = \frac{1}{T} = \frac{1}{0.005} = 200\ Hz$$

Figure 6-19



We know the frequency of household voltage is 60 Hz. What is the period?

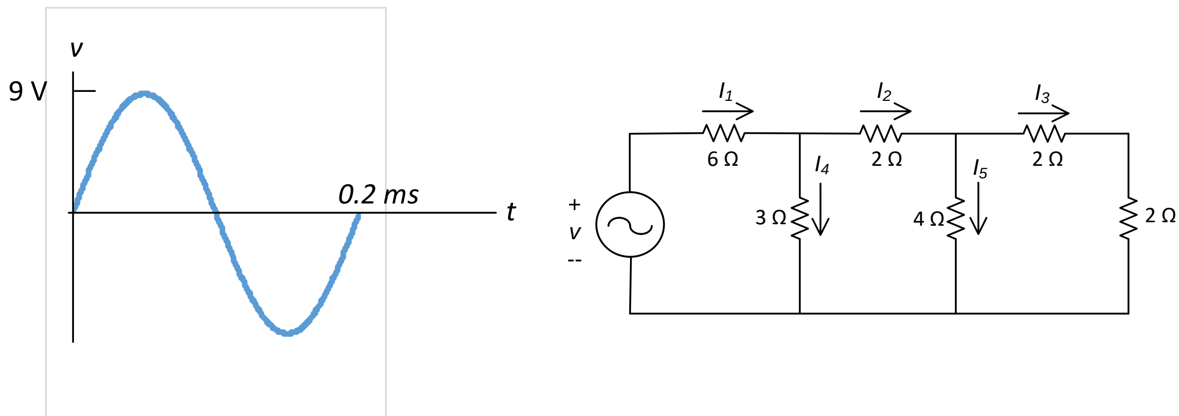
$$T = \frac{1}{f}$$

or

$$T = \frac{1}{60} = 16.7\ ms$$

In the following Fig. 6-20, find all voltage and waveform peaks and frequencies:

Fig. 6-20



Use 9 V as a dc value to calculate all voltage and current values. The frequencies are all equal to the input frequency of the supply:

I_1 = 1.17 A
I_2 = 0.5 A
I_3 = 0.25 A
I_4 = 0.667 A
I_5 = 0.25 A
V_A = 2 V
V_B = 1 V

The frequency calculation for all currents and voltages above is:

$$f = \frac{1}{T} = \frac{1}{0.2(10^{-3})} = 5\ kHz$$

We can use the oscilloscope to display the sine wave on the display. The volts per division give the amplitude in the y direction while the time per division give the frequency in the x direction.

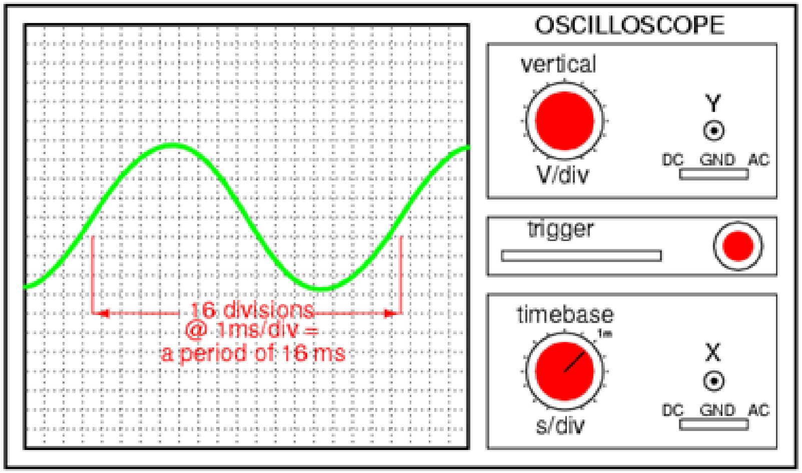


Fig. 6-21

Time period of sinewave is shown on oscilloscope.

Waves have common characteristics whether a sine wave or another type of wave. A violin produces a waveform that identifies it as a sound we recognize as a violin. Other instruments produce waveforms we can see on the oscilloscope as particular to that instrument. They all share a common characteristic of frequency. High frequencies can be identified as a high pitch and low frequencies as a low pitch. The analogy between sound and ac waveforms can be seen displayed on an oscilloscope very easily.

A table of musical notes and their frequency is shown below. We notice that an octave from one A to the next A gives a doubling of the frequency. This is what an octave means – doubling the frequency.

Note	Musical designation	Frequency (in hertz)
A	A_1	220.00
A sharp (or B flat)	$A^{\#}$ or B^{b}	233.08
B	B_1	246.94
C (middle)	C	261.63
C sharp (or D flat)	$C^{\#}$ or D^{b}	277.18
D	D	293.66
D sharp (or E flat)	$D^{\#}$ or E^{b}	311.13
E	E	329.63
F	F	349.23
F sharp (or G flat)	$F^{\#}$ or G^{b}	369.99
G	G	392.00
G sharp (or A flat)	$G^{\#}$ or A^{b}	415.30
A	A	440.00
A sharp (or B flat)	$A^{\#}$ or B^{b}	466.16
B	B	493.88
C	C^1	523.25

The frequency in Hertz (Hz) is shown for various musical notes.

The keyboard also shows the placement of the various notes and the repetition of the notes one octave higher or lower.

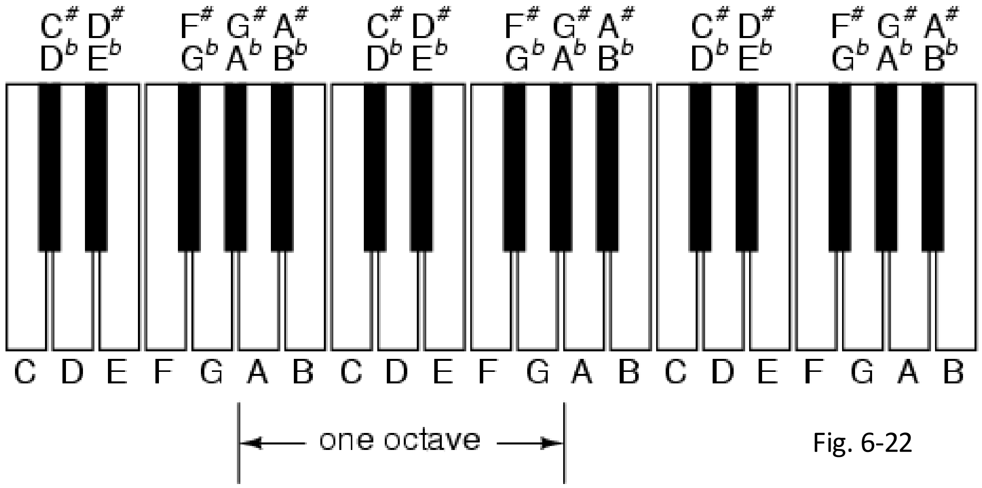


Fig. 6-22

Sine Wave Amplitude

We measure amplitude of the sine wave to the peak of the wave from the zero point of the waveform. It is measured as follows:

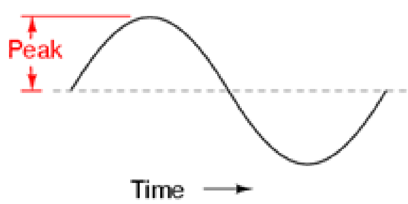


Fig. 6-23

Doubling the peak value gives the peak-to-peak value:

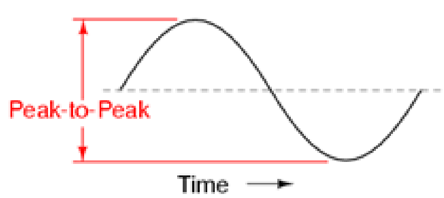


Fig. 6-24

Comparing the dc waveform to its equivalent ac waveform is necessary to calculate the equivalent power produced by the ac waveform. We can look at the equivalency of a bandsaw and a jigsaw. Both cut a piece of wood. Both have metal blades. The bandsaw uses a continuous motion to cut the wood while the jigsaw moves back and forth something like a sine wave. Both cut the wood.

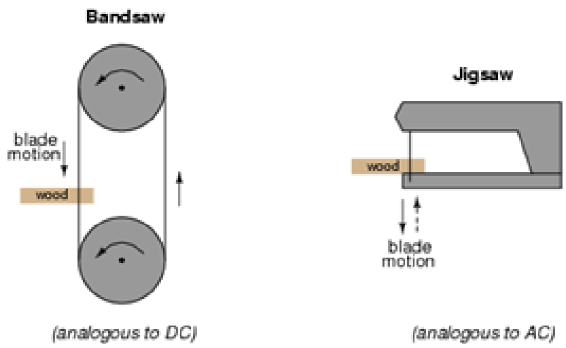
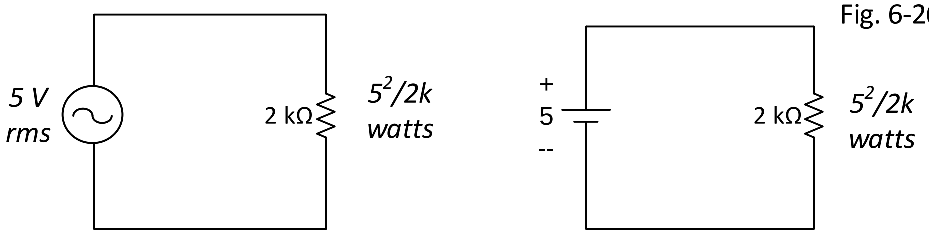


Fig. 6-25

Bandsaw-jigsaw analogy of DC vs AC

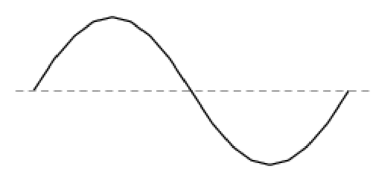
We can mathematically prove the method used to find the equal power in the sine wave as its dc equivalent. That will be left for another text. However, there is an equivalency just as there is in the cutting of a piece of wood with a jigsaw or a bandsaw. The ac equivalent voltage or current to the dc equivalent is the rms or root mean square value.

Fig. 6-26

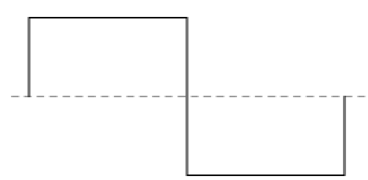


Note that the same power is dissipated through the ac circuit at left as the dc circuit at right

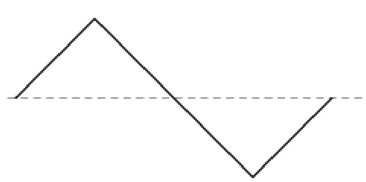
To find rms value if we have peak, we divide by 1.414 (square root of 2) or multiply by .707. The following table give some conversions for various wave types to their dc equivalent:



RMS = 0.707 (Peak)
AVG = 0.637 (Peak)
P-P = 2 (Peak)



RMS = Peak
AVG = Peak
P-P = 2 (Peak)

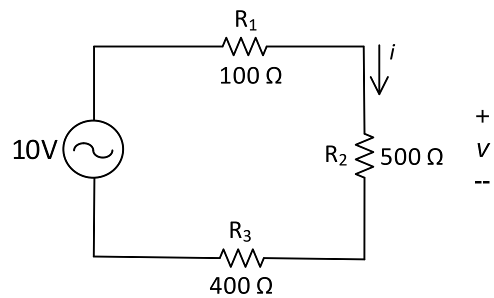


RMS = 0.577 (Peak)
AVG = 0.5 (Peak)
P-P = 2 (Peak)

Why do we care about rms values other than for power calculations. We are concerned about the rms value because this is what is read from the digital multimeter. This is the meter value!

Usually, the values of voltage and current are given as rms values. Unless otherwise labelled, the values are all rms.

RMS Voltage Circuit:



$$R_{Total} = R_1 + R_2 + R_3 = 1k\Omega$$

$$I = \frac{V}{R_{Total}} = \frac{10\ V}{1k\Omega} = 10\ mA$$

$$V_{R1} = IR_1 = 1\ V$$

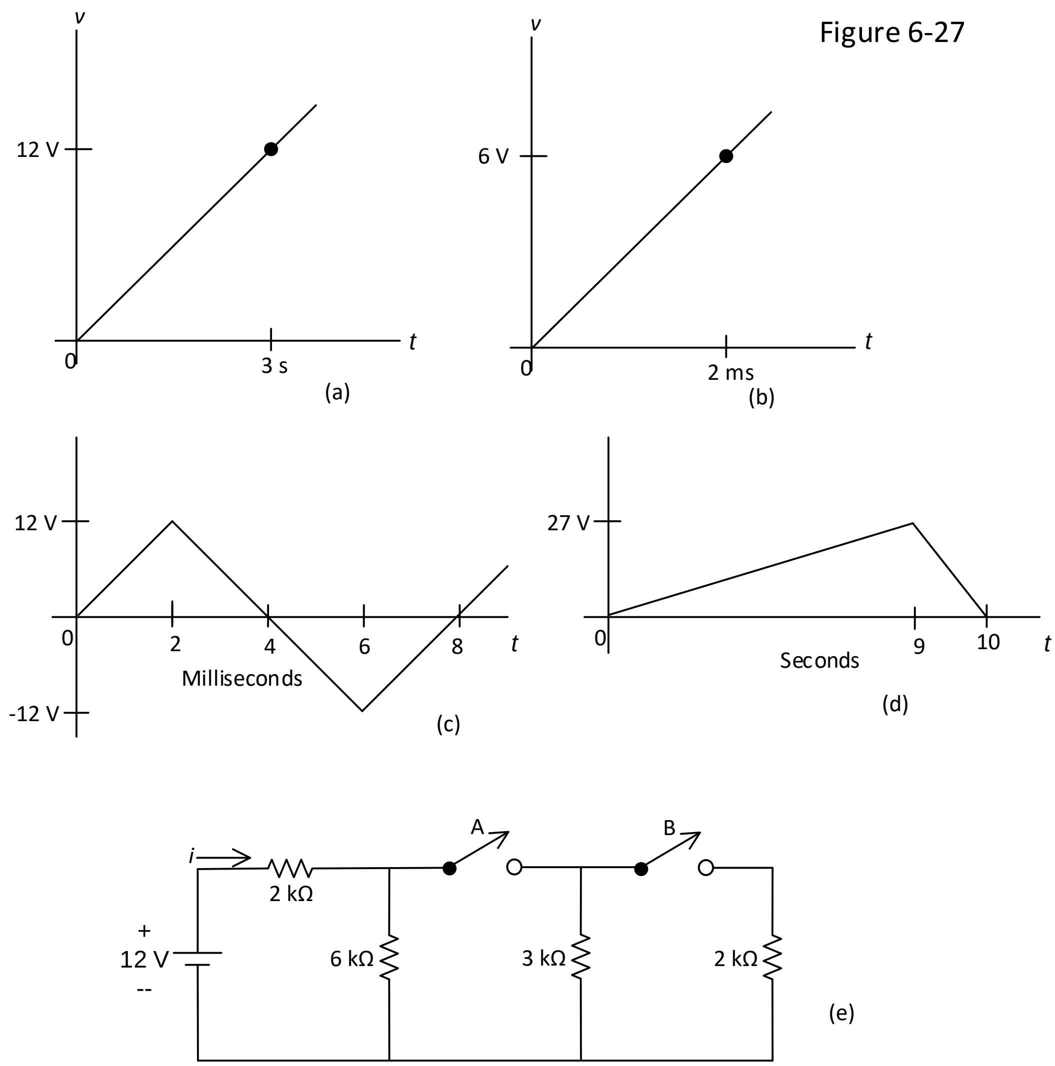
$$V_{R2} = IR_2 = 5\ V$$

$$V_{R3} = IR_3 = 4\ V$$

We work all ac circuits using rms values when using laws from the dc circuits, ie calculations from Ohm's Law, KCL, KVL, Thevenin, etc. We assume rms for all values unless otherwise stated.

Problems

6-1 In Fig. 6-27a, find V when t = 1 s? At t = 2 s?



6-2 In Fig. 6-27b, find v when t = 1 ms? At t = 10 ms?

6-3 Find the equation

$$y = mx$$

or

$$v = m \cdot i$$

a. For Fig. 6-27a?
b. For Fig. 6-27b?

6-4 For the waveform in Fig. 6-27c, find the voltage at: t = 2 ms, t = 4 ms, t = 6 ms and t = 8 ms?

6-5 For the waveform in Fig. 6-27d, find the voltage at: t = 1 s, t = 2 s, t = 9 s, t = 9.5 s.

6-6 In Fig. 6-27e A closes at t = 0 and B closes at t = 5 ms. What is the current waveform through the first 2- kΩ R.

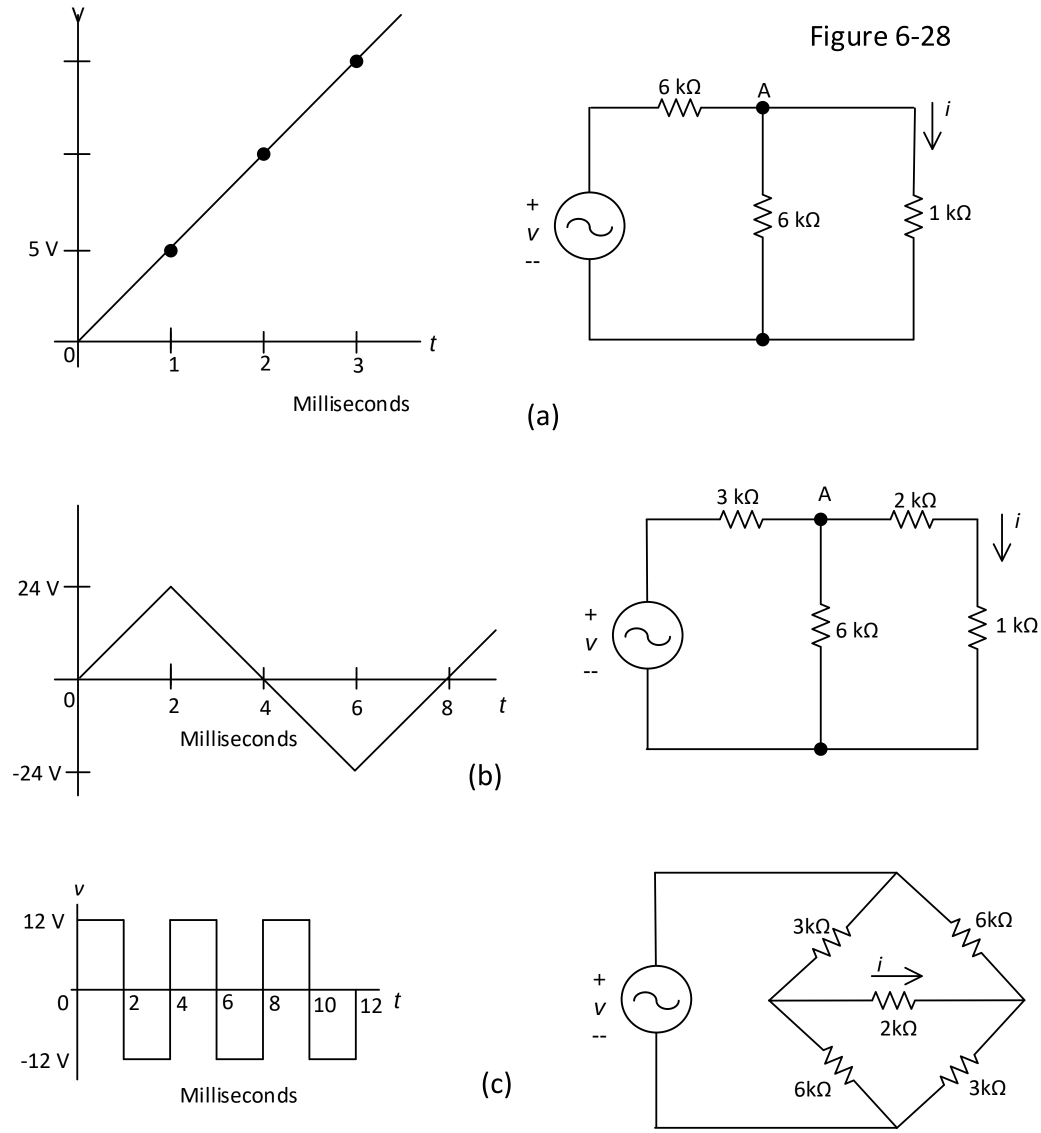
6-7 Sketch i in the 1-kΩ of Fig. 6-28a.

6-8 Sketch v at node A in Fig. 6-28a.

6-9 Sketch i in the 1-kΩ of Fig. 6-28b. Then sketch v across this resistor.

6-10 Sketch v at node A in Fig. 6-28b.

Figure 6-28

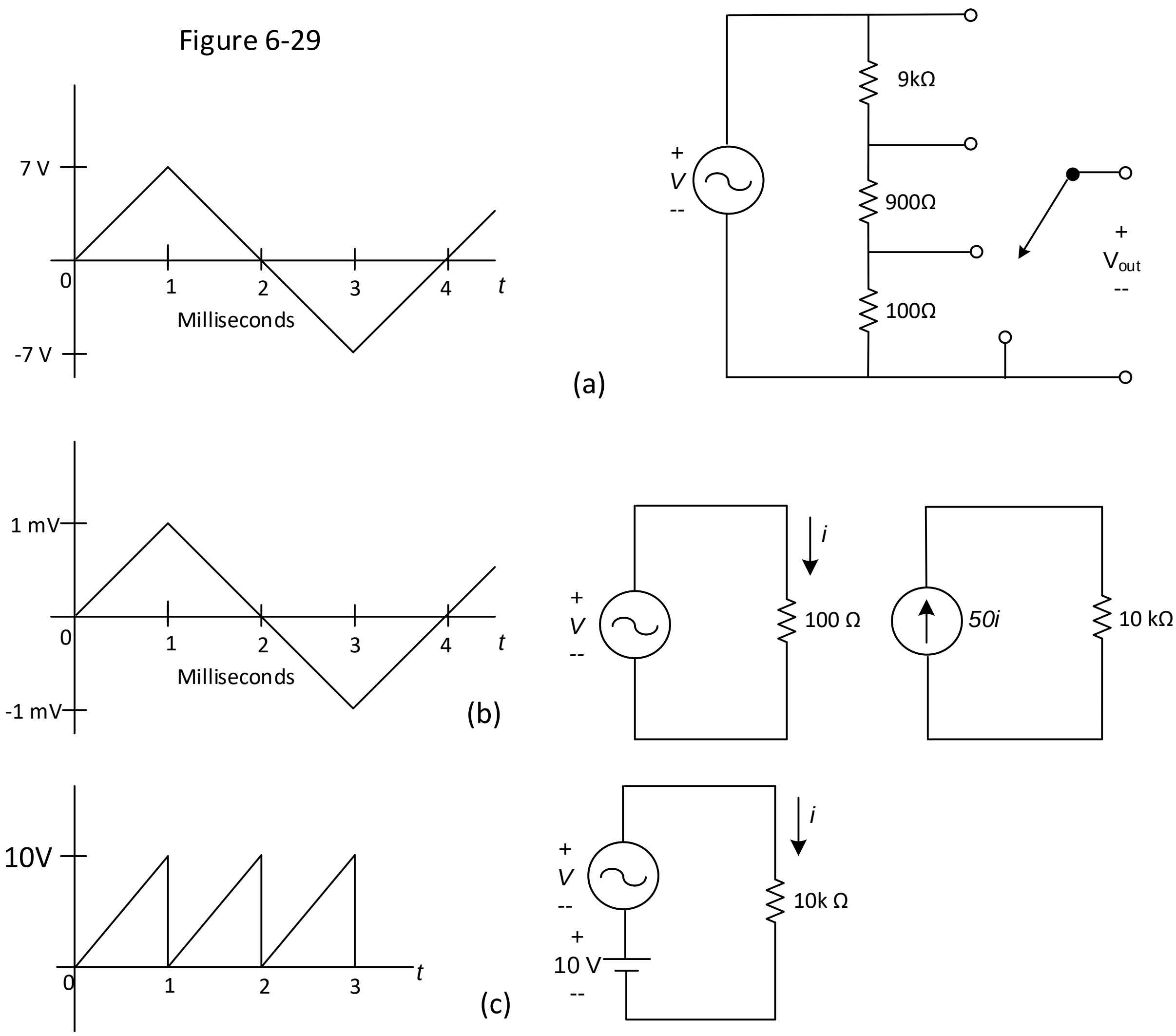


6-11 Use the square wave of Fig. 6-28c to find the current I through the 2-kΩ R. Then find the voltage across the resistor?

6-12 Sketch the waveform of each position of Fig. 6-29a?

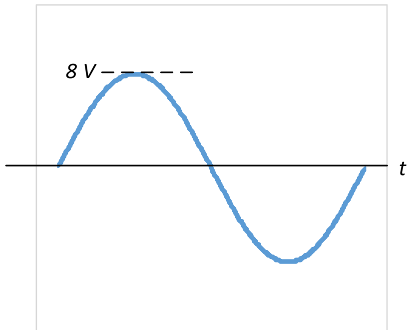
6-13 Sketch the waveform of the 10-kΩ R in Fig. 6-29b.

Figure 6-29

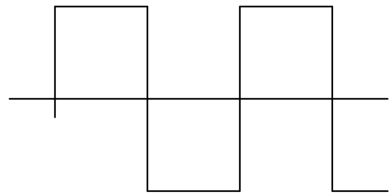


6-14 A time variable source shown in Fig. 6-29c and a dc source power a resistor. Sketch the current and voltage waveforms across this 10 kΩ resistor.

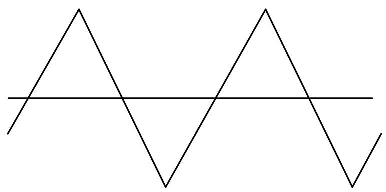
6-15 Use the waveforms of a, b, and c below to find the power dissipated in the two resistors and the rms value of current.



(a)



(b)



(c)