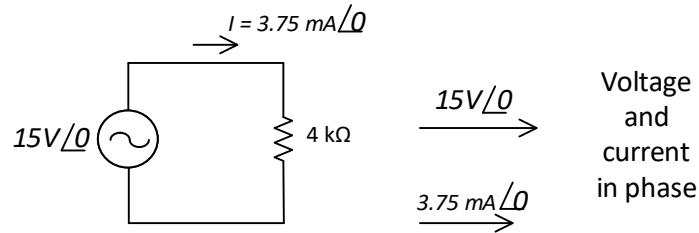


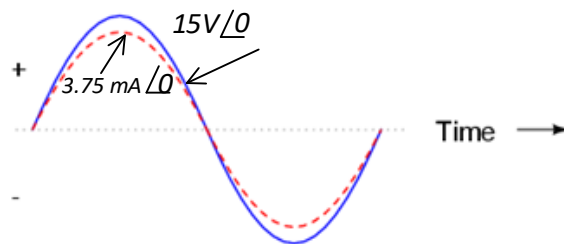
9 AC Resistor-Inductor-Capacitor Circuits

First, AC Resistive Circuits



Pure resistive AC circuit: resistor voltage and current are in phase.

A plot of the sinusoidal wave for voltage and current would be in-phase as shown below:

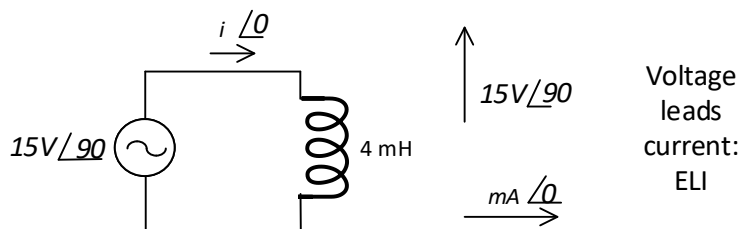


Voltage and current "in phase" for resistive circuit.

We define the relationship of an inductor as v or:

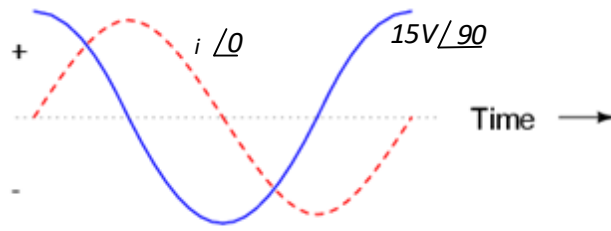
$$e = L \frac{di}{dt}$$

If the sine wave represents the current waveform, the derivative of the sine waveform is the cos waveform. We can apply this to the waveform above to observe that the voltage leads current by 90° in an inductor.



While we can find the phase angle of the current, we cannot find the magnitude unless we have the complete equation for the current waveform. In general:

$$i = a \sin(2\pi ft)$$



Pure inductive circuit

If we take the derivative of i , we find

$$\frac{di}{dt} = a \cdot 2\pi f \cdot \cos(2\pi f t)$$

So we must know the frequency f to find the amplitude of current for the graph above.

So, the difference between resistance and reactance is a phase shift and $2\pi f$. This is represented by the phase shift expression j and $2\pi f$.

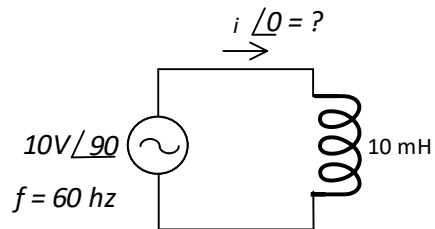
Therefore, when we write the resistance for an inductor, we must know the frequency and remember to include j :

$$X_L = j \cdot 2\pi f L$$

If we have a 10 mH, we can calculate its X_L (reactance):

Frequency (Hertz)	Reactance (Ohms)
60	3.7699
120	7.5398
2500	157.0796

We revisit the circuit from above to find the value of current. It is 2.6526 A at phase angle -90° :



$$I = \frac{V}{X_L} = \frac{10 V}{j3.7699} = -j2.6526$$

Resistance (impedance) of an inductor is: $X_L = j2\pi f \cdot L$

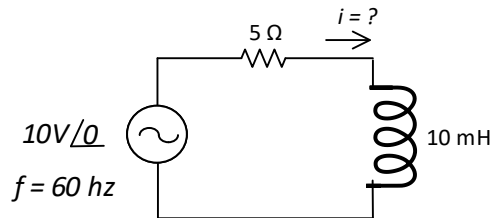
Resistance (impedance) of a capacitor is: $X_C = 1/(j2\pi f \cdot C)$

We will work through several problems using these values to find the unknown value in phasor form. Remember these values:

m = milli or 10^{-3}
 μ = micro or 10^{-6}
n = nano or 10^{-9}
p = pico or 10^{-12}

Series Resistor-Inductor Circuits

Now, we add a resistor to the inductor and keep the inductor's value of impedance:



We calculate the total impedance as the series combination of the resistance of the resistor and the impedance of the inductor. We call the total impedance Z :

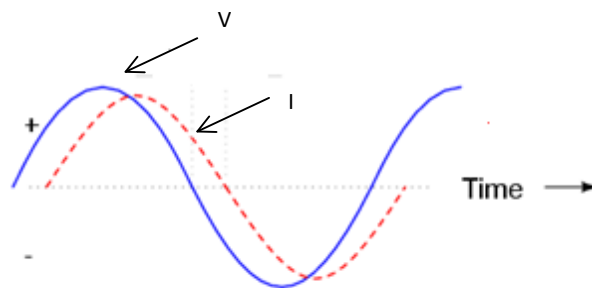
$$Z = (5 + j0) + (0 + j3.7699)\Omega$$

or

$$6.262 \Omega / 37.016$$

$$I = \frac{V}{Z} = \frac{10 V}{6.262 \Omega / 37.016} = 1.597 A / -37.016$$

The relationship of V and I are shown in the figure below. Notice the phase shift with voltage leading and current following:



We can now find the voltage across the resistor and the inductor by multiplying the current by the resistor or inductor's value of impedance. For the resistor, voltage =

$$V = IR = (1.597 A / -37.016)(5 \Omega) = 7.9847 V / -37.016)$$

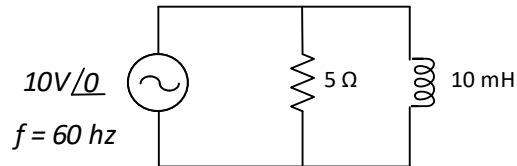
For the inductor, voltage =

$$V = IZ_L = (1.597 A / -37.016)(3.7699 \Omega / 90) = 6.0203 V / 52.984)$$

If we add the two vectors for V_R and V_L , we will find they add to 10 V.

Parallel Resistor-Inductor Circuits

Let's take the same components for our series example circuit and connect them in parallel:

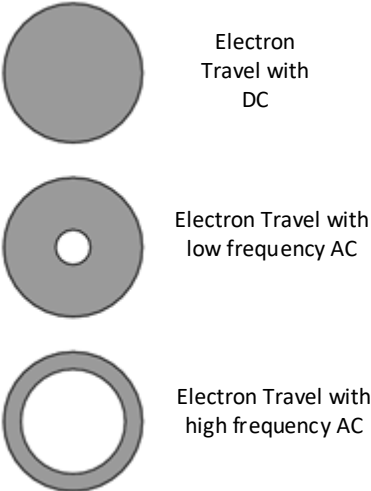


We use the same rules from above but use the parallel rules for Z instead of series rules.

$$Z = \frac{1}{\frac{1}{R} + \frac{1}{X_L}} = \frac{1}{\frac{1}{5\Omega} + \frac{1}{j3.7699}}$$

$$Z = 3.0101 / 52.984$$

We learn from physics that electrons travel uniformly down a wire if the current is dc. As frequency increases, there is a skin effect that only allows electrons to travel closer to the skin. This is seen in the following diagram:



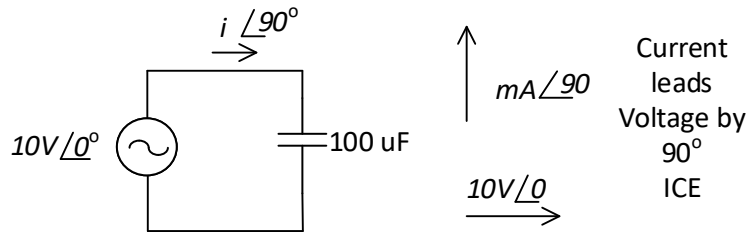
This can be seen with high frequency machinery in which hollow tubing replaces wire for transmission of high frequency currents.



We define the relationship of a capacitor as i:

$$i = C \frac{dv}{dt}$$

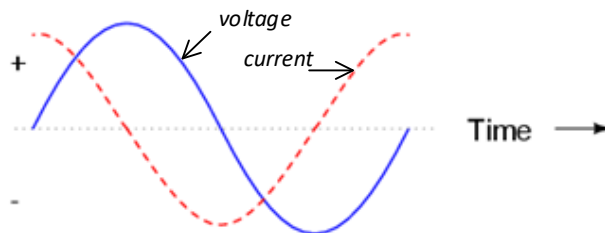
If the sine wave represents the current waveform, the derivative of the sine waveform is the cos waveform. We can apply this to the waveform above to observe that the current leads voltage by 90° in a capacitor.



While we can find the phase angle of the current, we cannot find the magnitude unless we have the complete equation for the current waveform. In general:

$$i = a \frac{d(\sin(2\pi ft))}{dt} = a \cdot 2\pi f \cdot \cos(2\pi ft)$$

We can see the sine waves below with current leading voltage through an ideal capacitor.



So we must know the frequency f to find the amplitude of current for the graph above.

The difference between resistance and reactance is a phase shift and $2\pi f$. This was true for an inductor and also now for a capacitor. This is represented by the phase shift expression j and $2\pi f$.

Therefore, when we write the resistance for an inductor, we must know the frequency and remember to include j :

$$X_C = \frac{1}{j \cdot 2\pi f C} = \frac{-j}{2\pi f C}$$

For a capacitor similar to the one above, we can calculate the magnitude of the reactance, X_C , using the formula above. For the 100 uF capacitor, at various frequencies, the reactance is:

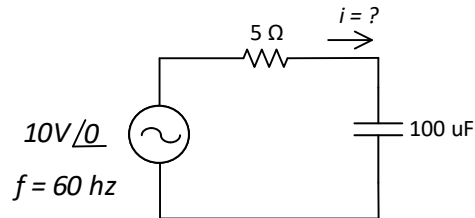
Frequency (Hertz)	Reactance (Ohms)
60	26.5258
120	13.2629
2500	0.6366

Do not forget to include the j in the final solution unless the magnitude is all that is being required. In this case at 60 Hz, we have for the circuit above:

$$X_C = 26.5258$$

$$I = \frac{V}{X_C} = \frac{10\text{ V}}{-j26.5258\Omega} = 0.3770\text{ A}$$

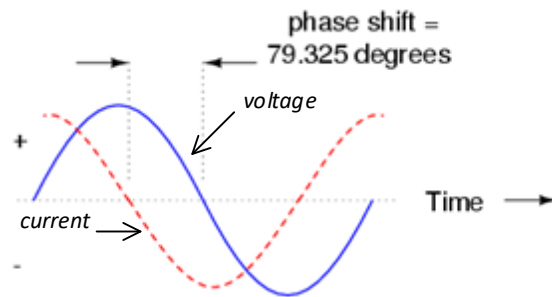
If we add a resistor to the circuit, we have results similar to those of the inductive circuit above.



$$Z_{Total} = 5\Omega - j26.5258$$

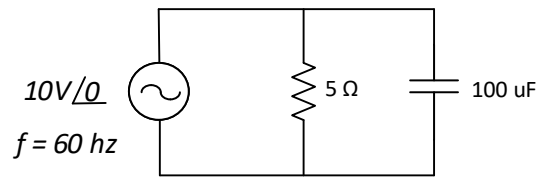
To find i given a voltage, we use:

$$I = \frac{V}{Z} = \frac{10\text{ V}}{5\Omega - j26.5258} = 370.5\text{ mA} / 79.325^\circ$$



Parallel Resistor-Inductor Circuits

Let's take the same components for our series example circuit and connect them in parallel:

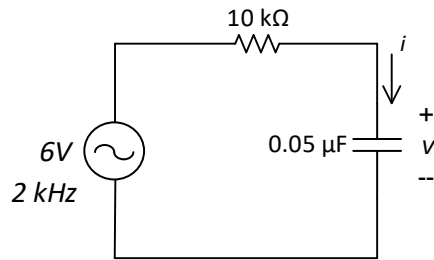


We use the same rules from above but use the parallel rules for Z instead of series rules.

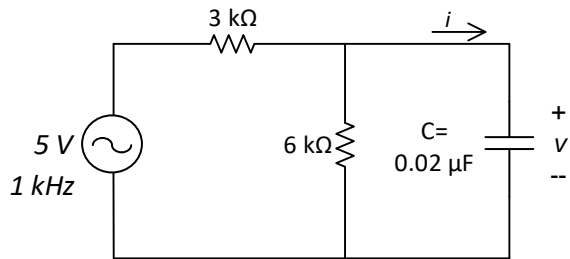
$$Z = \frac{1}{\frac{1}{R} + \frac{1}{X_L}} = \frac{1}{\frac{1}{5\Omega} + \frac{1}{-j26.5258}}$$

The solution is left to the student.

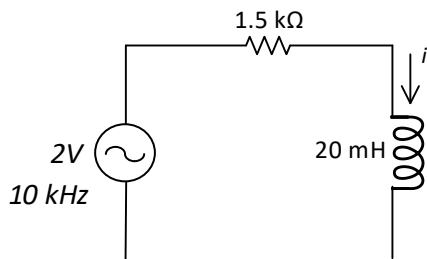
9.1 Find the voltage and current in phasor form for the circuit below:



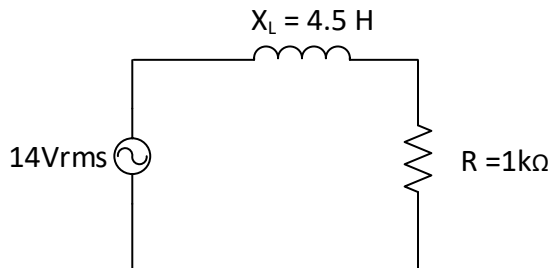
9.2 Find the voltage and current in phasor form for the circuit below:



9.3 Find the voltage and current in phasor form for the circuit below:



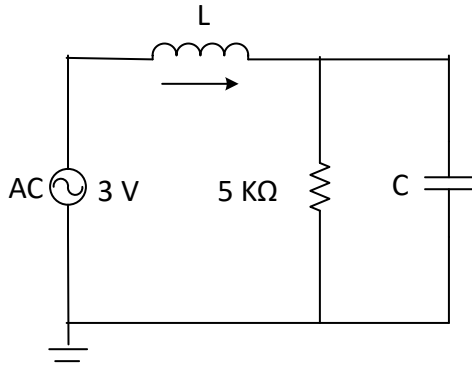
9.4 Use the following values of L , f and R to find the voltage in phasor form across the resistor (V_R) if frequency $f = 140$ Hz



9.5 For the following, find the current through the inductor. Write your answer in polar form.

$$X_L = j 2 \text{ K}\Omega$$

$$X_C = -j 2.5 \text{ K}\Omega$$



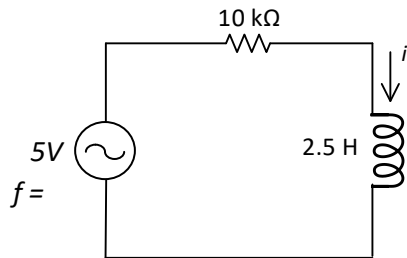
9.6 Find the voltage and current in phasor form for the circuit below if:

$$f = 100 \text{ hz}$$

$$f = 500 \text{ hz}$$

$$f = 1 \text{ kHz}$$

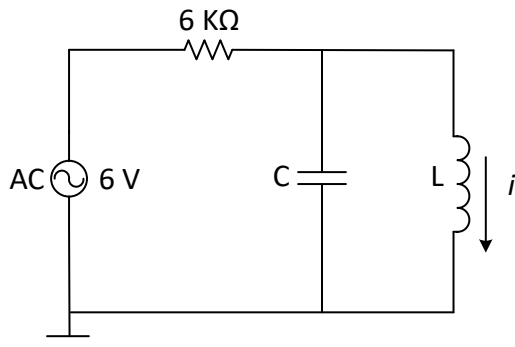
(this is the calculations for Lab 9 in the lab manual)



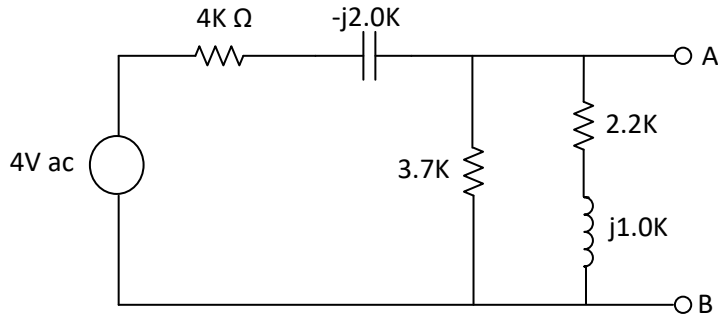
9.7 For the circuit below, find the voltage and current through the inductor L, the capacitor C:

$$X_L = j 4 \text{ K}\Omega$$

$$X_C = -j 3.5 \text{ K}\Omega$$



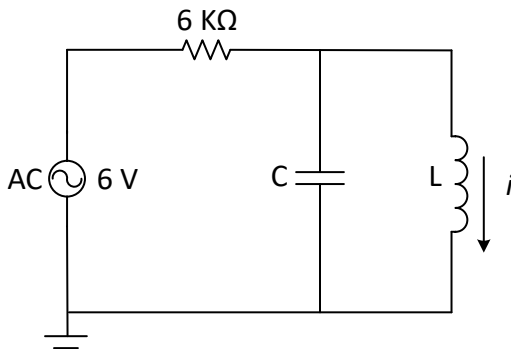
9.8 For the circuit below, find the voltage and current for a 5K resistor attached across A-B.



9.9 For the circuit below, find the current through the inductor L, the capacitor C:

$$X_L = j 4 \text{ K}\Omega$$

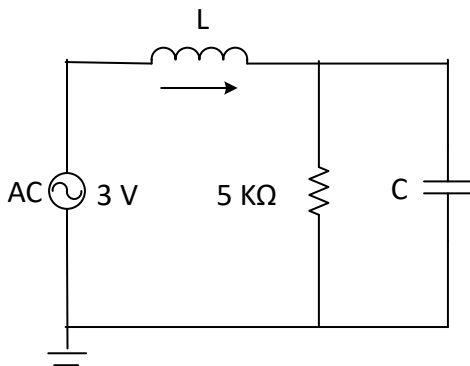
$$X_C = -j 3.5 \text{ K}\Omega$$



9.10 For the circuit below, find the current through the resistor, the capacitor C:

$$X_L = j 2.2 \text{ K}\Omega$$

$$X_C = -j 2.7 \text{ K}\Omega$$



AC Power

Electrical Motor HP Equation

1 hp (English horse power) = 745.7 W = 0.746 kW = 550 ft lb/s = 2.45 Btu/h = 33.000 ft lb/m

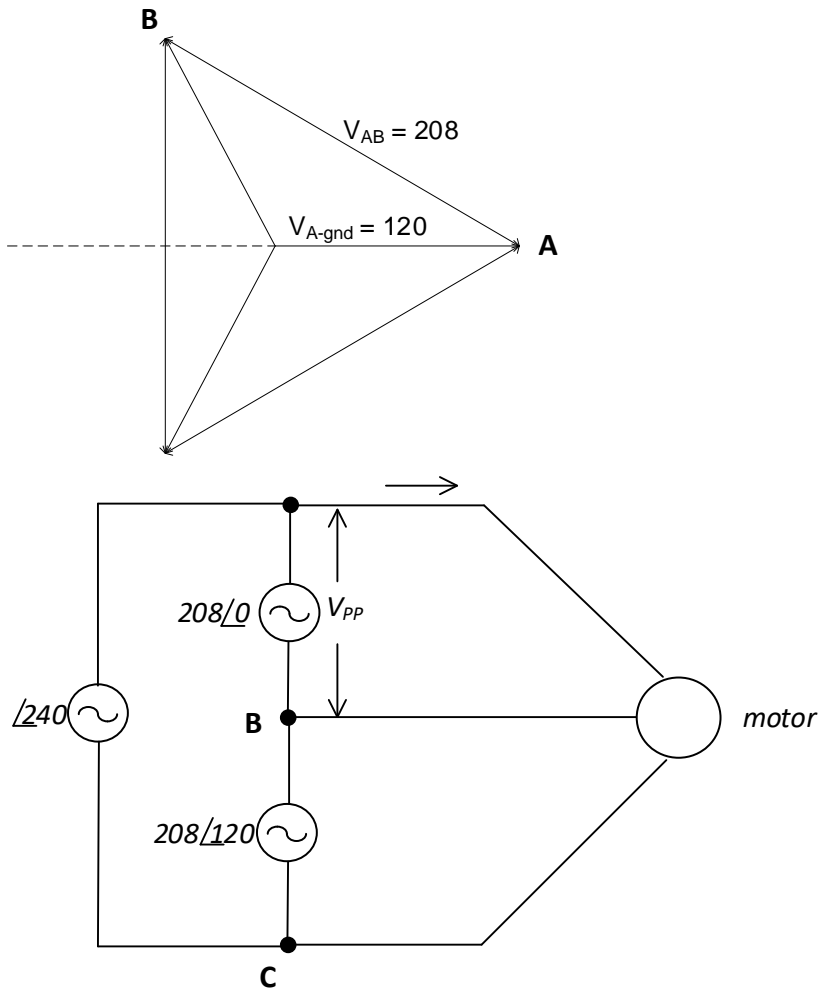
Real Power

$$P_{real} = \sqrt{3} \times V_{pp} \times I_p \times \cos\theta$$

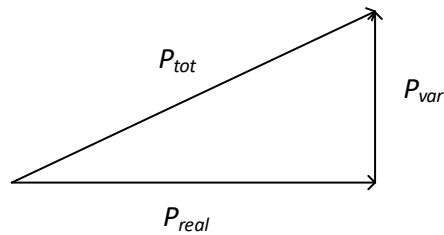
For purely resistive load: PF = $\cos\theta = 1$

Brake Horsepower

$$P_{BHP} = (\sqrt{3} \times V_{pp} \times I_p \times \cos\theta) / 746$$



Electrical Power Equations for Single Phase:

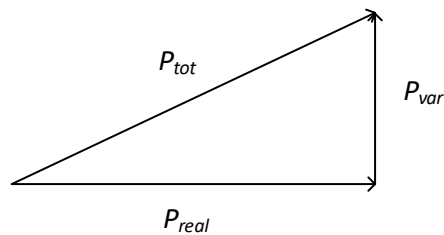


$$P_{real} = V \cdot I \cos \theta$$

$$P_{var} = V \cdot I \sin \theta$$

$$P_{tot} = V \cdot I$$

Electrical Power Equations for Three Phase:



$$P_{real} = \sqrt{3} V \cdot I \cos \theta$$

$$P_{var} = \sqrt{3} V \cdot I \sin \theta$$

$$P_{tot} = \sqrt{3} V \cdot I$$

- 9.11 For a single-phase circuit, the voltage is 150 V, current is 10 A and phase angle θ is 35° . Find the three values:

$$P_{tot}$$

$$P_{real}$$

$$P_{var}$$

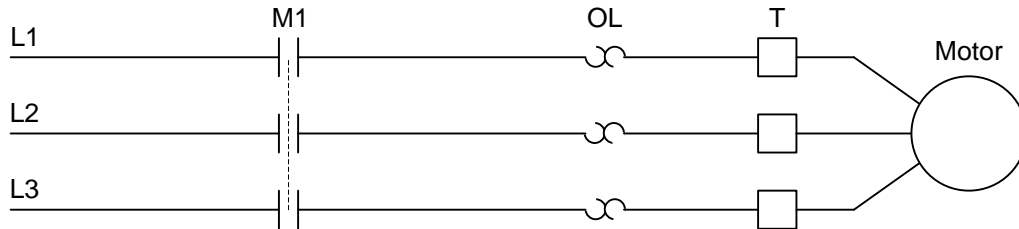
- 9.12 For a three-phase circuit, the phase-to-phase voltage is 460 V, phase current is 15 A and phase angle θ is 25° . Find P_{real} :

Find P_{BHP} for the circuit:

- 9.13 For a single-phase ac circuit with $V = 50$ v, $I = 4.5$ A and phase angle $\theta = 35^\circ$, find P_{real} , P_{tot} and P_{var} .
- 9.14 For a three-phase ac circuit with $V = 220$ v, $I = 12.5$ A and phase angle $\theta = 32^\circ$, find P_{real} , P_{tot} and P_{var} . Then find the horsepower (hp) of the circuit.

Motor Starters and Motor Starter Circuits

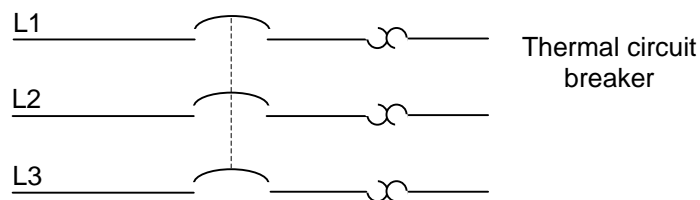
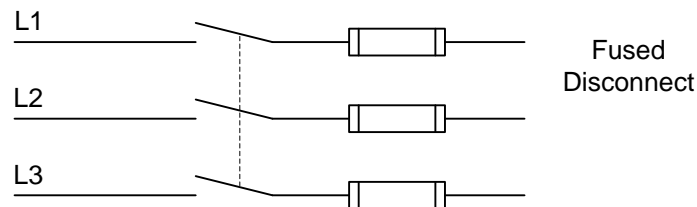
The inclusion of the motor starter circuit in a diagram includes many additional parts to the schematic. They deal with the power portion of the motor starter circuit. The basic circuit for starting a motor is as follows:

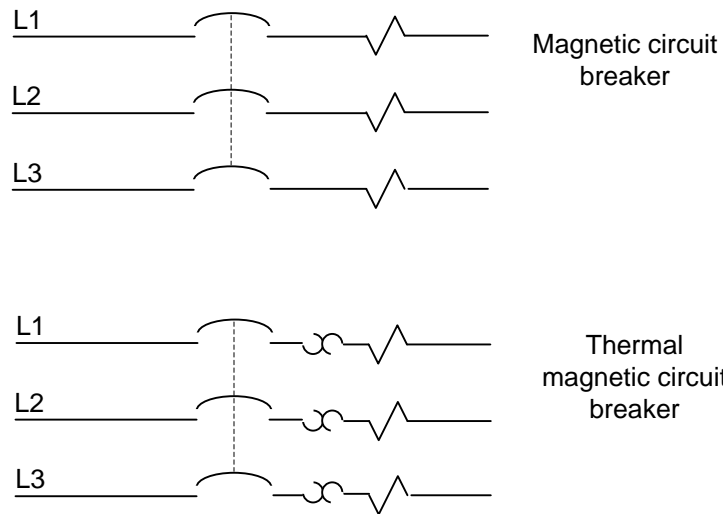


Typical Motor Starter Schematic

The motor contactor is labeled M1 in this example. The motor contactor is referenced from the coil of the motor starter circuit located in the ladder diagram for the motor. OL refers to the three overload circuits for this motor. Overloads are set to trip at a slightly higher value of current than the full load current of the motor. "T" refers to the T leads, marked T1, T2, and T3, the final termination points prior to the leads for the motor. Then the motor is shown. On most schematic drawings, the motor name and horsepower (HP) rating of the motor are included.

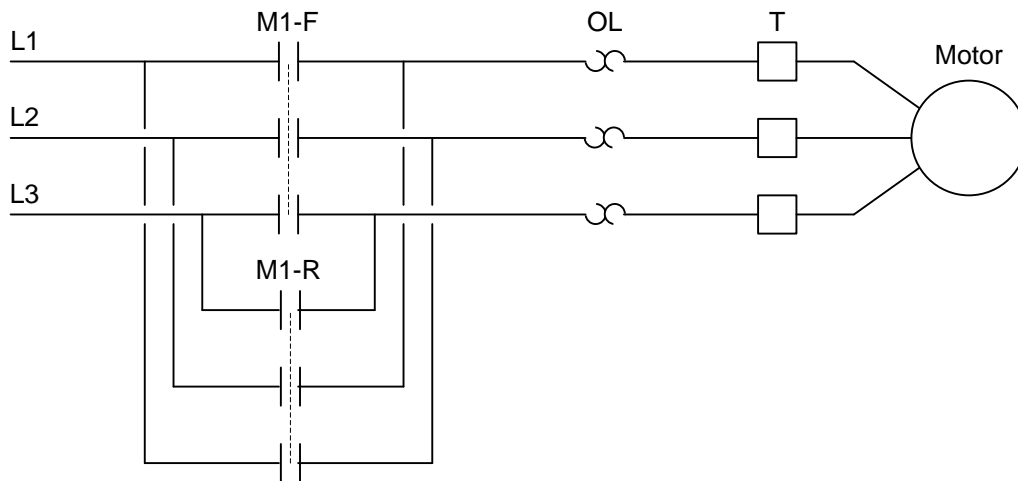
Prior to the motor contactor is a disconnect and either a fuse or circuit breaker. These elements are shown for each line of power L1, L2, and L3. They are shown with a tie element to show that if one of the three overload elements trip, all trip or disconnect.





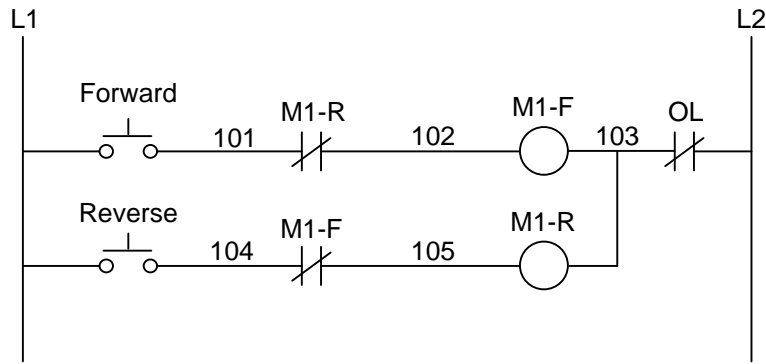
Types of Motor Disconnects

To reverse a three-phase squirrel cage motor, add the following to the motor control circuit:



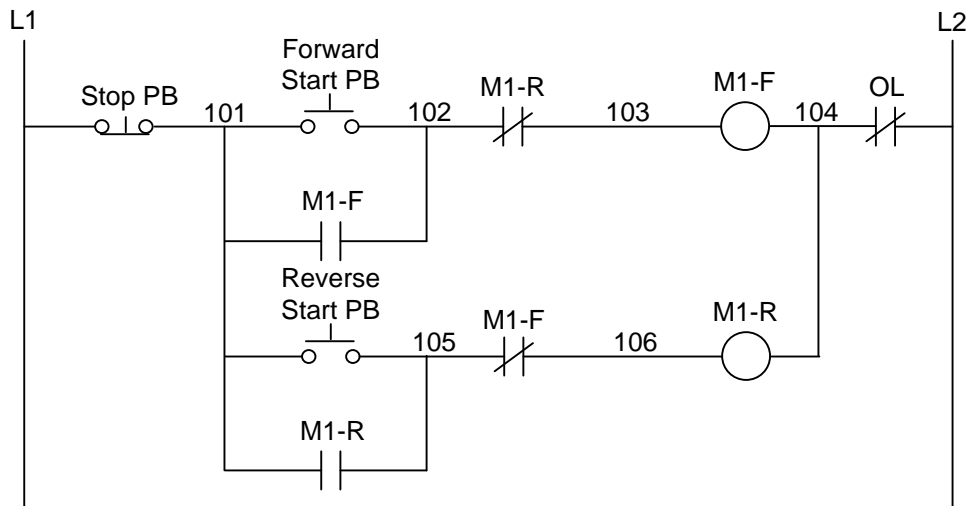
Reversing Motor Starter Example

Notice that to reverse a three-phase motor, switch any two leads to the motor. The motor starter must guarantee that both M1-R and M1-F are not turned on at the same time. If this happens, the contactors will short and a fuse will blow or the circuit breaker will trip. A motor circuit could be designed to start one contactor only as follows:



Reversing Motor Simple Control Example

The design uses a seal circuit. To incorporate the seal circuit, a holding contact must be used around the start push button for each the forward and reverse motor circuits.



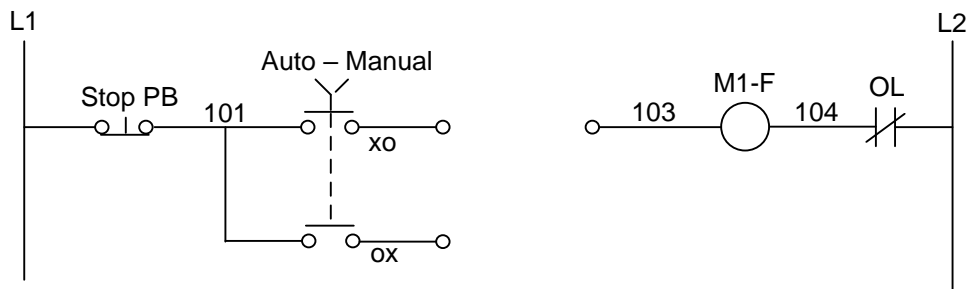
Reversing Motor Standard Control Example

Forward-Reverse motor starter circuits use a mechanical toggle bar so both contactors cannot be energized simultaneously. This provides a mechanical fail-safe to the electrical circuits.

While electrical students may never be required to actually wire a motor starter to control a motor, the experience of wiring a motor starter control circuit is of value. Wiring and debugging 120 VAC logic provides a greater challenge because of the higher possibility of electrical shock. Care must be taken to wire the circuit with no short-circuits in it. Troubleshooting of the circuit must only be done with the power off in a laboratory environment. It is acknowledged that most applications in the manufacturing environment are checked with the power on (the circuit is hot).

Various types of automatic and manual control circuits can be designed to start the motor

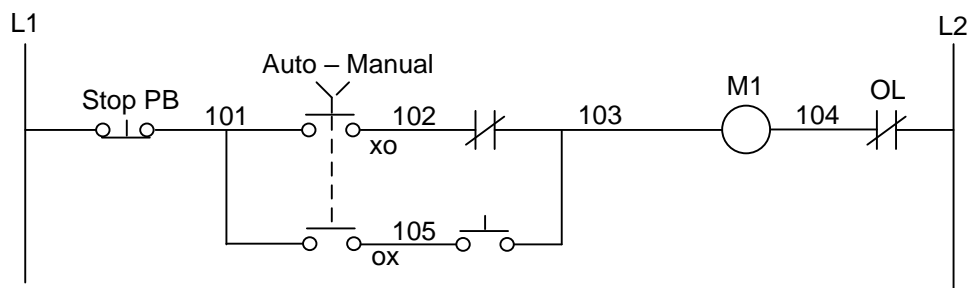
circuit. For example, the Auto/Manual selector switch is available to allow the end user to choose which mode is to be used when running the motor. It is constructed as follows:



Use of Auto/Manual Switch

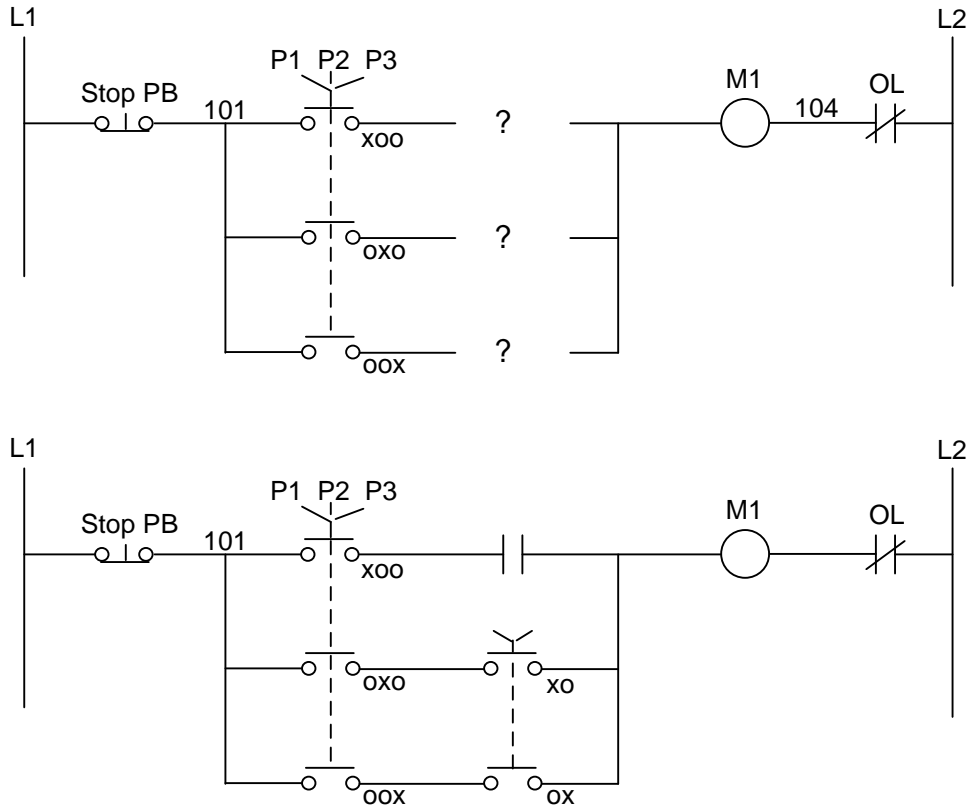
The Auto/Manual switch allows the user to choose a path for control. The xo and ox nomenclature is a handy means of labeling the position of the switch for which the contact is closed. In the figure below, the upper path is closed when the switch is in Auto and the lower path is closed in Manual.

The following schematic shows a possible design using the Auto/Manual selector switch. In automatic, the motor circuit responds when the relay contact turns off while in manual, the motor turns on when the push button is energized.



Use of Auto/Manual in Circuit

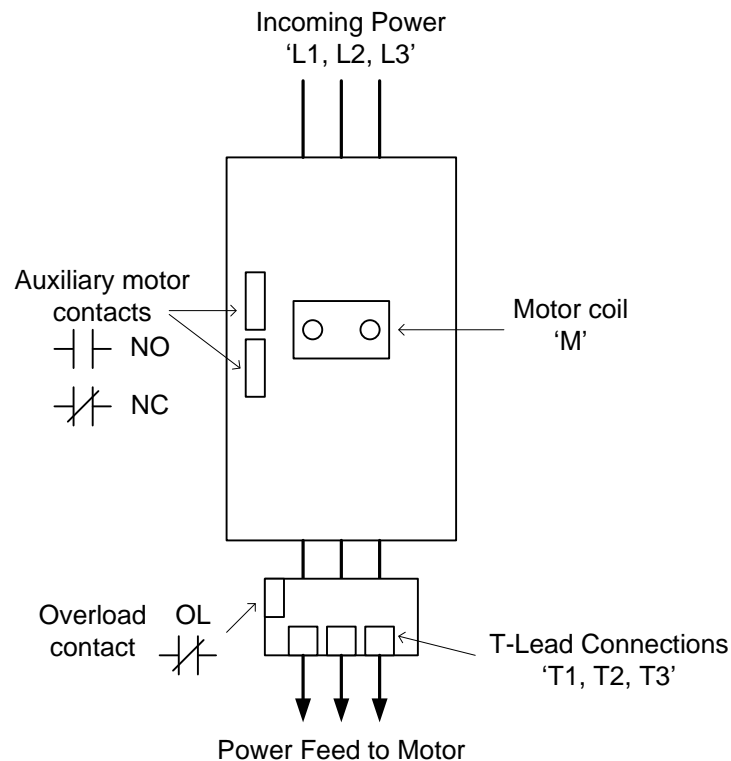
Multiple position switches, push buttons and other control contacts may be inserted to control the more complex motor circuit. Example circuits shown below are indicative of the kind of circuits designed in industry.



Use of Multiple Position Switches

To design a motor starter circuit, it must be decided whether or not a memory circuit is to be used. If a jog switch is used, no seal circuit is required since the jog switch implies holding the button down to move the motor. Later, a jog switch is incorporated into a memory circuit to allow either the motor to be started and stopped or jogged. Switches are used to choose modes of operation and separate one mode from another.

Wiring the starter:



Motor Starter Wiring Layout

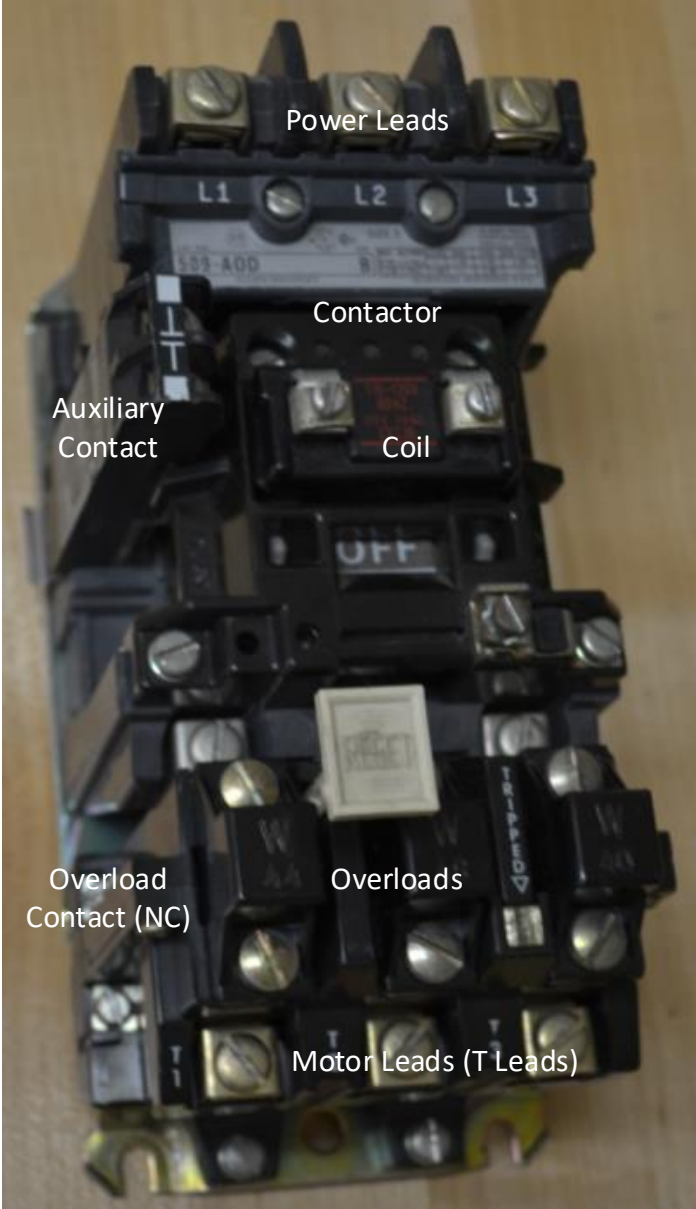
Three phase contactors have three contacts that simultaneously close to start the motor and open to stop the motor. To reverse the motor, the opposite contactor must be guaranteed to be open (checked mechanically and electrically) and any two of the three leads are reversed for reverse operation of the motor.

Motor contactors are part of an overall control circuit to guarantee that the motor is protected from burning up and that the wire is protected from short circuit conditions. Ahead of the motor starter contactor is found a set of three fuses or a circuit breaker capable of interrupting the current from a fault in the motor, the wiring or any part of the circuit downstream from the circuit breaker. Usually fuses are used since they tend to be cheaper. Fuses are also used due to their overall high fault interrupting ratings. Circuit breakers are chosen when it is critical that circuits be reset quickly and downtime searching for a replacement fuse is unacceptable. Circuit breakers are also used since fuses may allow a motor to burn up if a single fuse blows and the motor runs for a short while on a single phase. Overall fault current must be checked when selecting a circuit breaker for motor starting applications, however.

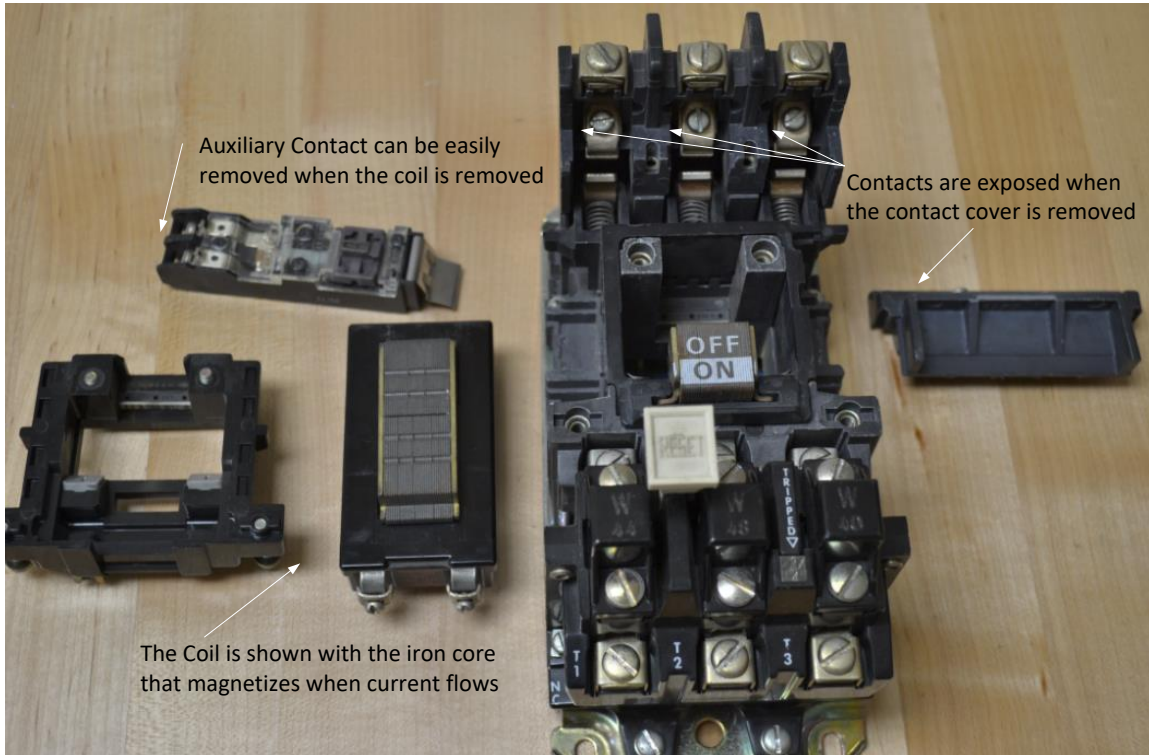
Overloads are used to protect the motor from low over-current conditions. The overloads are set to trip at a current rating slightly higher than the full load current of the motor. Overloads are calibrated to not trip with a rise in the ambient temperature but with a rise in the current of

the motor creating additional heat in the overload relay. In the figure below, an Allen-Bradley motor starter is shown. Labels indicate the various parts of the starter.

The following diagram shows the major components of a Motor Starter:

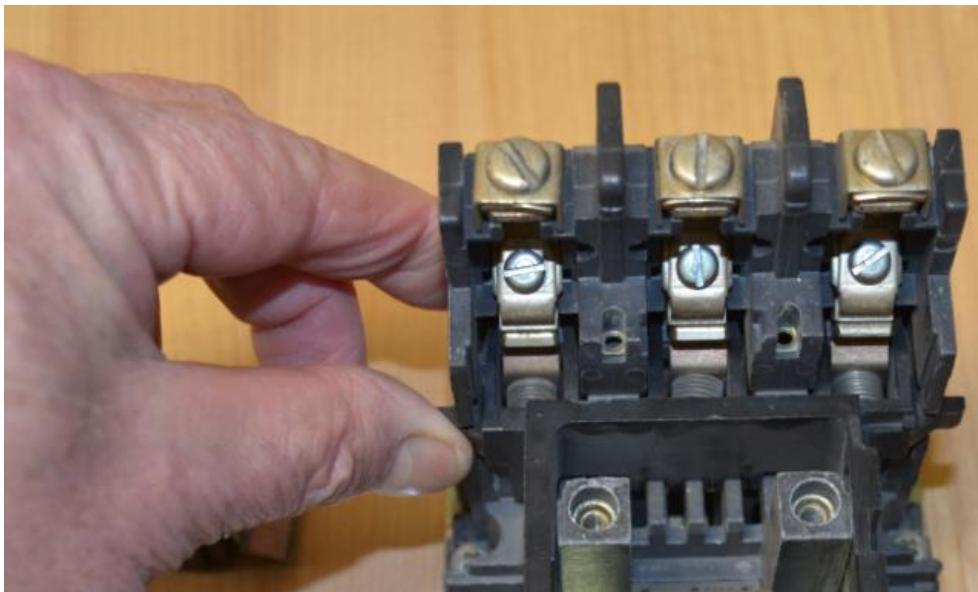


Motor Starter with several components removed and contactor exposed.



Motor Starter Exposed Above/Contacts Closed Manually Below

Contacts shown in engaged position. This happens when the coil conducts. The motor will start and run in this condition.



Aside from NEC

The National Electrical Code is not the focus of this chapter but should be read by every electrical student before entering the work force. This book deals with proper and safe installation of electrical equipment. Included are rules for installation and design as well as tables for design of electrical equipment. For example, Chapter 9 includes Table 8 defining the ohms per foot rating for different wire size.

The table lists various AWG (American Wire Gage) size wire, both solid and stranded and the ratings in ohms/km. Some are listed below:

#14 stranded	10.3 ohm/km
#12 stranded	6.50 ohm/km
#10 stranded	4.070 ohm/km
#8 stranded	2.551 ohm/km

Converted to ohm/ft,

#14 stranded	3.1212E-3 ohm/ft
#12 stranded	1.9697E-3 ohm/ft
#10 stranded	1.233E-3 ohm/ft
#8 stranded	7.730E-4 ohm/ft

Example Using NEC for Electrical Calculations

Use these values to calculate the voltage drop in the various wire choices for 1Amp. If the voltage drop is to be less than 10 V., then

$$10V = 1 A \times \text{ohm/ft} \times (\text{number of ft})$$

For a 5 A load, the formula changes to:

$$10V = 5A \times \text{ohm/ft} \times (\text{number of ft})$$

Note that if the circuit is 110 V and single phase, the length out and back to a device must be included.

For example, if a solenoid is capable of turning on with an inrush of 5 A and has a holding current of .2 A, what is the maximum length that #14 stranded may be used for a voltage drop of 10 V.

Answer: $10V = 5.0 \times 3.1212E-3 \times \text{Ft}$ Ft = 641.76 total or apx. 320 ft to the device



This work is licensed under a Creative Commons Attribution 4.0 International License.