7 Inductance and DC Response to Capacitors and Inductors

We are beginning a new phase of the book, the phase using differential equations. We will discuss things in a simple way but we will use derivatives. It is up to the student to understand the meaning of the equations. We will not use proofs but describe results and the problems that follow.

First, we will talk about ELI the ICE man. The ELI concerns inductors (L). The ELI refers to two different ideas. First:

$$E = L\frac{di}{dt}$$

E is voltage, *L* is inductance and *i* is current.

Also, Voltage (E) leads Current (I) in an inductive circuit.

ICE refers to capacitors. First:

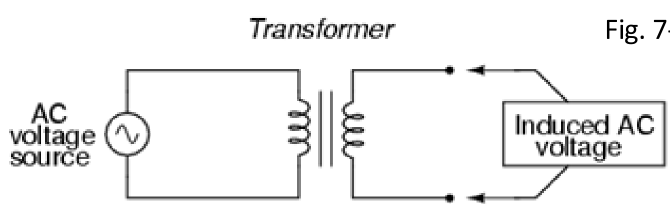
$$I = C\frac{de}{dt}$$

I is current, *C* is capacitance and *e* is voltage.

Also, Current (I) leads Voltage (V) in an capacitive circuit.

Transformers are an important to electrical discussion. They allow an ac voltage to be stepped up or down for use in another circuit that is isolated from the first. The voltage induces a magnetic waveform in a core which then induces a voltage waveform in a secondary winding. This second ac circuit then can be used to deliver power to a resistor or other device.

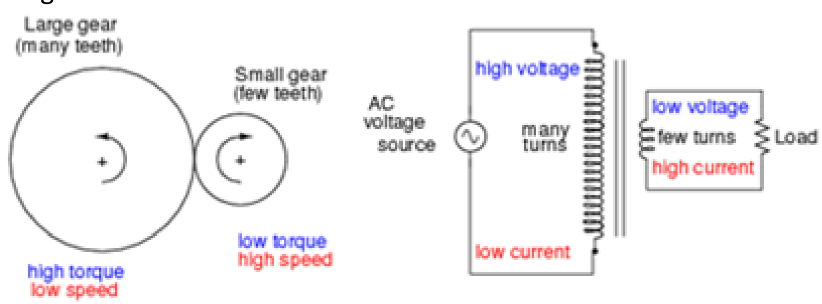
Fig. 7-1



Transformer "transforms" AC voltage and current.

Fig. 7-1 shows the basic design of a transformer. In Fig. 7-2, we see an analogy between transformers and a gear ratio. Fig. 7-2a shows a step-down transformer with the primary voltage high and secondary voltage low:

Fig. 7-2a



In this example, torque can be compared to voltage and speed to current. The large gear is compared to the side of the transformer with many windings which has relatively high voltage and low current when compared to the secondary side.

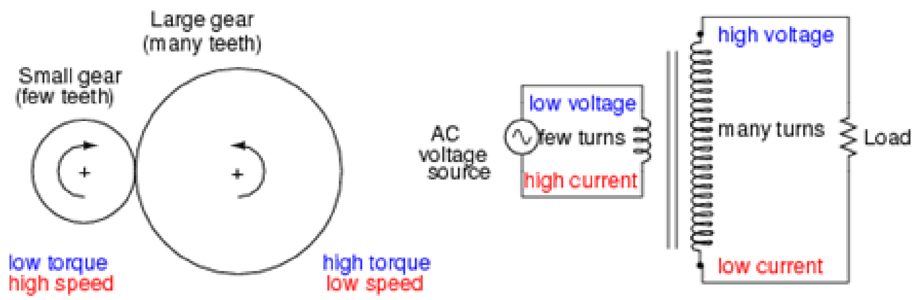


Fig. 7-2b

In the example of Fig. 7-2b, the opposite occurs with a small gear transmitting power to a second larger gear. The torque is low at left and high at right. The speed is high at left and low at right.

Transformers are used to deliver power from the power plant to the customer. This is shown in the following figure:

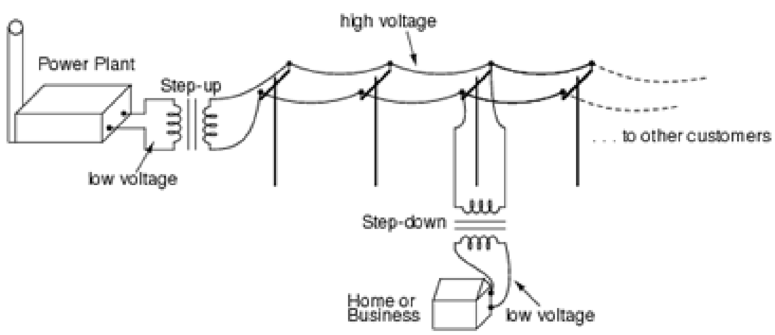


Fig. 7-3

With inductors, we have a time-varying flux which Faraday's law defines as a voltage produced in the inductor which may also produce a voltage in a second inductor. When this occurs, we have a transformer. Only time-varying signals generate a variable flux. This is defined as:

$$\frac{d\phi}{dt}$$

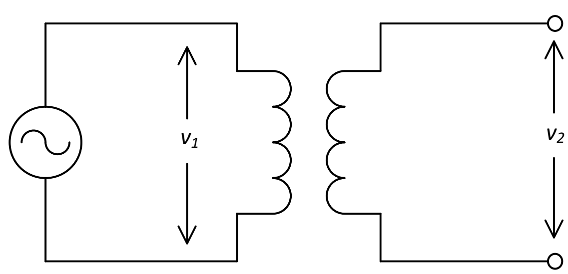


Figure 7-4

The number of turns of the coil around a iron core is defined as N. Voltage is defined as follows for an inductor in a transformer:

$$v = N\frac{d\phi}{dt}$$

The primary of a transformer gives:

$$v_1 = N_1\frac{d\phi_1}{dt}$$

For an ideal transformer, all flux is transferred to the secondary and:

$$v_2 = N_2\frac{d\phi_2}{dt}$$

If the transformer is ideal, all the flux from the primary is absorbed by the secondary and:

$$\frac{d\phi_1}{dt} = \frac{d\phi_2}{dt}$$

and

$$\frac{v_2}{v_1} = \frac{N_2}{N_1}$$

$$\frac{N_1}{N_2}$$

is the turns ratio.

In the following circuit, V_S = 10 V giving v_1 = 10 V. The turns ratio is 5:1 giving v_2 = 2 V. Then we find the current through a 4 kΩ resistor at .5 mA and the current through the primary at .1 mA.

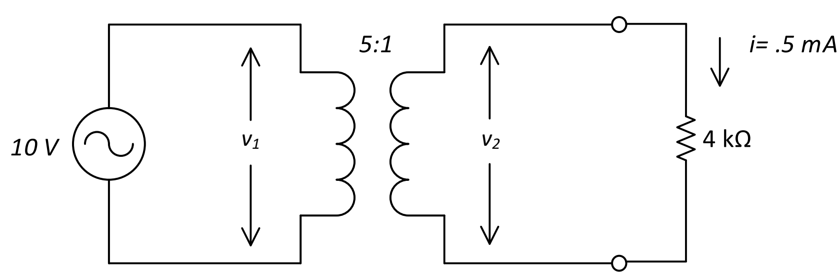


Figure 7-5

In the following circuit, V_S = 10 V giving v_1 = 10 V. The turns ratio is 1:2 giving v_2 = 20 V. Then we find the current through the 1 kΩ resistor at 20 mA. The primary current is then 40 mA.

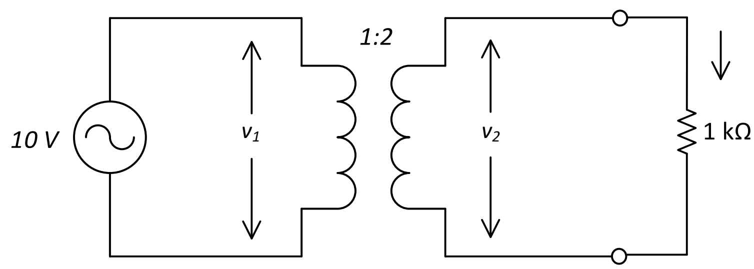


Figure 7-6

We notice that the ratio $i_1/\ i_2$ is:

$$\frac{i_1}{i_2} = \frac{N_2}{N_1}$$

Notice that the current ratio is opposite the turns ratio. Remember $v_1 \cdot i_1 = v_2 \cdot i_2$ for an ideal transformer. Various types of transformers

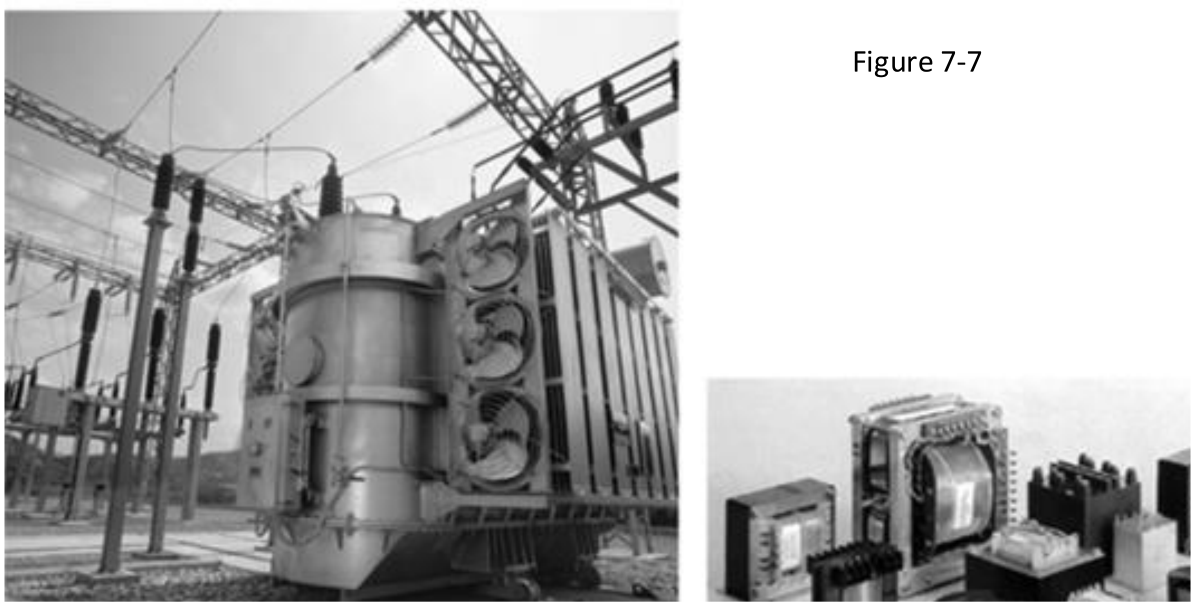


Figure 7-7

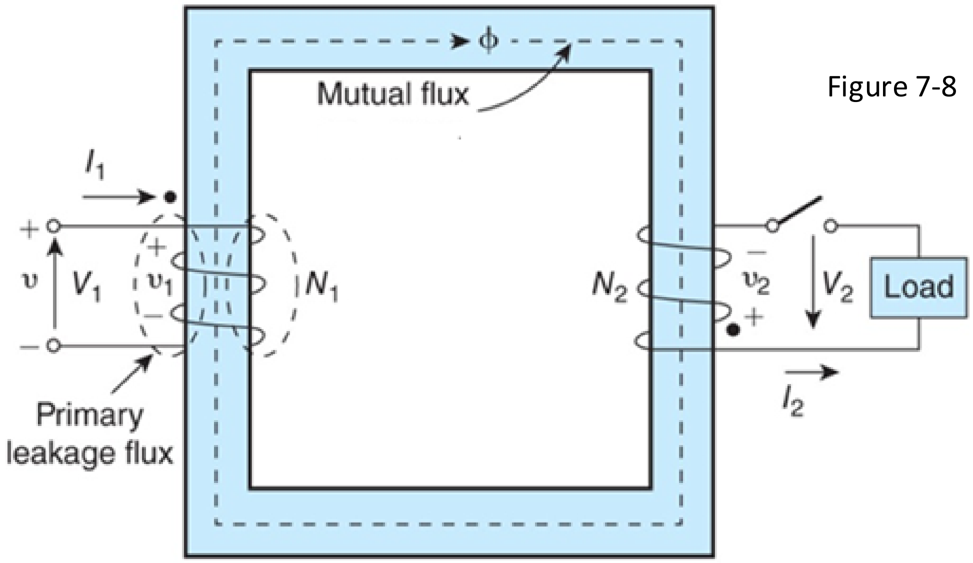


Figure 7-8

The single phase transformer pictured above is ideal. All the flux generated in the primary winding travels around the core and then to the secondary winding. For this transformer, primary power equals secondary power. Mutual flux carries all the flux with no leakage. While it is not possible for all primary power to be transferred to the secondary, large transformers are capable of about 95% efficiency. Voltage and current may be referenced as v_1 and i_1 or v_p and i_p for the primary windings. Voltage and current for the secondary are referenced as v_2 and i_2 or v_s and i_s.

Problems

7-1 For the following, find V_S, I_S, and I_P.

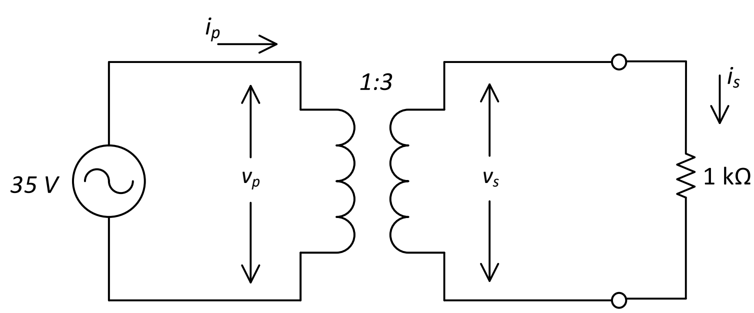


Figure 7-9

7-2 The following 5 kVA, 480-120 V single-phase transformer is connected as in Fig. 7-7. If losses are ignored, find transformer currents and V_S(Supply Voltage).

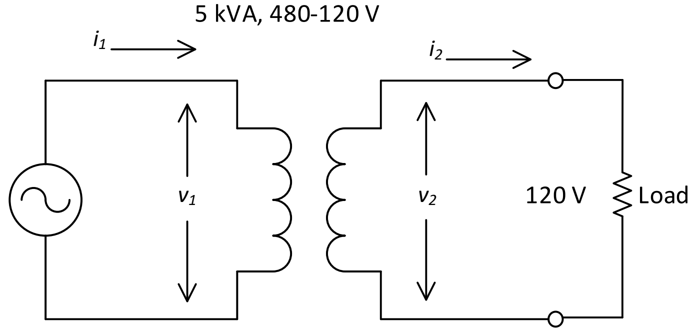


Figure 7-10

Solution

Secondary current is:

$$i_2 = \frac{P}{v} = \frac{5000}{120} = 41.67\ A$$

The windings ratio can be obtained by the ratio of the given windings:

$$\frac{N_2}{N_1} = \frac{V_2}{V_1} = \frac{120}{480} = \frac{1}{4}$$

Primary current is:

$$i_1 = \frac{1}{4}(41.67) = 10.42\ A$$

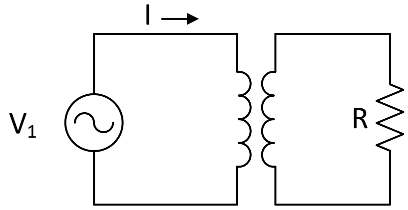
Primary voltage is:

$$v_1 = 480\ V$$

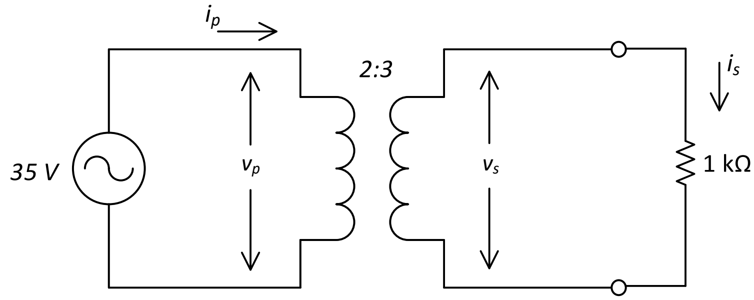
7-3 Use the following values of V_1 and R, assuming a primary-to-secondary turns ration of 5:1 to find I. The transformer is ideal.

V_1 = 25 V

R = 1.2 K Ω



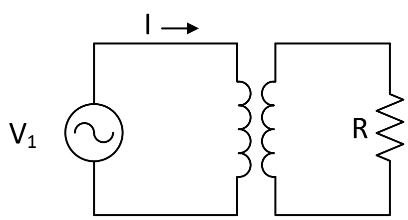
7-4 For the following, find V_S, I_S, and I_P.



7-5 Use the following values of V_1 and R, assuming a primary-to-secondary turns ration of 4:1 to find I. The transformer is ideal.

V_1 = 45 V

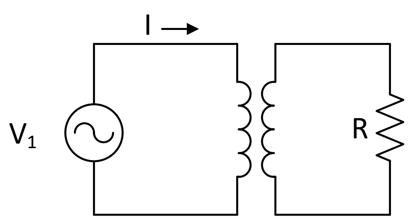
R = 1.2 K Ω



7-6 Use the following values of V_1 and R, assuming a primary-to-secondary turns ration of 1:12 to find I. The transformer is ideal.

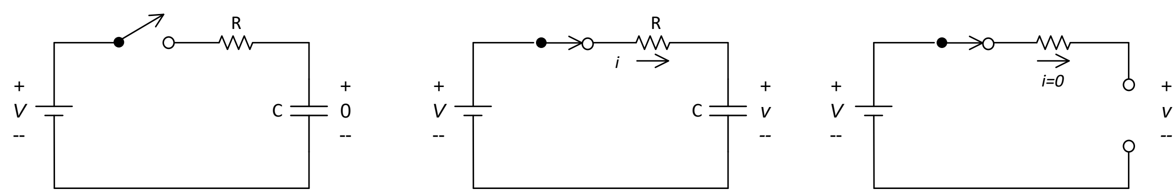
V_1 = 25 V

R = 1.5 K Ω



DC Transient R and C Circuits

With DC circuits including a switch and either a capacitor and resistor or inductor and resistor, we see a predictable response including an exponential rise or decay response. The response is related to the equations at the beginning of the chapter (remember ELI and ICE). These equations set up a first order differential equation for which the only response is an exponential response. We can observe the response by using charge to explain the circuit. When the circuit below closes, current flows until the capacitor is fully charged. That means, fully saturated with charge or electrons. Then the current stops. It doesn't stop immediately but gradually stops based on an exponential decay. Likewise, the voltage increases to the value of the battery gradually following an exponential rise. When the circuit settles, the circuit at right shows the final value of current and voltage. We see the current reduced to zero and the capacitor acting as an open circuit.



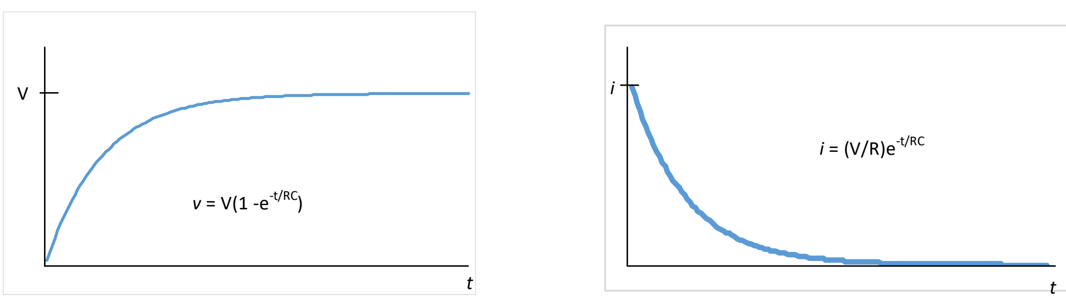


Figure 7-11

We graph the responses as found in the graphs above. The voltage across the capacitor gradually rises to the final value of the battery and the current is reduced to zero.

The equation for the KVL involves a derivative function (ICE).

$$i = C\frac{dv}{dt}$$

or

$$v = \frac{1}{C}\int i dt$$

For the equation around the loop, when the switch closes:

$$-V + iR + \frac{1}{C}\int i dt = 0$$

from which we get the equations above, $v = V(1 - e^{-t/RC})$ and $i = (V/R)e^{-t/RC}$:

If the circuit is more complicated, it may be simplified using the Thevenin rules to obtain the circuit at right below:

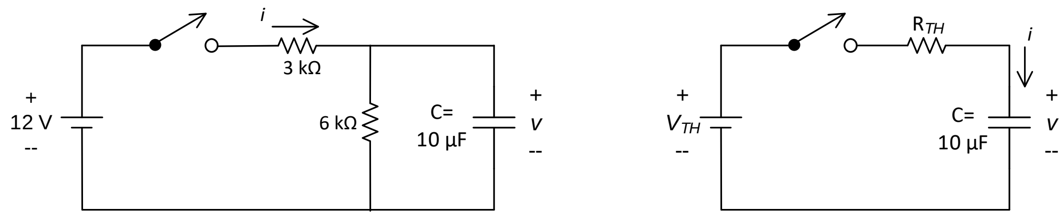


Figure 7-12

The time constant RC is referred to as τ or tau. The equation is written $e^{-t/RC}$ or $e^{-t/\tau}$.

The capacitor also may appear in a discharge circuit. The circuits below show a switch that may either in the left or right position. In the left position, the capacitor is being charged as in the circuits above. With the switch in the right position, the capacitor discharges and the voltage dissipates through the resistor. The resistor heats a little and the current and voltage result at 0.

The voltage equation through the capacitor in the discharge circuit shown in the figure below. First when the switch is closed to the left, the capacitor charges as before. When the switch is closed to the right, the capacitor discharges with the time constant of the second resistor R_2. Equations and graphic representation of voltage through the capacitor are shown as well as the equations for voltage across the capacitor are shown below:

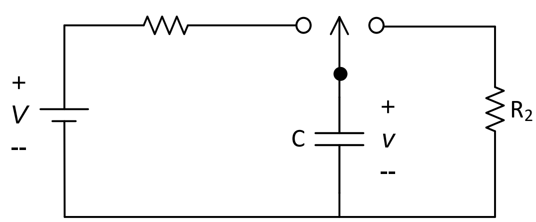
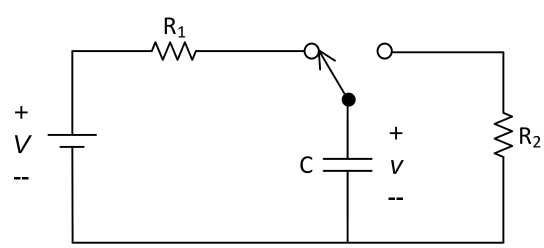
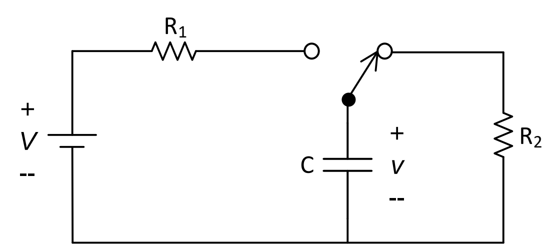
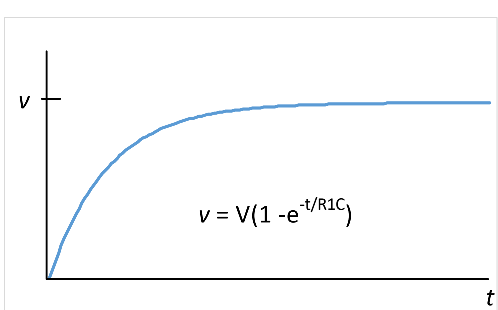
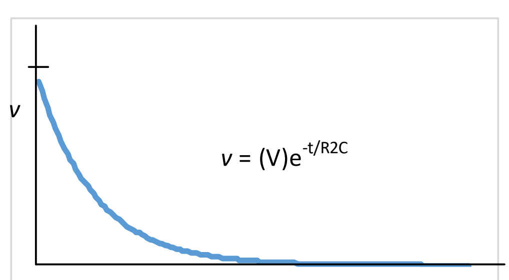


Figure 7-13



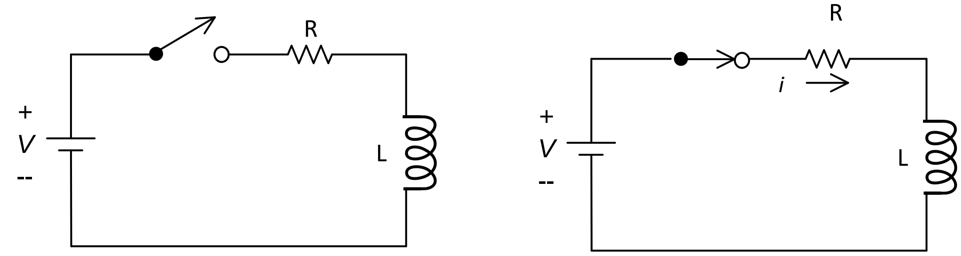


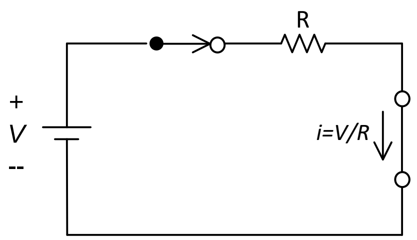




DC Transient R and L Circuits

What happens when a switch closes allowing an inductor to charge? The following shows the circuit following switch closure. Current begins to flow through the resistor until the inductor finally acts as a closed circuit or wire.





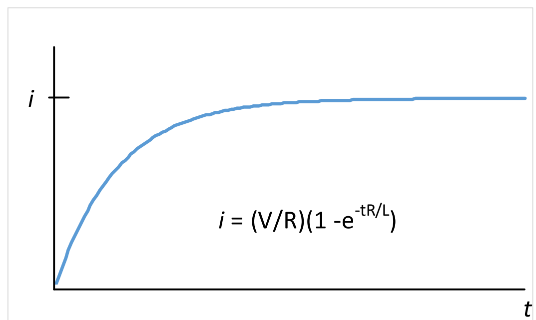


Figure 7-14

The figure above shows a simple circuit with voltage source, resistor and inductor. This circuit has a switch that closes at t = 0. The results of voltage and current are shown in the graph of i. The second circuit shows the circuit with the switch just closed while the third shows the circuit after much time has passed. The current of the circuit flows through the inductor as the graph shows.

The equation for the KVL involves a derivative function (ELI).

$$v = L\frac{di}{dt}$$

For the equation around the loop, when the switch closes:

$$-V + iR + L\frac{di}{dt} = 0$$

from which we get the equation above, $i = (V/R)(1 - e^{-tR/L})$:

The time constant L/R is referred to as τ or tau. The equation is written $e^{-tR/L}$ or $e^{-t/\tau}$.

If the circuit is more complicated, it may be simplified using the Thevenin rules to obtain the circuit at right below:

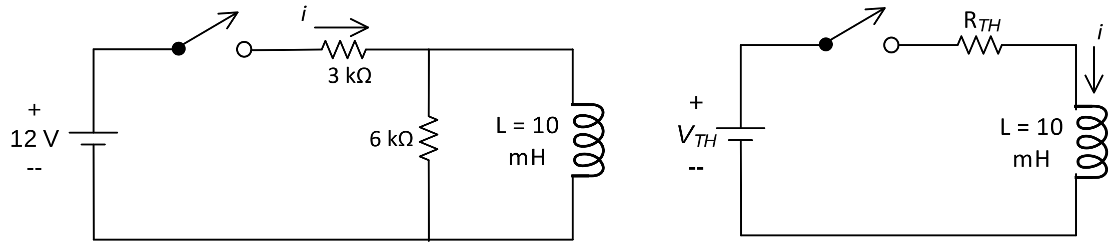


Figure 7-15

The inductor also may appear in a discharge circuit. The circuits below show a switch that may either in the left or right position. In the left position, the inductor is being charged as in the circuits above. With the switch in the right position, the inductor discharges and the current dissipates through the resistor. The resistor heats a little and the current and voltage result at 0.

The current equation through the inductor in the charge and discharge circuits are shown in the figure below. First the switch is moved to the left and after a period of time, switched to the right. Current charges in the inductor as the charging graph shows. Then the switch flips to the right and the inductor discharges through the resistor at the right. The equation for the discharge of current is shown in the graph and equation below the discharge circuit.

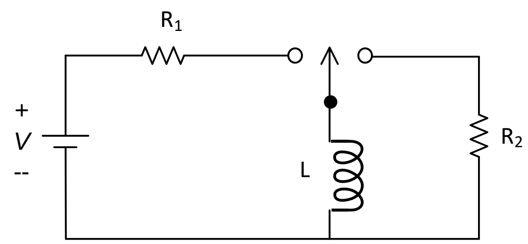
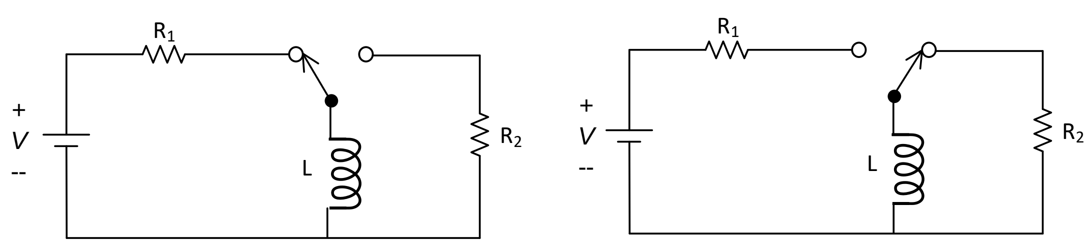
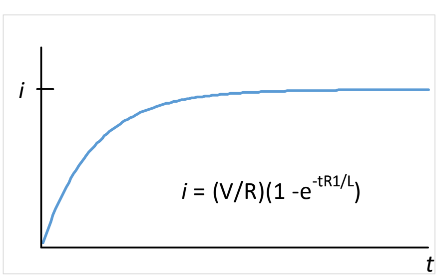


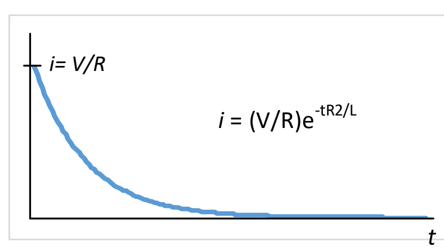
Figure 7-16



Current through the inductor during the charging cycle resembles this graph

Current through the inductor during the discharging cycle resembles this graph





Problems

7-7 For the circuit of Fig. 7-17a, assume the capacitor is uncharged before the switch closes. Write the equation of the voltage across this capacitor and sketch the waveform after t = 0. What is the time constant tau? Also, write the equation for current, *i*, through the circuit.

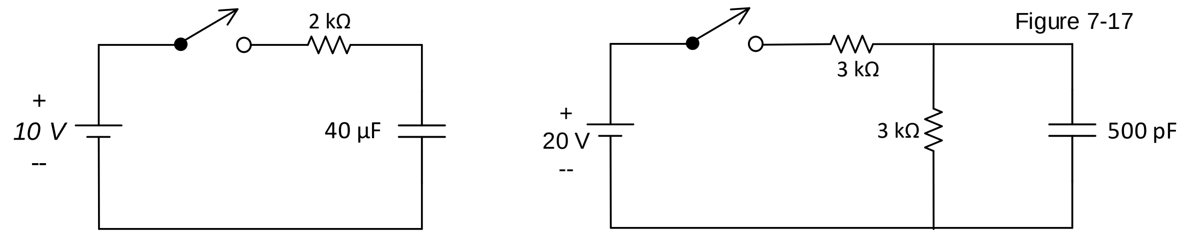


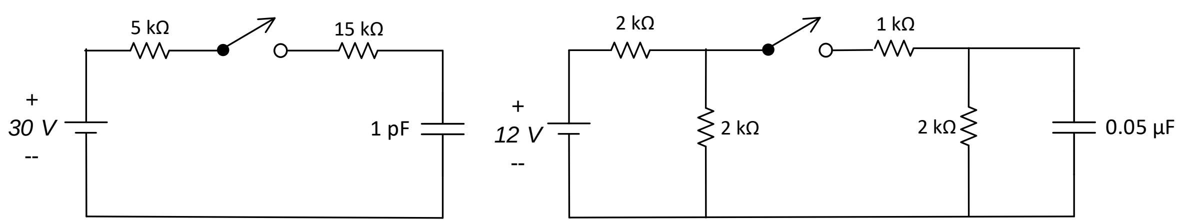
Figure 7-17

(a) (b)

7-8 For the circuit of Fig. 7-17b, assume the capacitor is uncharged before the switch closes. Write the equation of the voltage across this capacitor and sketch the waveform after t = 0. What is the time constant tau? Also, write the equation for current, *i*, through the capacitor. Next write the equation for the voltage across the capacitor and sketch the waveform after the switch re-opens.

7-9 For the circuit of Fig. 7-18a, assume the capacitor is uncharged before the switch closes. Write the equation of the voltage across this capacitor and sketch the waveform after t = 0. What is the time constant tau? Also, write the equation for current, *i*, through the capacitor.

Figure 7-18

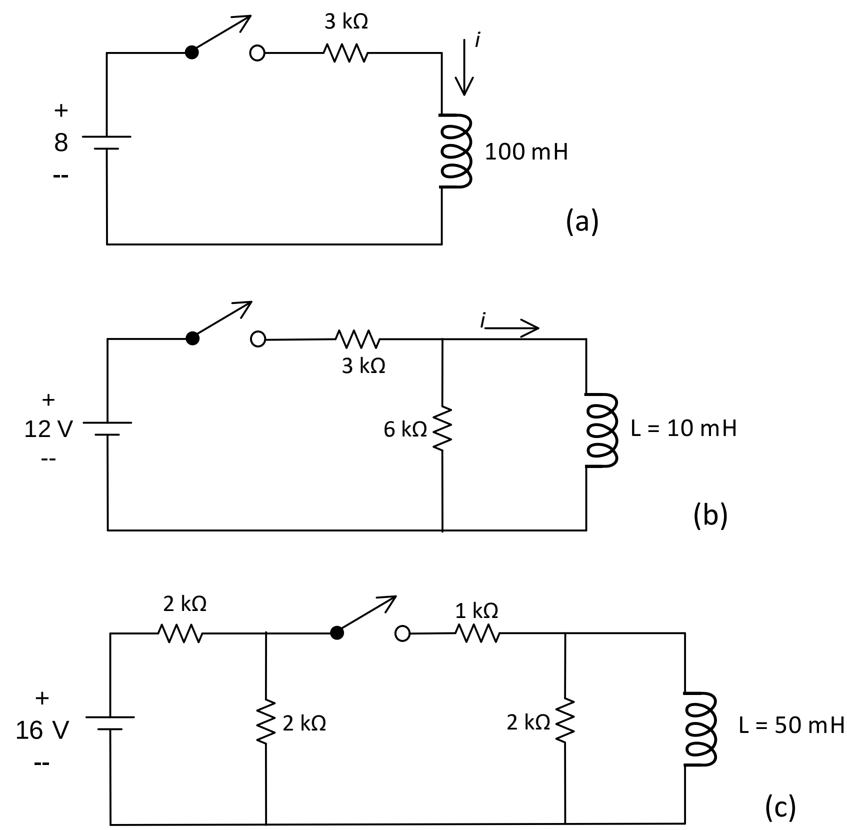


(a) (b)

7-10 For the circuit of Fig. 7-18b, assume the capacitor is uncharged before the switch closes. Write the equation of the voltage across this capacitor and sketch the waveform after t = 0. What is the time constant tau? Also, write the equation for current, *i*, through the capacitor.

7-11 For the circuit of Fig. 7-18b, write the equation of the voltage across this capacitor and sketch the waveform after the switch opens after fully charging. What is the time constant tau? Also, write the equation for current, *i*, through the capacitor.

Figure 7-19



7-12 For the circuit of Fig. 7-19a, write the equation of the current through the inductor and sketch the waveform after t = 0. What is the time constant tau?

7-13 For the circuit of Fig. 7-19b, write the equation of the current through the inductor and sketch the waveform after t = 0. What is the time constant tau? Then write the equation of the current through the inductor and sketch the waveform after the switch re-opens.

7-14 For the circuit of Fig. 7-19b, write the equation of the current through the inductor and sketch the waveform after the switch has been closed for a long time and then opens. What is the time constant tau?

7-15 For the circuit of Fig. 7-19c, write the equation of the current through the inductor and sketch the waveform after t = 0. What is the time constant tau.

7-16 For the circuit of Fig. 7-19c, write the equation of the current through the inductor and sketch the waveform after the switch has been closed for a long time and then opens. What is the time constant tau?