

Introduction

1.1. Electrical transmission systems.

The electrical transmission of signals and power is perhaps the most vital single contribution of engineering technology to modern civilization.

Among its visible manifestations, the most impressive are the high voltage transmission lines on tall steel towers that cross the countryside in all directions. Carrying thousands of megawatts of power, these lines link remote generating stations to urban load centers, or unite into cooperative complexes the power production facilities of large geographical areas.

Equally obvious and important, if neither impressive nor attractive, are the millions of miles of pole-lines that parallel city streets, rural highways and railways everywhere. The conductors of these lines may be delivering kilowatts or megawatts of power to domestic and industrial users, or may be transmitting telephone, teletype and data signals, at milliwatt power levels and kilohertz frequencies, over distances usually not exceeding a few dozen miles.

Every radio and television receiver is a terminal of another kind of electrical transmission system, in which the signal power at megahertz frequencies propagates freely from a transmitting antenna through the earth's atmosphere, guided only by the conductivity of the earth's surface, or by that of the ionospheric layers of the upper atmosphere. An essential but unobtrusive component of most television receiver installations is an electrical transmission line that conveys signal power at the picowatt or nanowatt level from the antenna to the receiver terminals.

At microwave frequencies of several gigahertz, wavelengths are small and antennas in the form of arrays, horns or paraboloidal "dishes" can have apertures many wavelengths wide. This makes possible the confinement of microwave signals or power into directed beams with small divergence angles. Towers supporting such antennas can be seen at 20 to 50 mile intervals in most regions of the United States. The electrical transmission systems of which they are a part carry all types of communication signals, including television video, telephone, data and control, at power levels of a few watts, over distances up to a few thousand miles. Similar antennas with even greater directivity are employed in microwave radar stations, in the uhf scatter-propagation circuits of the Arctic, and for communication, control and telemetering in all satellite projects, whether terrestrial, or exploratory in outer space.

Unknown to most laymen, a substantial amount of the electrical transmission of signals and power occurs on buried transmission circuits. The greatest mileage of these is in the form of twisted pairs of small wires, paper or plastic insulated, which are packed by the hundreds into cables in underground conduits. Carrying telephone, teletype and data signals within the exchange areas of cities, some of these lines operate at voice frequencies, others at carrier frequencies as high as 100 kilohertz and still others are multiplexed with pulse code modulation utilizing frequencies up to 1 or 2 megahertz. Also in city areas, partly buried and partly airborne are the vhf coaxial or shielded pair transmission lines handling the signals of multichannel cable television services.

Between cities, in the United States and in many parts of Europe, buried coaxial transmission lines operating at frequencies up to several megahertz carry all types of communication signals from teletype to television over distances of hundreds or thousands of miles, providing better security against enemy action, natural catastrophes and various kinds of signal interference than is offered by microwave link circuits.

The reliable worldwide intercontinental communication provided via terrestrial satellites is backed up by thousands of miles of less romantic but equally sophisticated carrier frequency circuits on submarine coaxial cable, with built-in amplifiers every few miles, that cross most of the oceans of the world.

Finally, in addition to all the relatively long transmission systems that have been enumerated, the totality of electrical transmission engineering includes the endless variety of shorter transmission line segments that perform many different functions within the terminal units of the systems. Ranging in length from a few millimeters in microwave circuits to inches or feet or hundreds of feet in devices at lower frequencies, and serving not only as transmission paths but in such other applications as resonant elements, filters and wave-shaping networks, many of these transmission line circuit components present more challenging design problems than all the miles of a long uniform line.

From the above summary, it can be seen that electrical transmission systems fall structurally into two distinct groups, according to whether the signals or power are guided along a path by a material "line", or propagate in the earth's atmosphere or space.

The attention of this book is confined solely to systems in the first of these two groups, and more specifically to systems consisting of two uniform parallel conductors, as defined in Chapter 2. The methods developed are applicable equally to two-wire low frequency electric power transmission lines, to telephone and data transmission lines of all types at all frequencies, and to all of the vhf, uhf and microwave two-conductor lines used in contemporary electrical engineering, at frequencies up to many gigahertz.

1.2. Analytical methods.

Electrical engineers are fully aware that, in principle, a complete analysis of any electrical problem involving time-varying signals can be made only through the use of electromagnetic theory as expressed in Maxwell's equations, with explicit recognition that the electric and magnetic fields throughout the region of the problem are the primary physical variables. In studying the modes of propagation of microwaves in hollow metal pipes, the radiation properties of antennas, or the interaction of electromagnetic waves with plasmas, for example, electrical engineering students always make direct use of electromagnetic theory.

However, it is not possible to solve the electromagnetic integral or differential equations either conveniently or rigorously in regions containing or bounded by geometrically complicated metal or dielectric structures, and the basic analytical discipline of electrical engineering curricula has from the beginning been that of lumped element circuit analysis, not electromagnetic theory. This employs the idealized concepts of two-terminal resistances, inductances and capacitances to represent the localized functions of energy dissipation, magnetic-field energy storage, and electric-field energy storage, respectively. Voltages and currents, which are related by integral or differential expressions to electric and magnetic fields, are the primary electrical variables.

The method is an adequate substitute for electromagnetic theory when the occurrences of the three functions mentioned can be separately identified, and when the dimensions of a circuit are sufficiently small that no appreciable change will occur in the voltage or current at any point during the time electromagnetic waves would require to propagate through

the entire circuit. The size criterion is obviously a function of frequency. At the power line frequency of 60 hertz, the methods of lumped element circuit analysis are applicable with high accuracy to circuits several miles long, while at microwave gigahertz frequencies the same methods may be useless for analyzing a circuit less than an inch across.

In addition to the techniques of electromagnetic theory and of lumped element circuit analysis, electrical engineers make use of a third analytical procedure for electrical problems, which combines features that are separately characteristic of each of the other two methods. It extends the application of the concepts of lumped element circuit analysis to circuits which can be indefinitely long in one dimension, but which must be restricted and uniform in the other dimensions throughout their length. The analysis discloses propagating waves of the voltage and current variables, analogous to the waves of electric and magnetic fields that are solutions of Maxwell's equations. The method is known as distributed circuit analysis.

The principal subject matter of this book is the application of distributed circuit analysis to uniform two-conductor transmission lines. Frequent use is necessarily made of lumped-element circuit concepts and methods, particularly when dealing with situations at transmission line terminals, and electromagnetic theory is required to develop expressions relating the distributed circuit coefficients of a uniform line to its materials, geometry and dimensions.

By judicious conversion of symbols, much of the theory presented in this book can also be applied directly to the analysis of any other physical forms of uniform one-dimensional transmission systems. Examples are the propagation of plane transverse electromagnetic waves in homogeneous media, and mechanical wave transmission systems including physical and architectural acoustics, underwater sound, and vibrations of strings, wires and solid rods.

1.3. The evolution of electrical transmission systems.

A brief review of the historical development of electrical transmission engineering can help to explain several features of the present scene.

The subject had an improbable beginning in 1729 when Stephen Gray, a 63 year old pensioner in a charitable institution for elderly men, discovered that the electrostatic phenomenon of attraction of small bits of matter could occur at one end of a damp string several hundred feet long when an electrostatically charged body (a rubbed glass tube) was touched to the other end. He concluded that "electric effluvia" were transmitted along the line.

Sixty years earlier, councillor Otto von Guericke of Magdeburg, Germany, (famous for the evacuated Magdeburg hemispheres that teams of horses could not pull apart) had noted that short threads connected at one end to his primitive electrostatic machine became charged throughout their length, but no one deduced the concept of electrical transmission from this observation until after Gray's time.

Gray established that electrostatic transmission occurred along his moist packthread lines if they were supported by dry silk threads, but not if they were supported by fine brass wires. This first distinction between electric conductors and electric insulators was further developed in the succeeding five years by French botanist Charles DuFay, who also reported the existence of two different kinds of electricity, eventually labeled positive and negative by Benjamin Franklin in 1747.

Only 24 years elapsed after Gray's experiments before the inevitable armchair inventor proposed in the "Scots Magazine" of Edinburgh for February 17, 1753, that an electrical communication system for use over considerable distances might be constructed by employing a transmission line of 26 parallel wires (each wire identified at each end for one letter of the alphabet) supported at 60-foot intervals by insulators of glass or "jewellers' cement". A sequence of letters was to be transmitted, using Gray's technique, by touching a charged

object to each of the appropriate wires in turn. At the receiving end, bits of paper or straw would be seen jumping successively to the ends of the corresponding wires. Between 1770 and 1830 several electrostatic telegraph systems were constructed in various parts of the world, over distances up to a few miles. None of these achieved any practical success, but the earlier ones may be said to have firmly established the transmission line concept.

Volta's discovery of the chemical pile in 1800 and Oersted's discovery of the magnetic effect of a current in 1820 resulted in the experimental magnetic telegraphs of Gauss, Henry and others in the early 1830's and these were followed by the first commercial electromagnetic telegraphs of Wheatstone and Cook in England in 1839, and of Morse in America in 1844. In both cases, transmission lines of buried insulated wires were tested first, but with poor results, and open wire lines on poles or trees were quickly adopted. By 1850 there were thousands of miles of fairly crude telegraph transmission lines in operation in the United States and Europe. Their design and construction was purely empirical. No test instruments existed, and even Ohm's law was unknown to most of the "electricians" of the time.

As telegraph land-lines were extended, the need arose for underwater cables across rivers, lakes, and larger expanses of water. In spite of the inadequacies of the insulating material available at the time, a successful 40-mile submarine cable was laid across the English Channel in 1851, and within two or three years submarine cables up to 300 miles in length had been laid in various parts of Europe. The operation of these long underwater telegraph circuits soon revealed a new transmission phenomenon, that of signal distortion. The received signals, recorded as pen traces on paper tapes, lost the squareness familiar on land-line circuits and became blurred attenuated waverings of a jittery baseline.

Promoters were naturally stimulated by the prospect that a transatlantic submarine cable might provide the first instantaneous communication link between Europe and America, but they hesitated to risk the large amount of capital required without some reasonable assurance that useful signals could traverse an underwater cable many times longer than the longest then in use. For advice they turned to William Thomson, professor of natural philosophy at the University of Edinburgh. This may be the first instance of the employment of a professional consultant on a major commercial venture in electrical engineering. Thomson (later Sir William and ultimately Lord Kelvin) carried out in 1855 the first distributed circuit analysis of a uniform transmission line. He represented the cable by series resistance and shunt capacitance uniformly distributed along its length. He fully appreciated that a more complete investigation of the cable's signal transmission properties might require attributing distributed series inductance and distributed shunt conductance (leakage) to the cable. By trial calculations, however, he found that at telegraph-signal frequencies the effects of the inductance would be negligible, and from measurements on cable samples he satisfied himself that leakage conductance could be kept low enough to be unimportant. Thomson's published paper of 1855 makes profitable reading for electrical engineers even today.

Thomson provided additional assistance of a more practical nature to the cable promoters by inventing more sensitive receiving galvanometers than any in existence, and by directing the manufacture of purer copper, with conductivity several times greater than that of the commercial metal then available. After various mechanical difficulties were surmounted, a cable designed to Thomson's specifications was successfully laid across the Atlantic in 1858, and carried messages for a few weeks before the insulation failed. Further financing of the cable project was delayed by the Civil War in the United States, but in the decade after 1866 numerous cables were laid, some of which were still in operation as telegraph circuits until fairly recently.

The technological problems encountered in the design and operation of long submarine cables were so much more sophisticated than those associated with land-line telegraphs that a new class of technical personnel gradually developed, consisting of men with a substantial knowledge of electrical principles and techniques. The world's first professional association of electrical engineers was the Society of Telegraph Engineers, founded in England in 1871, and most of its charter members were submarine telegraph engineers. The Society added "and Electricians" to its title in 1880, and became the present day Institution of Electrical Engineers in 1889, five years after the formation of the American Institute of Electrical Engineers in the United States.

The invention of the telephone in 1876 immediately made evident further complications in the use of transmission lines for electrical communication. The frequencies required for voice reproduction were hundreds of times higher than those used in telegraphy. Attempts at inter-city telephony over the telegraph lines of the time, which generally consisted of single iron wires with the ground as a return circuit, were frustrated by the low level and garbled unintelligibility of the received signals. In the ensuing 40 years, improvements in long distance voice frequency telephony developed slowly but steadily through a combination of empirical discoveries and theoretical studies.

The man chiefly responsible for a new and more complete mathematical analysis of signal propagation on transmission lines was Oliver Heaviside, one of the most unusual, and at the same time most productive engineer-mathematicians of all time. A nephew of Charles Wheatstone, the prominent electrical scientist and telegraph inventor, and brother of a well-known telegraph engineer, Oliver Heaviside worked for a few years in the British telegraph industry, then "retired" (according to a major encyclopedia) in 1874 at age 24, to spend the next fifty years of his life in almost total seclusion. During that half century his publications on transmission lines, electric circuit theory, vector analysis, operational calculus, electromagnetic field theory, and numerous other topics, did more to define the concepts and establish the theoretical methods of modern electrical engineering than has the work of any other one individual. Transmission line theory as developed in several of the chapters of this book is entirely the work of Oliver Heaviside, and was first published by him during the 1880's. Modern presentations of Maxwellian electromagnetic theory are also essentially in the form created by Heaviside.

By the end of the 19th century, experience and analysis had indicated that long voice frequency telephone circuits, still the most challenging transmission line problem, worked best when constructed of two large low-resistance copper wires, mounted as widely spaced, well insulated open-wire pole lines. The use of ground-return was abandoned. From his equations Heaviside had noted that on most practical lines, voice signals should travel with reduced loss and with greater fidelity if the distributed inductance of the line could be increased without adversely changing the other distributed circuit coefficients. In the United States, Michael Pupin of Columbia University and George Campbell of the Bell Telephone Laboratories conceived about 1900 that a practical alternative to the difficult process of increasing the uniformly distributed inductance of a line might be the insertion of low resistance lumped inductance coils at intervals of a mile or so along the line. The technique, known as "loading", is discussed briefly in Section 5.8 on page 60. It proved extremely successful, and loading coils were connected into tens of thousands of miles of open wire and cable telephone circuits in a period of about thirty years. Loading permitted the economy, in long telephone lines, of using smaller gauge copper wires than would otherwise have been needed to give the same electrical efficiency and quality of transmission.

The carbon microphone of telephony is an electromechanical amplifier, whose electrical power output can be a thousand times greater than its mechanical voice-power input. From about 1890 on, much effort was expended in attempts to develop this property into an

amplifier unit that could be inserted into long telephone lines to offset the effects of resistance and leakage. The results never became sufficiently satisfactory to be used on a commercial scale. The peak potential of passive voice-frequency telephone circuits is well exemplified by the historic occurrence in 1911 of a brief telephone conversation between New York City and Denver, Colorado. The circuit consisted of two large-conductor pole-mounted transmission lines between the two cities, connected in parallel. No amplifiers were involved, and the low-resistance, high-inductance lines could not benefit from any practical form of loading. The circuit could handle only a single voice frequency signal, and its operation proved that telephony over a distance of two thousand miles was an economic impossibility for the technology of the time. In a country more than three thousand miles across, this was an unpleasant conclusion.

At this critical juncture in the progress of electrical communication, Lee De Forest in 1912 offered the telephone industry his primitive erratic triode amplifier, incorporating the thermionic "audion" he had invented in 1907. Two years of intensive research in the industry's laboratories improved the device to the point of making transcontinental telephony a realized achievement in 1915. With vacuum tube amplifiers the losses of small conductor lines could be offset inexpensively by stable amplifier gain, and distortion could be reduced to any desired value by networks "equalizing" the characteristics of a line over any range of signal frequencies.

In possession of a practical solution to the problems of distance and signal quality, telephone engineers looked for new methods of cost reduction. It was obvious that the greatest rewards lay in the possibility of using the expensive transmission lines for several voice frequency signals simultaneously. Pursuit of this goal, in many different ways, became a major activity of the telephone industry for the next fifty years and still receives considerable attention.

Military interest in electronic tubes and their associated circuitry during World War I greatly accelerated the development of amplifiers, oscillators, filters and other devices, and helped make feasible by 1919 the first installations of long-distance carrier-frequency telephone systems, in which several voice frequency channels of bandwidth about 4 kilohertz are translated to different higher frequency intervals for transmission. Early carrier systems (the technique is now known as "frequency division multiplexing") handled three telephone channels in each direction on a single two-wire cable pair or open-wire transmission line, using frequencies up to about 30 kilohertz.

It is perhaps the greatest irony of electrical engineering history that the loading technique, which was the salvation of the long distance telephone industry in the first quarter of the century, had converted every loaded transmission line into a low-pass filter, incapable of transmitting any frequencies above 3 or 4 kilohertz, and hence useless for carrier frequency systems. From 1925 to 1940, most of the millions of loading coils previously installed were removed.

While the wire-line communication industry was evolving from the very limited capabilities of passive voice-frequency transmission line circuits to the virtually unlimited potentialities, for continental purposes at least, of multi-channel carrier-frequency circuits using vacuum tube amplifiers, the technology of communication by freely propagating electromagnetic waves was developing simultaneously. Marconi's experiments of 1895 to 1902 showed that local and intercontinental telegraphy could be accomplished without wires, using hertzian waves. Wireless telegraphy quickly became a glamorous and highly publicized activity, but in spite of dire forebodings it offered no significant competitive threat to any of the established telephone and telegraph services using land lines or submarine cables until after the development, during World War I, of medium power thermionic

vacuum tubes as transmitters and various sensitive vacuum tube circuits as receivers. (It did create still another distinctive body of technical personnel, however, and the Institute of Radio Engineers, founded in 1912, resulted from the merger of two small societies of wireless telegraph engineers. In 1963 the I.R.E. merged with the A.I.E.E. to form the Institute of Electrical and Electronics Engineers.)

Wartime progress led to the founding in the early 1920's of the radio broadcasting industry, using frequencies around one megahertz, whose technological principles have undergone only minor changes since that time. Subsequent exploration of higher frequencies discovered the extraordinary world-wide propagation characteristics of "short waves" between 3 and 30 megahertz, and radio telephone circuits at these frequencies, adopted by the telephone industry before 1930 for the first intercontinental telephone service, remained the only commercial solution to that problem for the next thirty years.

During this decade the rapidly increasing proportion of electrical engineers employed in carrier frequency telephony and high frequency radio found it necessary to deal with transmission circuits many wavelengths long, whose analysis required the use of distributed circuit methods. The study of Heaviside's transmission line theory began to appear in a few electrical engineering curricula, generally at the graduate level, using textbooks by Steinmetz, Dwight, Fleming, Pierce, Johnson and Kennelly, among others.

The 1930's witnessed the extension of the technology of electronic devices and circuits from frequencies in the tens of megahertz to frequencies of several hundred megahertz, with such applications as FM broadcasting and mobile telephone service, and the beginnings of television and radar. The theory of uniform transmission lines was incorporated into several undergraduate curricula, often as an optional subject. Popular textbooks dealing with the subject included those by Terman, Everitt and Guillemin.

Finally, the concentrated attention given to the whole field of uhf and microwave engineering during World War II, with the subsequent development of commercial television, microwave communication links, radio and radar astronomy, and the innumerable applications of these frequencies in space exploration and continuing military uses, made it obvious that transmission line theory must become a basic topic in all electrical engineering curricula.

The realization of this result was marked by the publication in the period 1949-1954 of a large number of widely used textbooks. Among the authors were Skilling, Johnson, Kimbark, Cramer, Ryder, Jackson and Karakash. These were the successors to a few textbooks that appeared during the war period, of which the most influential were probably the two by Sarbacher and Edson and by Ramo and Whinnery. Since 1955 new textbooks on transmission line theory have appeared less frequently.

The "high frequency frontier" of electrical engineering has in the last two decades been pushed to frequencies beyond one hundred gigahertz, using solid state and free electron devices, and with quantum-electronic lasers able to generate power at frequencies all the way from hundreds of gigahertz through hundreds of terahertz in the infra-red, visible and ultra-violet regions of the spectrum, the very concept of a frontier has lost most of its meaning.

At frequencies for which the free space wavelength is less than a few millimeters, two-conductor electrical transmission systems operating in the mode implicit in distributed circuit theory are little used. Other analytical formulations, such as those of electromagnetic theory, or of geometrical or physical optics, become appropriate. To the extent that the interests of electrical communication engineers are scattered over a far wider region of the electromagnetic spectrum than was the case a few years ago, the relative importance of transmission line theory in their total study program has begun to diminish. Two-conductor lines, however, will always remain a basic transmission technique, and for their

analysis Heaviside's theory will always be as fundamental as Maxwell's equations are to electromagnetics, since the theory is in fact a direct application of the primary electromagnetic relations to the principal mode of propagation on such lines.

1.4. References.

The following is a list of a few historic references and some of the more recent textbooks in the field of distributed circuit transmission line theory.

An exposure to the pithy personality of Oliver Heaviside, through the introductory Chapter 1 of his *Electromagnetic Theory*, is an interesting experience.

King's *Transmission Line Theory* contains a complete bibliography of transmission line textbooks published before 1955.

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10. Smith, P. H., "Transmission Line Calculator", *Electronics*, vol. 12, 1939, p. 29-31.
11. Smith, P. H., "An Improved Transmission Line Calculator", *Electronics*, vol. 17, 1944, p. 130-133 and 318-325.
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Chapter 2

Postulates, Symbols and Notation

2.1. Postulates of distributed circuit analysis.

The distributed circuit analysis of uniform transmission lines, begun by William Thomson (Lord Kelvin) in 1855 and completed by Oliver Heaviside about 1885, is derived by applying the basic laws of electric circuit analysis to systems described by the following postulates.

Postulate 1. The uniform system or line consists of two straight parallel conductors.

The adjective “uniform” means that the materials, dimensions and cross-sectional geometry of the line and its surrounding medium remain constant throughout the length of the line. Typically, a signal source is connected at one end of the system and a terminal load is connected at the other end, as shown in Fig. 2-1.

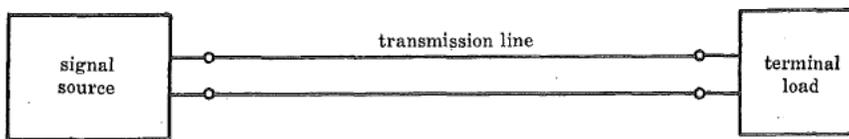


Fig. 2-1. Basic transmission line circuit.

This postulate does not require that the two conductors be of the same material or have the same cross-sectional shape. The analysis is therefore valid for a conductor of any material and cross section enclosing another conductor of any material and cross section, for a wire parallel to any conducting plane or strip, and for many other useful constructions in addition to the simple example of two parallel wires of circular cross section and of the same diameter and material.

The analysis is applicable to systems with more than two parallel conductors, provided these are interconnected in such a way as to present only two terminals at the points of connection of source and load. Systems may also involve shielding conductors that are not connected to the line at any point.

Fig. 2-2 shows the cross-sectional configurations of the conductors for several uniform two-conductor transmission lines used in engineering practice.

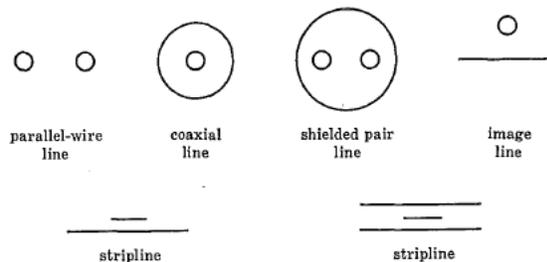


Fig. 2-2. Conductor cross sections for several practical transmission lines.

In general, twists or bends in a transmission line violate the "uniformity" postulate and create effects not explainable by the distributed circuit theory. These effects will be negligible if the rate of the twist or bend does not exceed about one degree in a length of line comparable with the separation of the line conductors.

The uniformity postulate is also violated by any discontinuities in a line, such as the termination points of an otherwise uniform system, or the point of connection between two uniform lines that differ physically in some respect. In the vicinity of discontinuities like these, phenomena will occur which are not in accord with distributed circuit theory. The anomalous behavior is usually confined to distances not greater than a few times the separation of the line conductors, on either side of the discontinuity.

Postulate 2. The currents in the line conductors flow only in the direction of the length of the line.

This is a basic premise of elementary electric circuit analysis, and it may seem unnecessary that such a requirement should have to be stated in making a distributed circuit analysis of transmission lines. It is a fact, however, that under certain conditions signals can propagate on any uniform transmission line with either the whole of the line current or a component of it flowing *around* the conductors rather than along them. These cases are known as "waveguide" modes of propagation. They are discussed further in Section 2.2.

Postulate 2 means that distributed circuit transmission line theory does not recognize the existence of waveguide modes.

Postulate 3. At the intersection of any transverse plane with the conductors of a transmission line, the instantaneous total currents in the two conductors are equal in magnitude and flow in opposite directions.

In elementary network theory the postulate equivalent to this for a simple loop circuit such as Fig. 2-1 would be that the current is the same at *all* points of the circuit at a given instant. Postulate 3 allows the instantaneous currents to be different at different cross sections of the line at the same instant. Clearly this is not possible without violating Kirchhoff's current law unless currents can flow transversely between the two conductors at any region of the line's length. Provision for such transverse currents is made in postulate 5.

Postulate 4. At the intersection of any transverse plane with the line conductors there is a unique value of potential difference between the conductors at any instant, which is equal to the line integral of the electric field along all paths in the transverse plane, between any point on the periphery of one of the conductors and any point on the periphery of the other.

Like postulate 3, this postulate has the consequence of ruling out waveguide modes, for which the line integral of the electric field is in general not independent of the path.

Postulate 5. The electrical behavior of the line is completely described by four distributed electric circuit coefficients, whose values per unit length of line are constant everywhere on the line. These electric circuit coefficients are resistance and inductance uniformly distributed as series circuit elements along the length of the line, together with capacitance and leakage conductance uniformly distributed as shunt circuit elements along the length of the line.

It is an essential part of this postulate that the values of these distributed circuit coefficients at a given frequency are determined only by the materials and dimensions of the line conductors and the surrounding medium. They do not vary with time, nor with line voltage or current. The line is thus a linear passive network.

Postulate 5 has a direct relation to postulates 3 and 4. The line currents of postulate 3 are accompanied by a magnetic field. The distributed line inductance is a measure of the energy stored in this magnetic field for unit length of line, per unit current. There is power loss as the line currents flow in the conductors. The distributed line resistance is a measure of the power loss in unit length of line per unit current.

Similarly the line potential difference of postulate 4 is associated with an electric field. The distributed line capacitance is a measure of the energy stored in this electric field per unit length of line, for unit potential difference. There is power loss in the medium between the conductors because of the potential difference. The distributed line conductance is a measure of the power loss in unit length of line, for unit potential difference.

The existence of distributed *shunt* circuit coefficients accounts for the possibility discussed in connection with postulate 3, that the conductor currents can be different at different cross sections of the line. Conduction currents or displacement currents will flow transversely between the conductors, as a function of the potential difference between them or its time rate of change, respectively. The line currents at two separated cross sections of the line will differ by the amount of the transverse current in the intervening length of line.

2.2. Waveguide modes and electromagnetic theory.

In Chapter 1 it was noted that a complete analysis of the transmission properties of any transmission line system can be made by starting with Maxwell's equations, and seeking a solution subject to the boundary conditions imposed by the line conductors. Such an analysis reveals all the "waveguide" modes mentioned under postulates 2 and 4. These fall into two categories known as TE (*transverse electric*) and TM (*transverse magnetic*) waves, distinguished respectively by field distributions with components of magnetic or electric field parallel to the length of the line. For any transmission line structure there is an infinite number of these modes, each with its own specific patterns of electric and magnetic fields.

Any TE or TM mode can be propagated on a particular transmission line only at frequencies above some minimum cutoff frequency, which is calculable for each separate mode from the dimensions and materials of the transmission line. For lines whose conductor separations do not exceed a few inches, these cutoff frequencies range from thousands to tens of thousands of megahertz. Hence in most practical uses of transmission lines, at frequencies from d-c to uhf there is no possibility of TE or TM modes being propagated. When transmission lines are used at microwave or millimeter-wave frequencies, care must sometimes be taken to avoid the occurrence of such modes, since their presence will invariably result in excessive line losses and other undesirable consequences.

No useful applications have yet been made of the TE and TM modes that can propagate on two-conductor transmission lines at extremely high frequencies. Within single conductors in the form of hollow metal pipes, however, the TE and TM modes are the basis of the invaluable microwave technique of waveguide transmission.

Although the TE and TM modes cannot propagate in any transmission system at frequencies below a cutoff frequency which is in the microwave frequency range for typical line constructions, the field patterns of one or more of these modes are invariably generated by irregularities and discontinuities in a system. When the frequency is below the cutoff frequency for the modes excited, the field patterns are unable to propagate as waves. They do, however, diffuse or penetrate a short distance from their point of origin, a distance not greater than a few times the conductor separation of the transmission system. They are responsible for the anomalous behavior of transmission lines, mentioned in Section 2.1, postulate 1, that occurs in the vicinity of discontinuities and is not explainable by distributed circuit theory.

2.3. The TEM mode.

The analysis of any uniform two-conductor transmission line by the methods of electromagnetic theory reveals one other unique mode in addition to the TE and TM infinite sequences of modes. This single mode differs from the others in that its electric and magnetic fields are everywhere transverse to the direction of the conductors' length, and there is no cutoff frequency other than zero. Designated TEM (*transverse electromagnetic*), this mode has the field representation that corresponds to the voltages and currents of the distributed circuit theory of transmission lines.

The electric and magnetic field patterns for the TEM mode in a circular coaxial line are shown in Fig. 2-3. They are the only possible distribution of electric and magnetic fields that can simultaneously satisfy the postulates listed in Section 2.1 and the basic laws of electromagnetism, for this geometry of conductors.

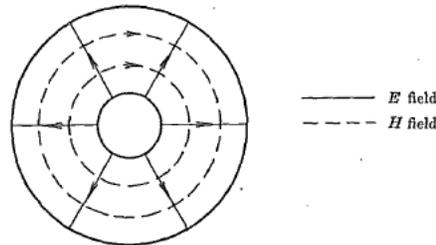


Fig. 2-3. Electromagnetic field pattern for the TEM (transverse electromagnetic) mode in a coaxial transmission line.

2.4. Distributed circuit analysis and electromagnetic theory.

The analysis of transmission lines by distributed circuit methods is not independent of the analysis by field methods, since the calculation of the circuit coefficients used in the former can be made only from a knowledge of the electric and magnetic fields associated with the line voltages and currents.

The practical advantage of the distributed circuit method of analysis is that it uses the circuit analysis language of voltages, currents, impedances, etc. It is customary in engineering practice, even at frequencies up through the microwave region, to designate the sources and loads used with transmission systems by their equivalent circuits, rather than by a statement of the spatial conditions they impose on electric and magnetic fields.

The combination of an analysis of transmission line behavior in circuit terms with the equivalent circuit specification of sources and loads, permits studying the overall system of source, line and load with the help of all the powerful techniques that have been developed in electric network theory.

A complete analysis by electromagnetic theory, on the other hand, of the detailed distributions of the electric and magnetic fields associated with a source-line-load system would raise insuperable mathematical difficulties for any structure with other than exceedingly simple and continuous geometry throughout.

2.5. Coordinates, coefficients and variables.

Before the postulates of equivalent circuit transmission line analysis stated in Section 2.1 can be embodied in equations, it is necessary to select symbols for coordinates, variables and physical coefficients, and to establish a few sign conventions. There are "standard" symbols recommended by major electrical professional societies for a few of the quantities to be

labelled, but for most of the decisions involved there are no generally accepted guides. The result is that current textbooks and journal articles on transmission line topics contain a regrettable variety of notation, to which the reader must adjust in each context.

A brief discussion of the reasons for the choices used in this book, together with comments on the possible merits of alternatives, is offered in the next few sections for the benefit of readers who like to compare several accounts of the same topic from different sources.

2.6. Choice of coordinate notation.

So far as transmission is concerned, the transmission line problem is one-dimensional, with a single coordinate axis parallel to the length of the line. In this book the symbol z is selected for this coordinate.

It is tempting to choose x for this coordinate, as in the one-dimensional problems of elementary mathematics. However, the analysis of many other transmission systems of interest to engineers, such as waveguides, vibrating strings, transversely vibrating rods, electromagnetic wave beams, etc. requires the use of coordinates in the transverse plane as well as in the direction of propagation. The use of x and y for these transverse coordinates has been adopted quite unanimously, and it is therefore appropriate to select z as the universal symbol for the longitudinal coordinate of all transmission systems. Supporting this choice is the applicability of cylindrical coordinates, with their longitudinal z -coordinate, to the analysis of the many forms of transmission line that have circular elements or circular symmetry.

2.7. Choice of origin for longitudinal coordinate.

An unexpected development that arises in setting up the distributed circuit theory of transmission lines is that, for certain important purposes, more significance attaches to the distance of a point on the line from the terminal load than to its distance from the signal source. This has led some writers to base their entire analysis on a longitudinal coordinate whose origin is at the load. Some take the additional step of locating this load-origin at the left end of the line, with the signal source at the right.

On the whole it seems more appropriate that a signal moving from a source to a load should be moving in the direction of an increasing coordinate, and since it is universal practice mathematically to have the coordinates of a one-dimensional problem increase from left to right, the analysis in this book uses the convention that the signal source is at the left-hand end of the line, the terminal load is at the right-hand end, and the longitudinal coordinate z has its origin at the signal source.

When occasion demands, the distance of a point on the line from the terminal load is indicated by a coordinate d , with origin at the load and increasing from right to left.

The symbol l is used at all times for the total length of the line.

Fig. 2-4 shows a complete transmission line system, with all the symbols for longitudinal coordinates and distances according to these chosen conventions.

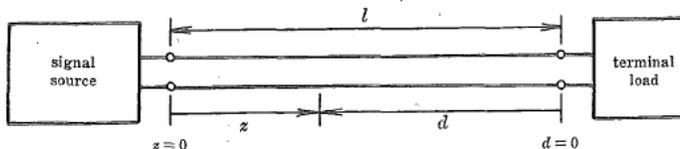


Fig. 2-4. Coordinates on a transmission line circuit.

2.8. Symbols for current and voltage.

The dependent variables in the distributed circuit analysis of transmission lines are current and voltage, which are functions of time at any point on the line, and functions of position on the line at any instant. The functional expressions describing these relations are determined by the signal source, the terminal load, and the distributed circuit coefficients and total length of the line.

In elementary circuit analysis it is generally accepted notation to use lower case letters as symbols for the instantaneous values of time-varying dependent variables. Capital letters are used for d-c quantities, and for the complex number or phasor values of a-c quantities which have constant amplitude harmonic variation with time.

In the analyses in this book, d-c currents and voltages seldom occur. Capital letter symbols for current and voltage therefore represent complex number phasor values. Their magnitudes are rms magnitudes, not peak values. Unless specifically designated as quantities at the signal source or terminal load ends of the line, they are functions of position along the line. Lower case symbols for current or voltage represent instantaneous values, which must also be understood to be functions of the coordinate z .

As a symbol for current the letter i, I (from the French word *intensité*) replaced all competitors long ago.

For voltage or potential difference the letter e, E (presumably for *electromotive force*) has had majority approval for several decades.

As long as physicists paid little attention to electric circuit theory, and engineers even less to electromagnetic field theory, only minor confusion resulted from using the same symbol E for the two important concepts of potential difference and electric field strength. This segregation no longer exists and the confusion is now serious. It is sometimes avoided by using special symbols, such as script letters, but these are awkward for note-takers and typists. The situation can be satisfactorily resolved by using the symbol v, V for voltage. In technical writing V is widely used for the concept of volume, and v for the concept of velocity. Neither of these occurs frequently enough in the same context with the concept of voltage to constitute a hazard.

In summary, the notation for dependent variables in the transmission line theory of this book is as follows:

i or $i(z, t)$ = instantaneous current at a specific point on a transmission line, i.e. the current at time t at coordinate z ;

I or $I(z)$ = complex rms value of a constant amplitude harmonically varying current at coordinate z ;

$|I|$ or $|I(z)|$ = rms magnitude of I or $I(z)$.

The symbols $v, v(z, t), V, V(z), |V|$ and $|V(z)|$ have corresponding meanings for voltage or potential difference.

A sign convention relating current directions and voltage polarities must be adopted, to avoid ambiguities when a circuit analysis is made of a transmission line section. The convention used is standard in elementary circuit theory, for two-terminal networks.

Thus at a coordinate z on a transmission line as shown in Fig. 2-5(a) below, an instantaneous voltage $v(z, t)$ in the time domain may be represented by an arrow drawn from one line conductor to the other in the transverse plane at z . The head of the arrow has positive polarity, and the voltage $v(z, t)$ is positive when the arrow is directed from the lower conductor to the upper, as in the figure.

Similarly, the line currents at coordinate z are indicated by *two* arrowheads, one in each line conductor, the two pointing in opposite directions according to postulate 3 of Section 2.1. The sign of the current is positive when the current in the *upper* line conductor flows in the direction of *increasing* z , as in the figure.

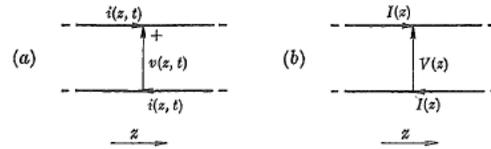


Fig. 2-5. Sign conventions for current and voltage in transmission line analysis: (a) in the time domain; (b) for phasors.

It can be noted that if the portion of the transmission line circuit to the left of the coordinate z is removed, the sign conventions adopted have the usual simple implication for the two-terminal passive network to the right of the coordinate z , that a positive applied voltage causes a positive current to flow.

Fig. 2-5(b) illustrates the corresponding conventions for phasor notation. Here the implication is that if $V(z)$ in the direction shown is chosen as a real reference phasor, the direction shown for $I(z)$ represents a phasor current with a non-negative real part.

Example 2.1.

What meaning can be given to the symbols $i(t)$, $I(t)$, $|v|$, $V(z, t)$ in view of the definitions given above?

Three of these four symbols have no meaning within the definitions stated.

Since the symbol i is for a current which is a function of both z and t , to write it as $i(t)$, a function of t alone would require additional explanation.

The symbol I is for a current whose manner of time-variation is implicitly understood. To designate it as $I(t)$, explicitly a function of time, is either unnecessary or implies some additional form of time variation (such as modulation) superimposed on the harmonic variation. An auxiliary definition would have to be supplied to give meaning to this symbol. The same reasoning applies to the term $V(z, t)$.

Since the symbol v stands for an instantaneous value of voltage at a particular point on the line, its meaning is a scalar quantity, which could be either positive or negative. $|v|$ is the magnitude of v , and is a positive quantity by definition.

2.9. Symbols for distributed circuit coefficients.

These are the symbols about which there is the greatest measure of agreement in transmission line writings. The definitions are:

R = total series resistance of the transmission line per unit length, including both line conductors, or both combinations of conductors making up the two sides of the line. In mks units, R is in ohms/meter. (If the interconductor space of a transmission line is filled with a material that is *magnetically* lossy, i.e. which converts electromagnetic field energy into heat in proportion to the square of the magnetic field B in the medium, these losses will be represented by a contribution to R , in the equivalent circuit.)

L = total series inductance of the transmission line per unit length, including inductance due to magnetic flux both internal and external to the line conductors. In mks units L is in henries/meter.

G = shunt conductance of the transmission line per unit length. This is the circuit representation of losses that are proportional to the square of the voltage between the conductors or the square of the electric field in the medium. Usually G represents internal molecular lossiness of dielectric insulating materials, rather than an actual charge flow leakage current. In mks units G is in mhos/meter.

C = shunt capacity of the transmission line per unit length. In mks units C is in farads/meter.

It must be noted carefully that the symbols R , L , G and C as defined here have *different meanings and dimensions* from those with which the reader has become familiar in the study of lumped element networks. In that context the symbols mean the resistance, inductance, etc. of a two-terminal element postulated to have negligible dimensions. Here they mean resistance, inductance, etc. *per unit length* of a four-terminal or two-port circuit with non-zero length and with uniformly distributed circuit coefficients. Using mks units, R in the former case would be expressed in ohms, while in the transmission line case it is expressed in ohms/meter, and a similar distinction holds for the other three symbols.

To avoid this ambiguity, writers have occasionally used modified symbols for the distributed circuit quantities, such as R' , r , \bar{R} , R/l , \mathcal{R} , etc. Most, however, use the notational procedure followed in this book, that R , L , G and C when *not subscripted* are the distributed circuit coefficients. When subscripted they represent lumped circuit elements in two terminal networks, the subscript indicating the function of the network, as described in the next section.

2.10. Symbols for terminal quantities and elements.

Notation is needed to distinguish voltages, currents and connected impedances at the signal source and terminal load ends of a transmission line from the same quantities at arbitrary points along the length of the line. This book follows common practice in using subscripts for the purpose. Transmission line writings show much variety in the choice of these subscripts. For the signal source end of the line, subscripts s or S (for source or sending-end), g or G (for generator) and o or O (for origin) are found, while for the terminal load end of the line r or R (for receiver), l or L (for load) and t or T (for termination) are all in common use.

Because R , L and G occur prominently in the analysis as symbols for the distributed circuit coefficients, it seems unwise to use them as subscripts with a totally different reference. The symbols S and T seem to offer minimum opportunity for confusion, and are adopted in this book as the subscripts referring respectively to the signal source end of the line and the terminal load end.

Example 2.2.

What are the meanings of i_T , V_S , I_G , I_L , V_t , $|I_T|$, and L_T , in the notation defined above?

i_T is the instantaneous current (at time t) at the terminal load end of the line.

V_S is the complex rms value or phasor rms value of the time-harmonic voltage of a generator connected at the input or signal source end of the line. It is *not* the symbol for the phasor voltage at the input *terminals* of the line, which would be $V(z=0)$. If the connected generator had zero internal impedance, or if the line current were zero at $z=0$, V_S and $V(z=0)$ would have the same value, but as symbols they represent different quantities.

I_G , I_L and V_t have no meaning within the definitions stated. $|I_T|$ is the rms magnitude of the phasor current at the terminal load end of the line. It is identical with $|I(z=l)|$ where $I(z=l)$ is the phasor line current at $z=l$, since by definition the total length of the line is l .

L_T represents a lumped inductance connected at the terminal load end of the line. It could also be the inductance component of the simple series circuit equivalent to any more complicated linear network connected as terminal load.

2.11. Notation for impedance and admittance.

Three occurrences of impedance quantities (or admittance quantities) need to be distinguished in transmission line theory. These are:

1. Impedances or admittances connected at the line terminals. A terminal load impedance has the symbol Z_T in this text, with components R_T and jX_T . From previous definitions it is clear that $Z_T = V_T/I_T$ always, this being a complex number equation involving the ratio of two phasors.

Similarly, the internal impedance of a generator connected at the signal source end of the line has the symbol $Z_S (= R_S + jX_S)$. In general Z_S is *not* equal to V_S/I_S , since the latter ratio depends on the total transmission line circuit, while Z_S can have any arbitrary value, entirely independent of the transmission line.

2. A unique quantity appearing in transmission line equations, that has the dimensions of impedance and is determined solely by the distributed circuit coefficients of the line and the signal frequency. This is now universally called the "characteristic impedance" of a transmission line and given the symbol Z_0 (with components R_0 and X_0). In the past it has often been called the "surge impedance" or the "iterative impedance" of a line. Symbols Z_c , Z_s and Z_i have occasionally been used for it.

3. The impedance given by the ratio of the phasor line voltage to the phasor line current at any cross section of the line. The notation $Z(z)$ or $Z(d)$ is appropriate for this (with corresponding symbols for real and imaginary components) according to whether the particular point on the line is located by its coordinate relative to the signal source end or the terminal load end of the line as origin. The impedance at a specific numerical coordinate requires expanded notation such as $Z(z=3)$ for a point 3 meters from the source end of the line.

This text does not make explicit use of the concepts of distributed line impedance $Z = R + j\omega L$ or distributed line admittance $Y = G + j\omega C$ found in many treatments of transmission line theory.

The corresponding admittance definition and symbol can be substituted for any of the above occurrences of impedance language. Thus:

$$Y_T = 1/Z_T = G_T + jB_T, \quad Y_0 = 1/Z_0 = G_0 + jB_0, \quad Y(z) = 1/Z(z) = G(z) + jB(z)$$

Either impedance or admittance may of course also be expressed in polar form: $Z(z) = |Z(z)| e^{j\theta(z)}$.

Example 2.3. What are the meanings of $G(z=0)$, $B(z=l)$, $X(d=5)$, $Y(3)$, $Z(0)$?

$G(z=0)$ means the conductance component (i.e. the real part) of that admittance which is the ratio of the phasor current at the signal source end of the line to the phasor voltage at the same position. This is clearly entitled to the notation G_{inp} , i.e. the conductance component of the input admittance Y_{inp} of the line. (Note that the symbol G here has no connection with the symbol G used for one of the distributed circuit coefficients of the line. The duplication is unfortunate but is well entrenched in contemporary notation.)

$B(z=l)$ means the susceptance component (i.e. the imaginary part) of that admittance which is the ratio of the phasor current to the phasor voltage at the terminal load end of the line. Since this ratio is necessarily identically equal to the terminal load admittance *connected* to the line, it follows that $B(z=l) = B_T$.

$X(d=5)$ means the reactive component (i.e. the imaginary part) of that impedance which is the ratio of the phasor voltage to the phasor current at a point on the line 5 m from the terminal load end.

$Y(3)$ and $Z(0)$ have no meaning since they are ambiguous as to whether the coordinate referred to is z or d .

2.12. Notation for transient response.

In introductory textbooks on the theory of lumped element circuits it is common practice to use a generalized notation in which voltages, currents, impedances and admittances are shown as functions of the complex frequency variable s . Circuit response for steady state harmonic signals of angular frequency ω is then obtained by substituting $j\omega$ for s , while time-domain response is obtained by Laplace transform methods.

Since the theory of distributed circuits is no more than the extension of lumped element circuit theory by the addition of a space variable, it might seem reasonable to take the same generalized approach to transmission line theory by writing all equations in terms of the complex frequency variable s . That no textbooks have so far been written in this form is evidence that elementary methods have not yet proved profitable for studying the general transient response of transmission line circuits. It is therefore realistic to develop transmission line theory in terms of the steady state angular frequency variable ω .

Chapter 1

Magnetic Circuits

1.1 INTRODUCTION AND BASIC CONCEPTS

Electric machines and electromechanical devices are made up of coupled electric and magnetic circuits. By a *magnetic circuit* we mean a path for magnetic flux, just as an electric circuit provides a path for the flow of electric current. Sources of magnetic fluxes are electric currents and permanent magnets. In electric machines, current-carrying conductors interact with magnetic fields (themselves arising from electric currents in conductors or from permanent magnets), resulting in electromechanical energy conversion.

Consider a conductor of length l placed between the poles of a magnet. Let the conductor carry a current I and be at right angles to the magnetic flux lines, as shown in Fig. 1-1. It is found experimentally that the conductor experiences a force F , the direction of which is shown in Fig. 1-1 and the magnitude of which is given by

$$F = BIl \quad (1.1)$$

Here, B is the magnitude of the *magnetic flux density* \mathbf{B} , whose direction is given by the flux lines. The SI unit of \mathbf{B} or B is the *tesla* (T). (Another, equivalent unit will be introduced shortly.) Notice from (1.1) that B could be defined as the force per unit current moment. Equation (1.1) is a statement of *Ampere's law*; the more general statement, which holds for an arbitrary orientation of the conductor with respect to the flux lines, is

$$\mathbf{F} = I\mathbf{l} \times \mathbf{B} \quad (1.2)$$

where \mathbf{l} is a vector of magnitude l in the direction of the current. Again the force is at right angles to both the conductor and the magnetic field (Fig. 1-2). Ampere's law, (1.1) or (1.2), providing as it does for the development of force, or torque, underlies the operation of electric motors.

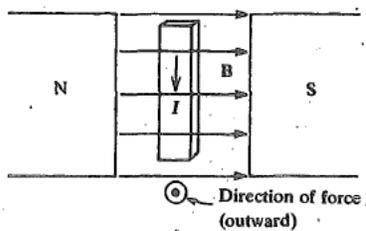


Fig. 1-1

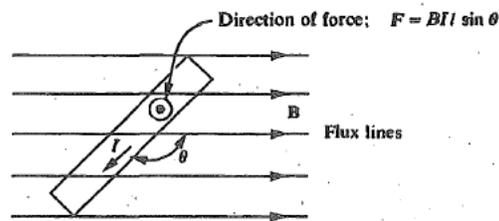


Fig. 1-2

The magnetic flux, ϕ , through a given (open or closed) surface is the flux of \mathbf{B} through that surface; i.e.,

$$\phi = \int_s \mathbf{B} \cdot d\mathbf{S} = \int_s \mathbf{B} \cdot \mathbf{n} dS \quad (1.3)$$

where \mathbf{n} is the unit outward normal to the elementary area dS of the surface (Fig. 1-3). In case \mathbf{B} is constant in magnitude and everywhere perpendicular to the surface, of area A , (1.3) reduces to

$$\phi = BA \quad (1.4)$$

from which

$$B = \frac{\phi}{A} \quad (1.5)$$

The SI unit of magnetic flux is the *weber* (Wb). We see from (1.5) that B or \mathbf{B} may be expressed in Wb/m^2 , i.e., $1 \text{ T} = 1 \text{ Wb/m}^2$.

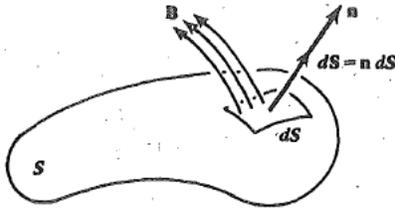


Fig. 1-3

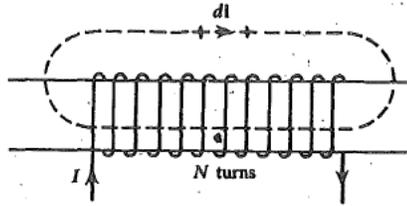


Fig. 1-4

The mutual relationship between an electric current and a magnetic field is given by *Ampere's circuital law*, one form of which is

$$\oint \mathbf{H} \cdot d\mathbf{l} = I \quad (1.6a)$$

where \mathbf{H} is defined as the *magnetic field intensity* (in A/m) due to the current I . According to (1.6a), the integral of the tangential component of \mathbf{H} around a closed path is equal to the current enclosed by the path. When the closed path is threaded by the current N times, as in Fig. 1-4, (1.6a) becomes

$$\oint \mathbf{H} \cdot d\mathbf{l} = NI \equiv \mathcal{F} \quad (1.6b)$$

in which \mathcal{F} (or NI) is known as the *magnetomotive force* (abbreviated mmf). Strictly speaking, \mathcal{F} has the same units, amperes, as I . However, in this book we shall follow the common convention of citing \mathcal{F} in *ampere turns* (At); that is, we shall regard N as carrying a dimensionless unit, the *turn*.

Magnetic flux, magnetic flux density, magnetomotive force, and (see Section 1.2) permeability are the basic quantities pertinent to the evaluation of the performance of magnetic circuits. The flux, ϕ , and the mmf, \mathcal{F} , are related to each other by

$$\phi = \frac{\mathcal{F}}{\mathfrak{R}} \quad (1.7)$$

where \mathfrak{R} is known as the reluctance of the magnetic circuit.

1.2 PERMEABILITY AND SATURATION

In an isotropic, material medium, \mathbf{H} , which is determined by moving charges (currents) only, and \mathbf{B} , which depends also on the properties of the medium, are related by

$$\mathbf{B} = \mu\mathbf{H} \quad (1.8)$$

where μ is defined as the *permeability* of the medium, measured in *henries per meter* (H/m). (For the *henry*, see Section 1.8.) For free space, (1.8) gives

$$B = \mu_0 H \tag{1.9}$$

where μ_0 , the permeability of free space, has the value $4\pi \times 10^{-7}$ H/m.

The core material of an electric machine is generally ferromagnetic, and the variation of B with H is nonlinear, as shown by the typical *saturation curve* of Fig. 1-5(a). It is clear that the slope of the curve depends upon the operating flux density, as classified in regions I, II, and III. This leads us to

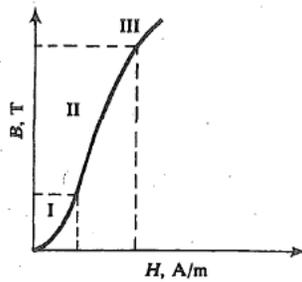


Fig. 1-5(a)

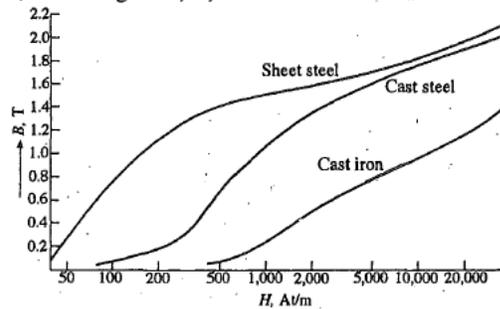


Fig. 1-5(b)

the concept of different types of permeabilities. We rewrite (1.8) as

$$B = \mu H = \mu_r \mu_0 H \tag{1.10}$$

in which μ is termed permeability and $\mu_r = \mu/\mu_0$ is called *relative permeability* (which is dimensionless). Both μ and μ_r vary with H along the B - H curve. In the following, relative permeability is assumed; that is, the constant μ_0 is factored out. The slope of the B - H curve is called *differential permeability*;

$$\mu_d \equiv \frac{1}{\mu_0} \frac{dB}{dH} \tag{1.11}$$

The *initial permeability* is defined as

$$\mu_i \equiv \frac{1}{\mu_0} \lim_{H \rightarrow 0} \frac{B}{H} \tag{1.12}$$

The (relative) permeability in region I is approximately constant and equal to the initial permeability. In all three regions, the ratio of B to H at a point on the curve is known as *amplitude permeability*:

$$\mu_a \equiv \frac{1}{\mu_0} \frac{B}{H} \tag{1.13}$$

Different ferromagnetic materials have different saturation curves, as shown in Fig. 1-5(b).

1.3 LAWS GOVERNING MAGNETIC CIRCUITS

In some respects, a magnetic circuit is analogous to a dc resistive circuit; the similarity is summarized in Table 1-1.

Table 1-1. Analogy between a dc electric circuit and a magnetic circuit

Electric Circuit	Magnetic Circuit
Ohm's law, $I = V/R$	$\phi = \mathcal{F}/\mathfrak{R}$
resistance, $R = l/\sigma A$	reluctance, $\mathfrak{R} = l/\mu A$
current, I	flux, ϕ
voltage, V	mmf, \mathcal{F}
conductivity, σ	permeability, μ
conductance, G	permeance, ϕ

In the table, l is the length and A is the cross-sectional area of the path for the flow of current in the electric circuit, or for the flux in the magnetic circuit. In a magnetic circuit, however, l is the mean length of the flux path. Because ϕ is analogous to I and \mathfrak{R} is analogous to R , the laws of resistors in series or parallel also hold for reluctances. The basic difference between electrical resistance, R , and magnetic reluctance, \mathfrak{R} , is that the former is associated with an energy loss (whose rate is I^2R), while the latter is not. Also, magnetic fluxes take leakage paths (Fig. 1-6), whereas electric currents normally do not.

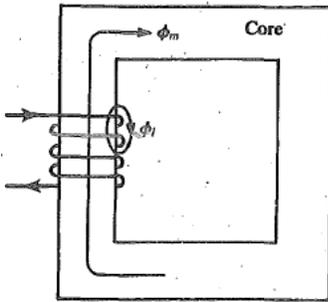


Fig. 1-6. Path of leakage flux, ϕ_l .

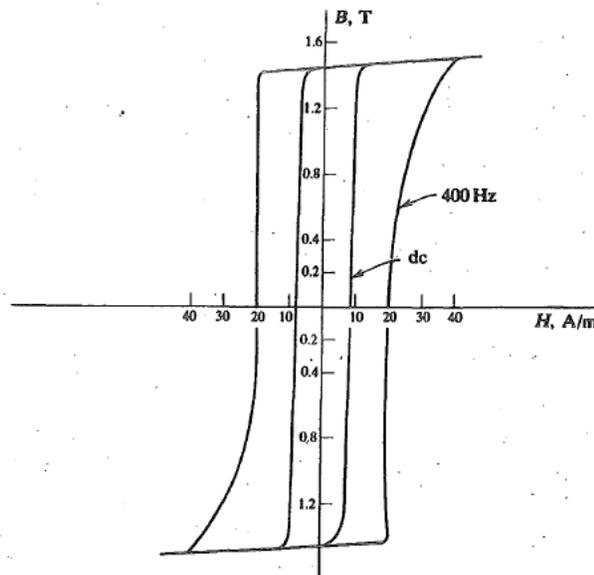


Fig. 1-7. Deltamax tape-wound core 0.002-in strip hysteresis loop.

1.4 AC OPERATION AND LOSSES

If the mmf is ac, then the B - H curve of Fig. 1-5 is replaced by the symmetrical *hysteresis loop* of Fig. 1-7. The area within the loop is proportional to the energy loss (as heat) per cycle; this energy loss is known as *hysteresis loss*.

Eddy currents induced in the core material (Fig. 1-8) constitute another feature of the operation of a magnetic circuit excited by a coil carrying an alternating current. The losses due to hysteresis and eddy currents—collectively known as *core losses* or *iron losses*—are approximately given by the following expressions:

$$\text{eddy-current loss: } P_e = K_e f^2 B_m^2 t^2 \text{ (W/kg)} \quad (1.14)$$

$$\text{hysteresis loss: } P_h = K_h f B_m^{1.5 \text{ to } 2.5} \text{ (W/kg)} \quad (1.15)$$

In (1.14) and (1.15), B_m is the maximum flux density, f is the ac frequency, K_v is a constant depending upon the material conductivity and thickness, and K_h is another proportionality constant. In addition, in (1.14), t is the lamination thickness (See Sec. 1.5).

1.5 STACKING FACTOR

To reduce eddy-current loss, a core may be constructed of laminations, or thin sheets, with very thin layers of insulation alternating with the laminations. The laminations are oriented parallel to the direction of flux, as shown in Fig. 1-8(b). Eddy-current loss is approximately proportional to the square of lamination thickness, which varies from about 0.05 to 0.5 mm in most electric machines. Laminating a core increases its volume. The ratio of the volume actually occupied by the magnetic material to the total volume of the core is known as the *stacking factor*; Table 1-2 gives some values.

Table 1-2

Lamination Thickness, mm	Stacking Factor
0.0127	0.50
0.0254	0.75
0.0508	0.85
0.10 to 0.25	0.90
0.27 to 0.36	0.95

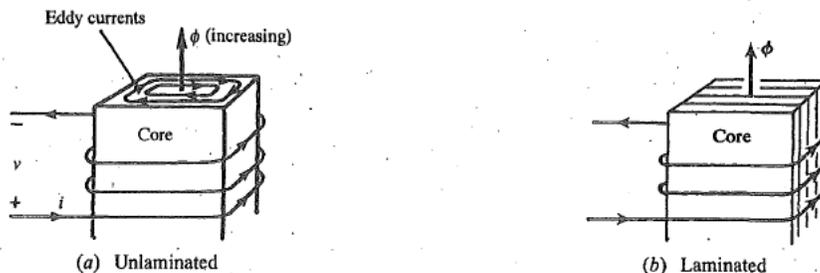


Fig. 1-8

Because hysteresis loss is proportional to the area of the hysteresis loop, the core of a machine is made of "good" quality electrical steel which has a narrow hysteresis loop. Tape wound cores also have lower losses. Magnetic properties of some core materials are given in Appendix C.

1.6 FRINGING

Fringing results from flux lines appearing along the sides and edges of magnetic members separated by air, as shown in Fig. 1-9; the effect increases with the area of the airgap. Fringing increases with the length of the airgap.

1.7 ENERGY STORED IN A MAGNETIC FIELD

The potential energy, W_m , stored in a magnetic field within a given volume, v , is defined by the volume integral

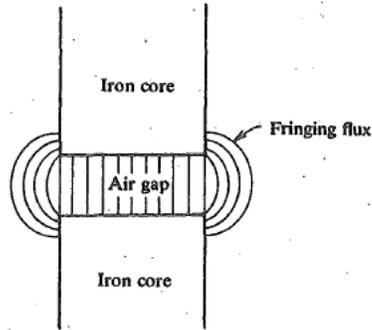


Fig. 1-9

$$W_m = \frac{1}{2} \int_v \mathbf{B} \cdot \mathbf{H} \, dv = \frac{1}{2} \mu \int_v H^2 \, dv = \frac{1}{2\mu} \int_v B^2 \, dv \quad (1.16)$$

1.8 INDUCTANCE CALCULATIONS

Inductance is defined as flux linkage per unit current;

$$L \equiv \frac{\lambda}{i} = \frac{N\phi}{i} \quad (1.17)$$

The unit of inductance is the *henry* (H). From (1.17) it is seen that 1 H = 1 Wb/A.

For a magnetic toroid wound with n distinct coils, as shown in Fig. 1-10, n^2 inductances may be defined:

$$L_{pq} \equiv \frac{\text{flux linking the } p\text{th coil due to the current in the } q\text{th coil}}{\text{current in the } q\text{th coil}} = \frac{N_p(k_{pq}\phi_q)}{i_q} \quad (1.18)$$

where k_{pq} , the fraction of the flux due to coil q that links coil p , is called the *coupling coefficient* between the two coils. By definition, $k_{pq} \leq 1$; a value less than 1 is attributable to leakage flux between the locations of coil p and coil q . When the two subscripts in (1.18) are equal, the inductance is termed *self-inductance*; when unequal, the inductance is termed *mutual inductance* between coils p and q . Inductances are symmetrical; that is, for all p and q ,

$$k_{qp} = k_{pq} \quad \text{and} \quad L_{qp} = L_{pq} \quad (1.19)$$

To express L_{pq} in terms of the magnetic-circuit parameters, we substitute $\phi_q = N_q i_q / \mathfrak{R}$ in (1.18), obtaining

$$L_{pq} = \frac{k_{pq} N_p N_q}{\mathfrak{R}} = k_{pq} N_p N_q \phi \quad (1.20)$$

where \mathfrak{R} is the reluctance of the magnetic circuit and ϕ is its permeance. We may replace \mathfrak{R} in (1.20) by $l/\mu A$ (for a circuit for which l and A can be defined) to get

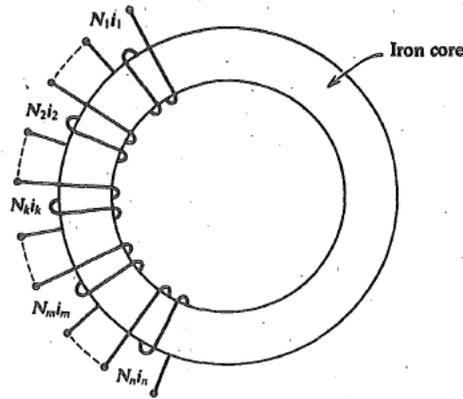


Fig. 1-10

$$L_{pq} = \left(\frac{\mu A}{l} \right) k_{pq} N_p N_q \tag{1.21}$$

Equations (1.17) through (1.21) can be used for inductance calculations. Alternatively, we may express the energy stored in an inductance L , carrying a current i , as

$$W_m = \frac{1}{2} Li^2 \tag{1.22}$$

and then obtain L by equating the right side of (1.22) to the right side of (1.16). For an n -coil system, the general relationship is

$$\frac{1}{2} \sum_{p=1}^n \sum_{q=1}^n L_{pq} i_p i_q = \frac{1}{2} \int_V \mathbf{B} \cdot \mathbf{H} dv \tag{1.23}$$

1.9 MAGNETIC CIRCUITS WITH PERMANENT MAGNETS

In Section 1.1 we mentioned that a permanent magnet is the source of a magnetic field. In a magnetic circuit excited by a permanent magnet, the operating conditions of the magnet are largely determined by its position in the circuit. The second-quadrant B - H characteristics (demagnetization curves) of a number of Alnico permanent magnets are shown in Fig. 1-11; Fig. 1-12 shows the characteristics of several ferrite magnets. Commercially available characteristics are still expressed in CGS units (which, if desired, may be converted to SI units by use of Appendix A). Through a point (H_d, B_d) of a demagnetization curve there pass a hyperbola giving the value of the energy product, $B_d H_d$, and a ray from the origin (of which only the distal end is shown) giving the value of the permeance ratio, B_d / H_d . The significance of the energy product is apparent from (1.16), and a permanent magnet is used most efficiently when the energy product is maximized.

Example 1.1 The remanence, B_r , of a permanent magnet is the value of B at zero H after saturation; the coercivity, H_c , is the value of H to reduce B to zero after saturation. Using Fig. 1-11, find B_r , H_c , and the maximum energy product, $(BH)_{max}$, for Alnico V. Compare with the value listed in Appendix C, Table C-1.

We read B_r and H_c from the vertical and horizontal intercepts, respectively, of the demagnetization curve.

$$B_r = 12.4 \times 10^3 \text{ gauss} = 1.24 \text{ T}$$

$$H_c = 630 \text{ oersteds} = 50 \text{ kA/m}$$

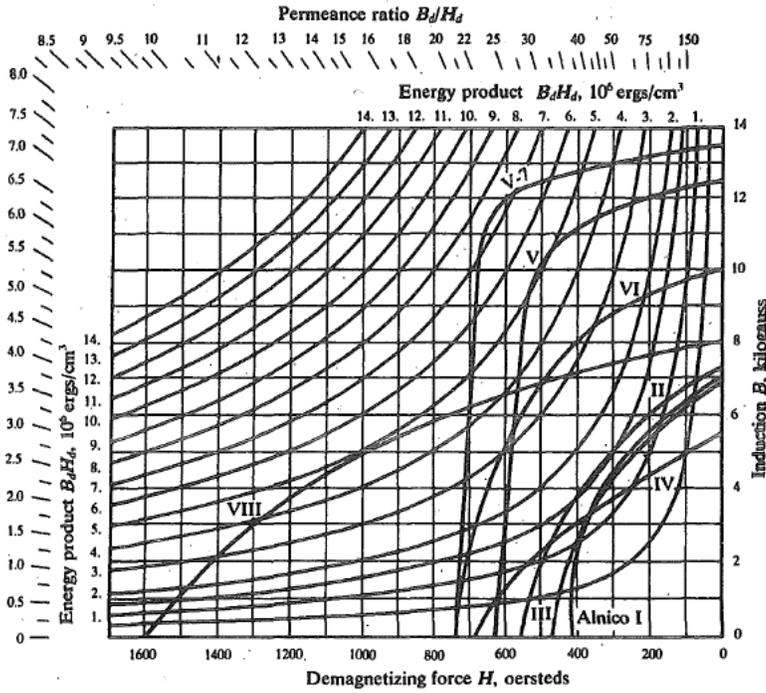


Fig. 1-11. Demagnetization and energy-product curves for Alnico magnets.

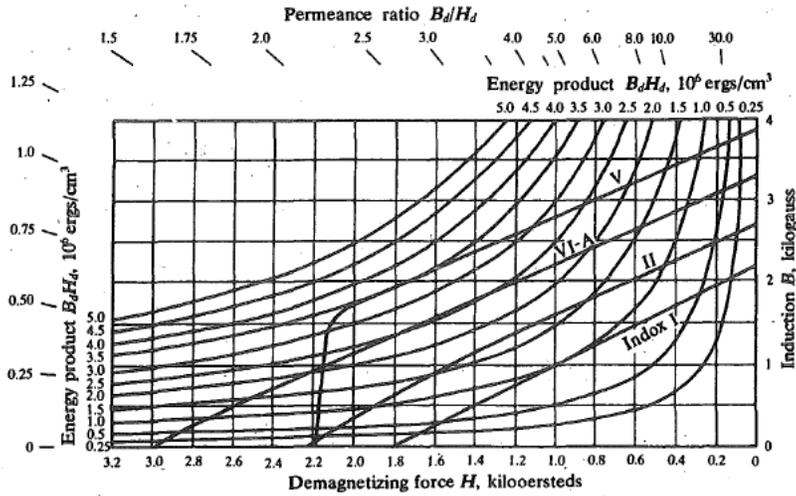


Fig. 1-12. Demagnetization and energy-product curves for Indox ceramic magnets.

where we use Appendix A to convert to SI units. These values are consistent with the ranges given for Alnico V in Table C-1.

The maximum energy product is read from the hyperbola that is just tangent at the knee of the demagnetization curve:

$$(BH)_{\max} = 5.2 \times 10^6 \text{ erg/cm}^3 = 520 \text{ kJ/m}^3$$

This value is roughly 10 times larger than the value inferred from Table C-1. However, we must remember that, in the CGS system, (1.16) is replaced by

$$W_m = \frac{1}{8\pi} \int_V \mathbf{B} \cdot \mathbf{H} \, dv$$

Thus, CGS energy products are expected to be 4π times as great as SI energy products. Making the adjustment:

$$\frac{1}{4\pi} (BH)_{\max} = \frac{5.2 \times 10^6}{4\pi} \text{ ergs/cm}^3 = 41 \text{ kJ/m}^3$$

and now the agreement with Table C-1 is very good.

Once the type of permanent magnet has been chosen, the design approach is as follows. From Ampere's law, for a circuit consisting of an airgap, a portion of a permanent magnet, and another ferromagnetic portion,

$$H_d l_m = H_g l_g + V_{ml} \quad (1.24)$$

where H_d = magnetic field intensity of the magnet, oersteds
 l_m = length of magnet, cm
 H_g = field intensity in the gap, oersteds = flux density in gap, gauss
 l_g = length of gap, cm
 V_{ml} = reluctance drop in the other ferromagnetic portion, gilberts

Observe that, because μ_0 is unity in the CGS system, H_g and B_g are numerically equal.

The cross-sectional area of the magnet is found from the flux required in the airgap via the relationship

$$B_d A_m = K B_g A_g \quad (1.25)$$

where B_d = flux density in the magnet, gauss
 A_m = cross-sectional area of magnet, cm^2
 B_g = flux density in the gap, gauss
 A_g = cross-sectional area of gap, cm^2
 K = dimensionless leakage factor

Formulas for the determination of leakage factors for some common configurations are given in Problem 1.17. Solving for A_m in (1.25) and for l_m in (1.24) (neglecting V_{ml}), and using $H_g = B_g$, we obtain

$$\text{volume} = A_m l_m = \frac{B_g^2 A_g l_g K}{B_d H_d} \text{ (cm}^3\text{)} \quad (1.26)$$

The permeance ratio, shown on Figs. 1-11 and 1-12, is, in the CGS system, the ratio of the equivalent permeance of the external circuit, $\Phi_{ge} \equiv A_g K / l_g$, to the permeance of the space occupied by the permanent magnet, $\Phi_m \equiv A_m / l_m$. This can be seen by solving (1.25) for B_d and (1.24) (neglecting V_{ml}) for H_d , and taking the ratio:

$$\frac{B_d}{H_d} = \frac{A_g I_m K}{A_m I_g} = \frac{\phi_{ge}}{\phi_m} \quad (\text{CGS units}) \quad (1.27)$$

Equation (1.27) is deceptively simple in appearance, for the task of obtaining analytical expressions for K —and therefore for ϕ_{ge} —is very difficult. Supposing the permeances known, (1.27) plots as a straight line (the *load line*) in the B - H plane, and the intersection of this line with the B - H curve gives the operating point of the magnet. See Problem 1.33(c).

Solved Problems

- 1.1. Find the magnetic field intensity due to an infinitely long, straight conductor carrying a current I amperes, at a point r meters away from the conductor.

From Fig. 1-13 and (1.6),

$$\oint \mathbf{H} \cdot d\mathbf{l} = 2\pi r H_\phi = I \quad \text{or} \quad H_\phi = \frac{I}{2\pi r} \quad (\text{A/m})$$

From the geometry of the problem, the radial and longitudinal components of \mathbf{H} are zero.

- 1.2. The conductor of Problem 1.1 carries 100 A current and is located in air. Determine the flux density at a point 0.05 m away from the conductor.

Since $B = \mu_0 H$, from Problem 1.1 we have

$$B_\phi = \mu_0 H_\phi = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 100}{2\pi \times 0.05} = 0.4 \text{ mT}$$

- 1.3. A rectangular loop is placed in the field of the conductor of Problem 1.1 as shown in Fig. 1-14. What is the total flux linking the loop?

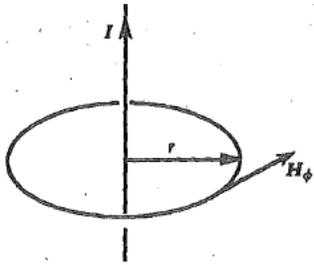


Fig. 1-13

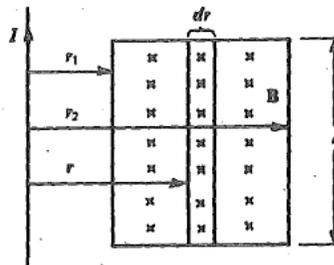


Fig. 1-14

Assuming a medium of permeability μ , we have from Problem 1.1

$$B_\phi = \mu H_\phi = \frac{\mu I}{2\pi r} \quad (\text{T})$$

The flux $d\phi$ in the elementary area $dA = l dr$ is given by

$$d\phi = B_{\phi} dA = \frac{\mu I l}{2\pi} \frac{dr}{r}$$

and

$$\phi = \frac{\mu I l}{2\pi} \int_{r_1}^{r_2} \frac{dr}{r} = \frac{\mu I l}{2\pi} \ln \frac{r_2}{r_1} \quad (\text{Wb})$$

- 1.4. A cast steel ring has a circular cross-section 3 cm in diameter and a mean circumference of 80 cm. The ring is uniformly wound with a coil of 600 turns. (a) Estimate the current in the coil required to produce a flux of 0.5 mWb in the ring. (b) If a saw cut creates a 2-mm airgap in the ring, find approximately the airgap flux produced by the current obtained in (a). Find the current which will produce the same flux in the airgap as in (a). Neglect fringing and leakage. Refer to Fig. 1-5(b) for the magnetization characteristic of cast steel.

(a) Ring cross section, $A = \frac{\pi}{4} \times 3^2 \times 10^{-4} = 7.07 \times 10^{-4} \text{ m}^2$

Core flux density, $B = \frac{\phi}{A} = \frac{0.5 \times 10^{-3}}{7.07 \times 10^{-4}} = 0.707 \text{ T}$

From Fig. 1.5(b), @ $B = 0.707 \text{ T}$, $H = 675 \text{ At/m}$

Mmf = $\mathcal{F} = Hl = 675 \times 0.8 = 540 \text{ At} = NI = 600 I$

Thus, $I = \frac{540}{600} = 0.9 \text{ A}$

(b) $\mathcal{R}_{\text{total}} = \mathcal{R}_{\text{core}} + \mathcal{R}_{\text{air}}$. From (a),

$$\mathcal{R}_{\text{core}} = \frac{\mathcal{F}}{\phi} = \frac{540}{0.5 \times 10^{-3}} = 1.08 \times 10^6 \text{ H}^{-1}$$

$$\mathcal{R}_{\text{air}} = \frac{g}{\mu_0 A} = \frac{2.0 \times 10^{-3}}{4\pi \times 10^{-7} \times 7.07 \times 10^{-4}} = 2.25 \times 10^6 \text{ H}^{-1}$$

$$\mathcal{R}_{\text{total}} = (1.08 + 2.25)10^6 = 3.33 \times 10^6 \text{ H}^{-1}$$

Airgap flux, $\phi_{\text{air}} = \frac{\mathcal{F}}{\mathcal{R}_{\text{total}}} = \frac{540}{3.33 \times 10^6} = 0.162 \text{ mWb}$

(c) To maintain $\phi = 0.5 \text{ mWb}$, $\mathcal{F} = R\phi = 3.33 \times 10^6 \times 0.5 \times 10^{-3} \text{ At}$

Or $\mathcal{F} = NI = 1665 \text{ At}$. Thus, $I = \frac{1665}{600} = 2.775 \text{ A}$.

- 1.5. For the magnetic circuit shown in Fig. 1-15, $N = 10$ turns, $l_g = 0.1 \text{ mm}$, $l_m = 100 \text{ mm}$, stacking factor = 0.9; the core material is M-19. Calculate I required to establish a 1-T flux density in the airgap. Neglect fringing and leakage.

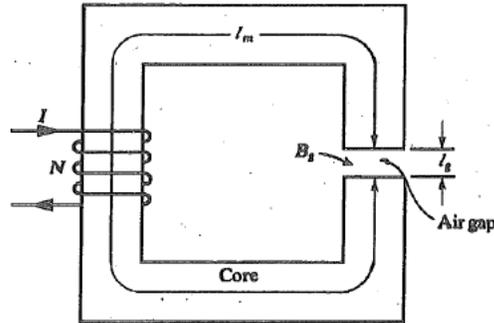


Fig. 1-15

For the airgap:

$$H_g = \frac{B_g}{\mu_0} = \frac{1.0}{4\pi \times 10^{-7}} = 7.95 \times 10^5 \text{ A/m}$$

$$\mathcal{F}_g = H_g l_g = (7.95 \times 10^5)(10^{-4}) = 79.5 \text{ At}$$

For the core:

$$B_m = \frac{B_g}{\text{stacking factor}} = \frac{1}{0.9} = 1.11 \text{ T}$$

and, from Appendix C, Fig. C-1, at 1.11 T we have

$$H_m = 130 \text{ A/m} \quad \text{and} \quad \mathcal{F}_m = (130)(0.0100) = 13 \text{ At}$$

Then the total required mmf is

$$\mathcal{F}_g + \mathcal{F}_m = 79.5 + 13 = 92.5 \text{ At}$$

from which

$$I = \frac{92.5}{10} = 9.25 \text{ A}$$

- 1.6. From Appendix C, determine the relative amplitude permeability for (a) AISI 1020 and (b) M-19, at a flux density of 1 T. ($\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$.)

$$(a) \quad \mu_a = \frac{1}{\mu_0} \left(\frac{1}{1600} \right) \approx 500$$

$$(b) \quad \mu_a = \frac{1}{\text{u.}} \left(\frac{1}{90} \right) \approx 8800$$

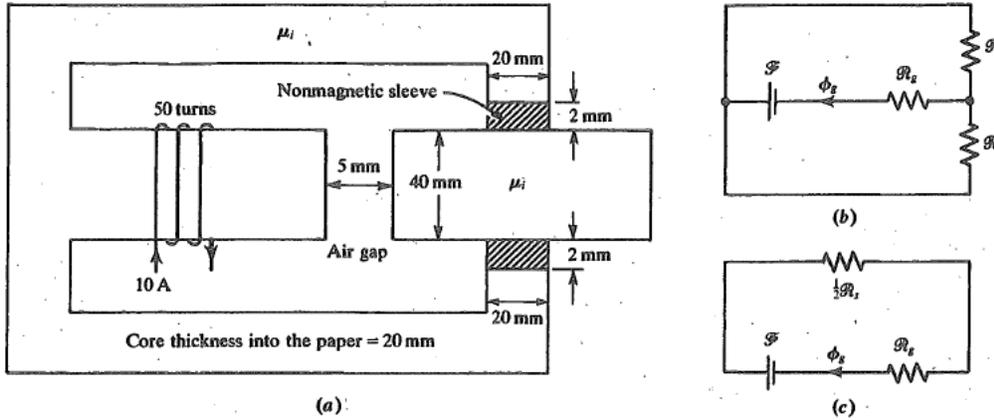


Fig. 1-16

1.7. Assuming an ideal core ($\mu_r \rightarrow \infty$), calculate the flux density in the airgap of magnetic circuit shown in Fig. 1-16(a).

The electrical analog, Fig. 1-16(b), may be reduced to the simpler Fig. 1-16(c). From the latter, and Table 1-1,

$$\mathfrak{R}_g \equiv \text{airgap reluctance} = \frac{5 \times 10^{-3}}{\mu_0(20 \times 40 \times 10^{-6})} = \frac{50}{8\mu_0}$$

$$\mathfrak{R}_s \equiv \text{sleeve reluctance} = \frac{2 \times 10^{-3}}{\mu_0(20 \times 20 \times 10^{-6})} = \frac{20}{4\mu_0}$$

$$\mathfrak{R}_t \equiv \text{total reluctance} = \mathfrak{R}_g + \frac{1}{2} \mathfrak{R}_s = \frac{70}{8\mu_0}$$

$$\phi_g \equiv \text{airgap flux} = \frac{\mathcal{F}}{\mathfrak{R}_t} = \frac{(50)(10)}{70/8\mu_0} = \frac{400\mu_0}{7}$$

$$B_g \equiv \text{airgap flux density} = \frac{\phi_g}{A_g} = \frac{400\mu_0/7}{20 \times 40 \times 10^{-6}}$$

Substituting $\mu_0 = 4\pi \times 10^{-7}$, we get $B_g = 90$ mT, or 900 gauss.

1.8. A composite magnetic circuit of varying cross section is shown in Fig. 1-17(a); the iron portion has the B - H characteristic of Fig. 1-17(b). Given: $N = 100$ turns; $l_1 = 4l_2 = 40$ cm; $A_1 = 2A_2 = 10$ cm²; $l_g = 2$ mm; leakage flux, $\phi_l = 0.01$ mWb. Calculate I required to establish an air-gap flux density of 0.6 T.

Corresponding to $B_g = 0.6$ T,

$$H_g = \frac{0.6}{\mu_0} = 4.78 \times 10^5 \text{ A/m}$$

$$\mathcal{F}_g = (4.78 \times 10^5)(2 \times 10^{-3}) = 956 \text{ At}$$

$$B_{II} = B_g = 0.6 \text{ T}$$

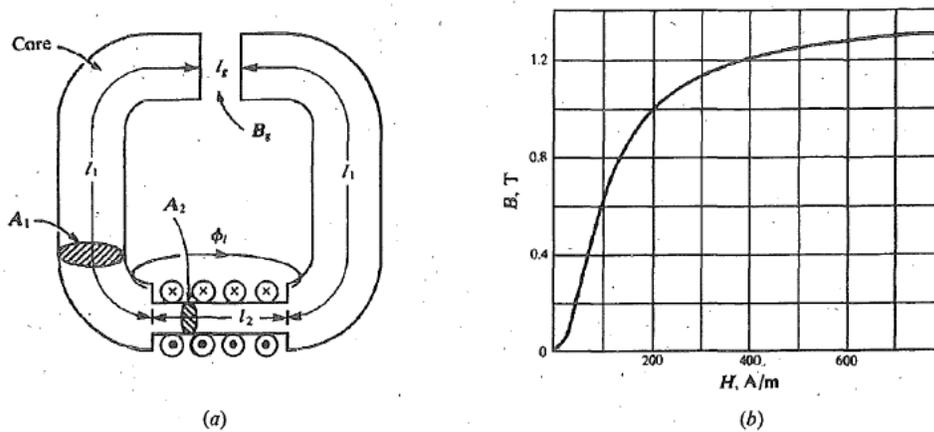


Fig. 1-17

From Fig. 1-17(b) at $B = 0.6$ T, $H = 100$ A/m. Thus, for the two lengths l_1 we have

$$\mathcal{F}_{l_1} = (100)(0.40 + 0.40) = 80 \text{ At}$$

The flux in the airgap, ϕ_g , is given by

$$\phi_g = B_g A_1 = (0.6)(10 \times 10^{-4}) = 0.6 \text{ mWb}$$

The total flux produced by the coil, ϕ_c , is the sum of the airgap flux and the leakage flux:

$$\phi_c = \phi_g + \phi_l = 0.6 + 0.01 = 0.61 \text{ mWb}$$

The flux density in the portion l_2 is, therefore,

$$B_2 = \frac{\phi_c}{A_2} = \frac{0.61 \times 10^{-3}}{5 \times 10^{-4}} = 1.22 \text{ T}$$

For this flux density, from Fig. 1-17(b), $H = 410$ A/m and

$$\mathcal{F}_{l_2} = (410)(0.10) = 41 \text{ At}$$

The total required mmf, \mathcal{F} , is thus

$$\mathcal{F} = \mathcal{F}_g + \mathcal{F}_{l_1} + \mathcal{F}_{l_2} = 956 + 80 + 41 = 1077 \text{ At}$$

For $N = 100$ turns, the desired current is, finally,

$$I = \frac{1077}{100} = 10.77 \text{ A}$$

1.9. Draw an electrical analog for the magnetic circuit shown in Fig. 1-17(a).

See Fig. 1-18.

1.10. Calculate the (total) self-inductance and the leakage inductance of the coil shown in Fig. 1-17(a).

From Problem 1.8 the total flux produced by the coil is $\phi_c = 0.61$ mWb and $I = 10.77$ A. Hence

$$L = \frac{N\phi_c}{I} = \frac{(100)(0.61 \times 10^{-3})}{10.77} = 5.66 \text{ mH}$$

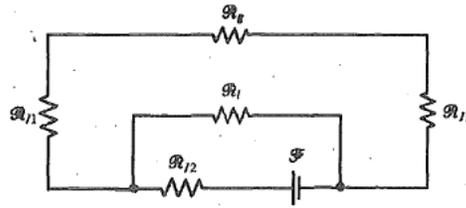


Fig. 1-18

and
$$L_l = \frac{N_{\phi l}}{I} = \frac{(100)(0.01 \times 10^{-3})}{10.77} = 0.093 \text{ mH}$$

- 1.11. Determine the magnetic energy stored in the iron and in the airgap of the magnetic circuit of Fig. 1-17(a).

From (1.16),

$$W_{\text{air}} = \frac{1}{2\mu_0} B_g^2 \times \text{vol}_{\text{gap}} = \frac{(0.6)^2}{2\mu_0} [(10 \times 10^{-4})(2 \times 10^{-3})] = 0.286 \text{ J}$$

From (1.22) and Problem 1.10,

$$W_{\text{iron}} = \frac{1}{2} LI^2 - W_{\text{air}} = \frac{1}{2} N\phi_c I - W_{\text{air}} = 0.328 - 0.286 = 0.042 \text{ J}$$

- 1.12. If the stacking factor is 0.8 and B_g remains 0.6 T, determine the flux densities in the various portions of the magnetic circuit of Fig. 1-17(a).

$$B_g = 0.6 \text{ T}$$

$$B_{11} = \frac{B_g}{\text{stacking factor}} = \frac{0.6}{0.8} = 0.75 \text{ T}$$

$$B_{12} = \frac{B_2}{\text{stacking factor}} = \frac{1.22}{0.8} = 1.525 \text{ T}$$

where the value of B_2 is taken from Problem 1.8.

- 1.13. A toroid of rectangular cross section is shown in Fig. 1-19. The mean diameter is large compared to the core thickness in the radial direction, so that the core flux density is uniform. Derive an expression for the inductance of the toroid, and evaluate it if $r_1 = 80 \text{ mm}$, $r_2 = 100 \text{ mm}$, $a = 20 \text{ mm}$, and $N = 200$ turns. The core relative permeability is 900.

flux linkage: $\lambda = N\phi$

flux:
$$\phi = \frac{Ni}{\mathfrak{R}} = \frac{\mu ANi}{2\pi r} = \frac{\mu a(r_2 - r_1)Ni}{\pi(r_2 + r_1)}$$

since $A = a(r_2 - r_1)$ and $r = (r_2 + r_1)/2$. Hence

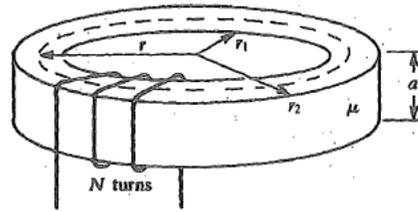


Fig. 1-19

$$L = \frac{\lambda}{i} = \frac{\mu a (r_2 - r_1) N^2}{\pi (r_2 + r_1)}$$

Substituting the numerical values, we obtain

$$L = \frac{(900\mu_0)(20 \times 10^{-3})(20 \times 10^{-3})(200)^2}{\pi(180 \times 10^{-3})} = 32 \text{ mH}$$

- 1.14. The density of the core material of the toroid of Fig. 1-19 is $7.88 \times 10^3 \text{ kg/m}^3$ and the core is wound with a round wire, AWG size 8. What is the total mass of the toroid?

$$\begin{aligned} \text{volume of core} &= (\pi r_2^2 - \pi r_1^2)a = \pi[(0.100)^2 - (0.080)^2](0.020) = 72\pi \times 10^{-6} \text{ m}^3 \\ \text{mass of core} &= (7.88 \times 10^3)(72\pi \times 10^{-6}) = 1.782 \text{ kg} \\ \text{mean length per turn} &= 2(a + r_2 - r_1) + (10\% \text{ for bending around corners}) = 0.088 \text{ m} \\ \text{total length of coils} &= (200)(0.088) = 17.6 \text{ m} \end{aligned}$$

From Appendix B, No. 8 wire weighs 50.2 lb/1000 ft, or 0.0747 kg/m. Thus

$$\begin{aligned} \text{mass of coil} &= (17.6)(0.0747) = 1.315 \text{ kg} \\ \text{total mass of toroid} &= 1.782 + 1.315 = 3.097 \text{ kg} \end{aligned}$$

- 1.15. For the toroid shown in Fig. 1-19, (a) derive an expression for the magnetic field intensity, $H(r)$. (b) What is the core flux if $\mu_r = 1$? (c) If the core flux density is assumed to be uniform, and equal to its value at the (arithmetic) mean radius, what percent error would be made in the computation of the core flux by this approximation as compared to the calculation in (b)? (d) If the geometric mean radius is used instead of the arithmetic mean, what is the percent error?

$$(a) \quad H = \frac{NI}{2\pi r}$$

$$(b) \quad \phi = \int_{r_1}^{r_2} \mu_0 \frac{NI}{2\pi r} a \, dr = \mu_0 \frac{aNI}{2\pi} \ln \frac{r_2}{r_1}$$

$$(c) \quad \text{Using the arithmetic mean, } B \approx \mu_0 NI / \pi(r_2 + r_1), \text{ and so}$$

$$\phi \approx \mu_0 \frac{aNl}{2\pi} \frac{2(r_2 - r_1)}{r_2 + r_1}$$

Let $r_2/r_1 = b$; then

$$\text{percent error} = 100 \left[1 - \frac{2(b-1)}{(b+1) \ln b} \right]$$

For example, if $b = 2$, percent error = 3.9%.

(d) Using the geometric mean, $B \approx \mu_0 NI / 2\pi \sqrt{r_2 r_1}$, and so

$$\phi \approx \mu_0 \frac{aNl}{2\pi} \frac{r_2 - r_1}{\sqrt{r_2 r_1}}$$

$$\text{percent error} = 100 \left[\frac{b-1}{\sqrt{b} \ln b} - 1 \right]$$

If $b = 2$, percent error = 2%.

1.16. Sometimes the B - H curve of a core material can be expressed by the *Froelich equation*,

$$B = \frac{aH}{b + H} \quad (1.28)$$

where a and b are constants of the material. Let $a = 1.5$ T and $b = 100$ A/m. A magnetic circuit consists of two parts (in series), of lengths l_1 and l_2 and cross-sectional areas A_1 and A_2 . If $A_1 = 25$ cm² = $2A_2$ and $l_1 = 25$ cm = $\frac{1}{2}l_2$, and if the core carries an mmf of 1000 At, calculate the core flux.

From (1.28),

$$B = \frac{1.5 H}{100 + H} \quad (\text{T})$$

and for the magnetic circuit,

$$\mathcal{F} = H_1 l_1 + H_2 l_2 = \frac{1}{4} H_1 + \frac{1}{2} H_2 = 1000$$

and

$$\phi = B_1 A_1 = B_2 A_2 \quad \text{or} \quad 2B_1 = B_2 \quad \text{or} \quad \frac{3.0 H_1}{100 + H_1} = \frac{1.5 H_2}{100 + H_2}$$

Eliminating H_1 between the above two equations yields

$$-3H_2^2 + 5250 + 1200000 = 0 \quad \text{or} \quad H_2 = 1954.6 \text{ A/m}$$

Thus

$$B_2 = \frac{(1.5)(1954.6)}{100 + 1954.6} = 1.427 \text{ T}$$

and
$$\phi = (1.427) \left(\frac{25}{2} \times 10^{-4} \right) = 1.784 \text{ mWb}$$

If the B - H characteristic had been given in graphical form, this problem could only be solved by trial and error. See Problem 1.31.

- 1.17. Determine the length, b , and cross-sectional area, A_m , of the magnet in Fig. 1-20(a) to produce a flux density of 2500 gauss in the airgap. The permanent magnet to be used is Alnico V; the dimensions of Fig. 1-20(a) are as follows: $l_g = 0.4$ cm, $c = 6.0$ cm, gap area = 4.0 cm^2 ($2 \text{ cm} \times 2 \text{ cm}$). We assume that the reluctance in the soft-iron parts of the circuit is negligible, giving a reluctance drop, V_{mp} , of zero; we estimate that the leakage factor is 4.0 and that the magnet is to be operated at maximum energy product (knee of the demagnetization curve in Fig. 1-11).

From (1.25),

$$A_m = \frac{B_g A_g K}{B_d} = \frac{(2500)(4)(4.0)}{10.5 \times 10^3} = 3.8 \text{ cm}^2$$

From (1.24), noting that $H_g = B_g$ in CGS units,

$$b = l_m = \frac{(2500)(0.4)}{450} = 2.22 \text{ cm}$$

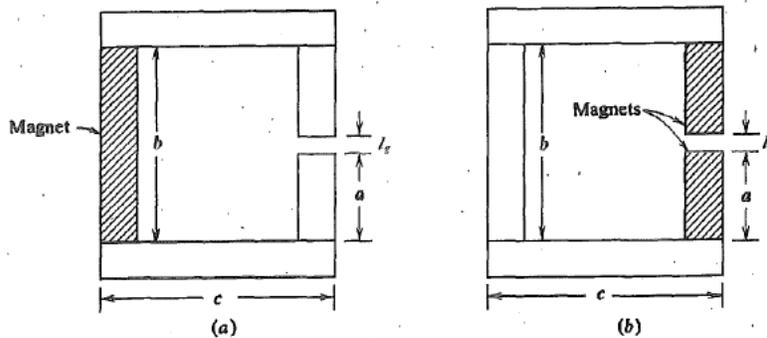


Fig. 1-20

We must now check our estimate of the leakage factor. A leakage factor for the configuration of Fig. 1-20(a) is given by

$$K = 1 + \frac{l_g}{A_g} \left[1.7 C_a \left(\frac{a}{a + l_g} \right) + 1.4 c \sqrt{\frac{C_c}{b}} + 0.67 C_b \right] \quad (1.29)$$

where C_a , C_b , and C_c are the cross-sectional perimeters of the circuit portions whose lengths are a , b , and c , respectively. The factor 0.67 in (1.29) arises from the fact that permanent magnets have a "neutral zone" that does not contribute to leakage. Substituting $b = 2.2$ cm, $a = (b - l_g)/2 = 0.91$ cm, $c = 6.0$ cm, $C_a = (4)(2) = 8$ cm, $C_c = 8.0$ cm, and $C_b = 4\sqrt{3.8} = 7.80$ cm into (1.29) gives

$$K = 4.062$$

This value could now be put back into (1.25), giving a slightly different value of A_m . This, in turn would change C_a in (1.29), resulting in a new value of leakage factor. A few iterations of these formulas are usually necessary to obtain a consistent set of dimensions for the total magnetic circuit.

The high value for leakage factor obtained for the configuration of Fig. 1-20(a) indicates that this is not a very efficient magnetic circuit. A much more efficient use of the permanent magnet is to locate it adjacent to the airgap, as shown in Fig. 1-20(b). The leakage factor for Fig. 1-20(b) is

$$K = 1 + \frac{l_g}{A_g} 0.67 C_a \left[1.7 \left(\frac{0.67 C_a}{0.67 a + l_g} \right) + \frac{l_g}{2a} \right] \quad (1.30)$$

Using the same dimensions for all sections of the circuit in Fig. 1-20(b) as were used in Fig. 1-20(a) (even though this might result in an oversized permanent magnet), we obtain from (1.30)

$$K = 1.624$$

- 1.18. An inductor, made of a highly permeable material, has N turns. The dimensions of the core and the coil are as shown in Fig. 1-21. Calculate the input power to the coil to establish a given flux density B in the airgap. The winding space-factor of the coil is k_r and its conductivity is σ .

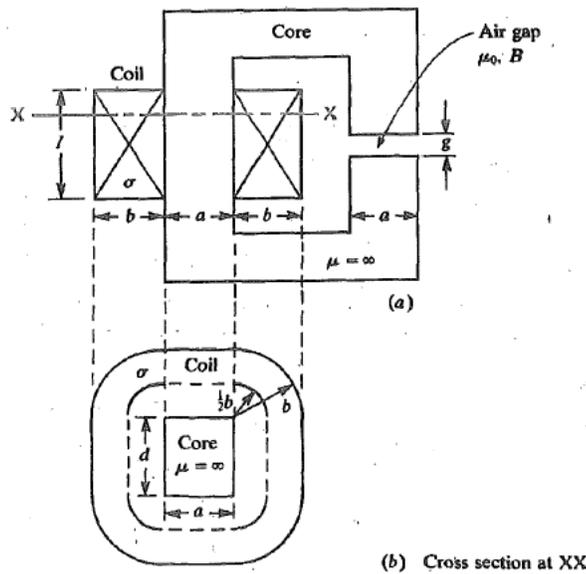


Fig. 1-21

From Fig. 1-21(b), the mean length per turn is

$$l = 2a + 2d + 4 \left(\frac{1}{4} \right) \left(2\pi \frac{b}{2} \right) = 2 \left(a + d + \pi \frac{b}{2} \right)$$

and the total length of wire making the coil is lN . Let A_c denote the cross-sectional area of the wire; then its resistance is

$$R = \frac{lN}{\sigma A_c}$$

giving an input power of

$$P_i = I^2 R = \frac{I^2 l N}{\sigma A_c}$$

But

$$\mathcal{F} = NI = \phi \mathcal{R} = BA \frac{g}{\mu_0 A} = \frac{Bg}{\mu_0} \quad \text{or} \quad I = \frac{Bg}{\mu_0 N}$$

Substituting above yields

$$P_i = \frac{B^2 g^2 l}{\mu_0^2 \sigma N A_c}$$

The total volume of the wire is $(bc)k_s = lNA_c$. Hence, finally,

$$P_i = \frac{B^2 g^2 l}{\mu_0^2 \sigma bck_g}$$

1.19. The inductor of Problem 1.18 is made of magnet wire. If the dimensions in Fig. 1-21 are

$$a = b = c = d = 25 \text{ mm} \quad g = 2 \text{ mm}$$

and the core flux density is 0.8 T, calculate the input power and the number of turns. Assume $k_s = 0.8$, $\sigma = 5.78 \times 10^7 \text{ S/m}$, and a coil current of 1 A. (Note: $1 \text{ S} = 1 \Omega^{-1}$.)

From Problem 1.18,

$$l = 2 \left(1 + 1 + \frac{\pi}{2} \right) (25 \times 10^{-3}) = 0.1785 \text{ m}$$

$$P_i = \frac{(0.8)^2 (2 \times 10^{-3})^2 (0.1785)}{(4\pi \times 10^{-7})^2 (5.78 \times 10^7) (25 \times 10^{-3})^2 (0.8)} = 10 \text{ W}$$

Also from Problem 1.18

$$N = \frac{Bg}{\mu_0 I} = \frac{(0.8)(2 \times 10^{-3})}{(4\pi \times 10^{-7})(1)} = 1273 \text{ turns}$$

1.20. For the inductor of Problem 1.19, find the area of the conductor cross section. What is the time constant of the coil and at what voltage may it be operated?

From $P_i = I^2 R = 10 \text{ W}$ and $I = 1 \text{ A}$, $R = 10 \Omega$. Then the operating voltage is

$$V = IR = (1)(10) = 10 \text{ V}$$

Also, from $R = lN/\sigma A_c$

$$A_c = \frac{lN}{\sigma R} = \frac{(0.1785)(1273)}{(5.78 \times 10^7)(10)} = 0.393 \times 10^{-6} \text{ m}^2 = 0.393 \text{ mm}^2$$

The flux linking the coil is $\phi = Bad$, and so

$$L = \frac{Bad}{I} = \frac{(0.8)(25 \times 10^{-3})^2}{1} = 0.5 \text{ mH}$$

and the time constant is

$$\tau = \frac{L}{R} = \frac{0.5 \times 10^{-3}}{10} = 50 \mu\text{s}$$

Notice that from the value of A_c , a suitable wire size can be determined.

Supplementary Problems

- 1.21. From Appendix C, Fig. C-1, determine the relative amplitude permeability at a flux density of 1.2 T for (a) M-19 and (b) 48 NI. *Ans.* (a) 5457; (b) 9550
- 1.22. Replot the B - H curve for M-19 on rectangular coordinate paper. (Figure C-1 of Appendix C is plotted on semilog paper.) Identify the three ranges of permeability, I, II, and III, of Fig. 1-5. *Ans.* For range II: $0.4 \leq B \leq 0.8$ T
- 1.23. The magnetic circuit of Fig. 1-22 has the B - H characteristic of Fig. 1-17(b). Calculate the mmf of the coil to establish a 1-T flux density in the airgap. *Ans.* 902 At

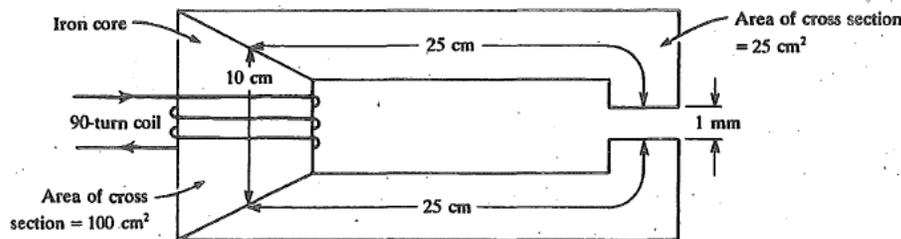


Fig. 1-22

- 1.24. The coil of Fig. 1-22 has 90 turns. For the data of Problem 1.23, determine (a) the energy stored in the coil, (b) the energy stored in the airgap, (c) the energy stored in iron. *Ans.* (a) 1.13 J; (b) 0.995 J; (c) 0.135 J
- 1.25. Calculate the inductance of the coil of Fig. 1-22, (a) excluding the effect of the iron core (i.e., assuming the core to be infinitely permeable) and (b) including the effect of the iron core. *Ans.* (a) 25.45 mH; (b) 22.45 mH
- 1.26. The toroid of Fig. 1-19 is cut to make an airgap 2 mm in length. Given: $r = 500$ mm; core composed of 0.2-mm strips of 48 NI magnetic material; core flux density is 0.6 T; and $N = 100$ turns. Including the effect of stacking factor, calculate the coil current for the given flux density. *Ans.* 8.66 A
- 1.27. A system of three coils on an ideal core is shown in Fig. 1-23, where $N_1 = N_3 = 2N_2 = 500$ turns, $g_1 = 2g_2 = 2g_3 = 4$ mm, and $A = 1000$ mm². Calculate (a) the self-inductance of coil N_1 and (b) the mutual inductance between coils N_2 and N_3 . *Ans.* (a) 62.83 mH; (b) 31.42 mH

- 1.28. If gap g_1 (Fig. 1-23) is closed, what are the mutual inductances between (a) N_1 and N_2 , (b) N_2 and N_3 , and (c) N_3 and N_1 ? *Ans.* (a) 78.54 mH; (b) 0; (c) 157.08 mH

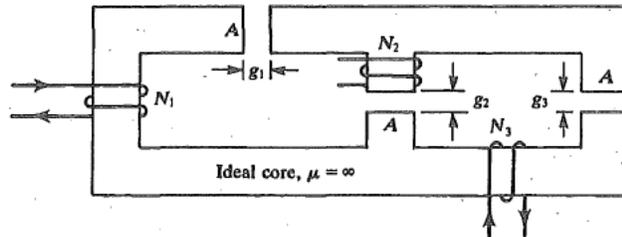


Fig. 1-23

- 1.29. The coils of Problem 1.28 are connected in series (with all mutual inductances being positive) and carry a current of 10 A. What is the total energy stored in the entire magnetic circuit? *Ans.* 37.3 J
- 1.30. The toroid of Fig. 1-19 is made of 0.019-in-thick silicon steel laminations having the characteristics shown in Appendix C, Fig. C-2. The density of the material is $7.88 \times 10^3 \text{ kg/m}^3$ and the dimensions of the core are $r_1 = 100 \text{ mm}$, $r_2 = 120 \text{ mm}$, and $a = 40 \text{ mm}$. If the maximum core flux density is 1 T at 150 Hz, determine the total core loss. *Ans.* 19.2 W

Permittivity is concerned with electric fields and is the "ability of a material to polarize in response to the (electric) field", while permeability is concerned with magnetic fields and is "the degree of magnetization of a material in response to a magnetic field"

Permittivity of free space: $\epsilon_0 = 8.8541878176 \times 10^{-12}$ F/m

Permeability of free space: $\mu_0 = 4(\pi) \times 10^{-7}$ H/m

Interestingly: $c = 1/\sqrt{\mu_0\epsilon_0}$, where c is the speed of light

Let's start by writing Maxwell's equations for the vacuum and without charges and currents:

$$\nabla \cdot \mathbf{E} = 0 \quad \nabla \cdot \mathbf{E} = 0$$

$$\nabla \times \mathbf{E} = -\partial \mathbf{B} / \partial t \quad \nabla \times \mathbf{E} = -\partial \mathbf{B} / \partial t$$

$$\nabla \cdot \mathbf{B} = 0 \quad \nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{B} = \mu_0 \epsilon_0 \partial \mathbf{E} / \partial t \quad \nabla \times \mathbf{B} = \mu_0 \epsilon_0 \partial \mathbf{E} / \partial t$$

Taking the curl of the second equation we have

$$\nabla \times (\nabla \times \mathbf{E}) = -\nabla \times (\partial \mathbf{B} / \partial t) \quad \nabla \times (\nabla \times \mathbf{E}) = -\nabla \times (\partial \mathbf{B} / \partial t)$$

Using a known identity ($\text{curl} = \text{grad div} - \nabla^2$), and interchanging second order derivatives, we have

$$\nabla (\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = -\partial \partial t (\nabla \times \mathbf{B}) \quad \nabla (\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = -\partial \partial t (\nabla \times \mathbf{B})$$

div E is zero (first equation), and curl B can be replaced from fourth equation:

$$\nabla^2 \mathbf{E} = 1/c^2 \partial^2 \mathbf{E} / \partial t^2, \quad \nabla^2 \mathbf{E} = 1/c^2 \partial^2 \mathbf{E} / \partial t^2,$$

where

$$1/c^2 = \mu_0 \epsilon_0$$

This is the wave equation for (transversal) waves propagating at velocity c .

The laws of Electromagnetism predict that should be (electromagnetic) waves moving at speed c . Some of them (those with wavelengths between roughly 0.4 and 0.8 μ m) are called light, and therefore c is called the speed of light.

Introduction to Maxwell's Equations

Maxwell's Equations are a set of 4 complicated equations that describe the world of electromagnetics. These equations describe how electric and magnetic fields propagate, interact, and how they are influenced by objects.

James Clerk Maxwell [1831-1879] was an Einstein/Newton-level genius who took a set of known experimental laws (Faraday's Law, Ampere's Law) and unified them into a symmetric coherent set of Equations known as Maxwell's Equations. Maxwell was one of the first to determine the speed of propagation of electromagnetic (EM) waves was the same as the speed of light - and hence to conclude that EM waves and visible light were really the same thing.

Maxwell's Equations are critical in understanding [Antennas](#) and Electromagnetics. They are formidable to look at - so complicated that most electrical engineers and physicists don't even really know what they mean. Shrouded in complex math (which is likely so "intellectual" people can feel superior in discussing them), true understanding of these equations is hard to come by.

This leads to the reason for this website - an intuitive tutorial of Maxwell's Equations. I will avoid if at all possible the mathematical difficulties that arise, and instead describe what the equations mean. And don't be afraid - the math is so complicated that those who do understand complex vector calculus still cannot apply Maxwell's Equations in anything but the simplest scenarios. For this reason, intuitive knowledge of Maxwell's Equations is far superior than mathematical manipulation-based knowledge. To understand the world, you must understand what equations mean, and not just know mathematical constructs. I believe the accepted methods of teaching electromagnetics and Maxwell's Equations do not produce understanding. And with that, let's say something about these equations.

Maxwell's Equations are laws - just like the law of gravity. These equations are rules the universe uses to govern the behavior of electric and magnetic fields. A flow of electric current will produce a magnetic field. If the current flow varies with time (as in any wave or periodic signal), the magnetic field will also give rise to an electric field. Maxwell's Equations shows that separated charge (positive and negative) gives rise to an electric field - and if this is varying in time as well will give rise to a propagating electric field, further giving rise to a propagating magnetic field.

To understand Maxwell's Equations at a more intuitive level than most Ph.Ds in Engineering or Physics, click through the links and definitions above. You'll find that the complicated math masks an inner elegance to these equations - and you'll learn how the universe operates the Electromagnetic Machine.

1. $\nabla \cdot \mathbf{D} = \rho_v$
2. $\nabla \cdot \mathbf{B} = 0$
3. $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
4. $\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}$

Here is a tutorial video explaining Maxwell's Equations intuitively:

[Top: Maxwell's Equations](https://www.feynmanlectures.caltech.edu/II_18.html)
https://www.feynmanlectures.caltech.edu/II_18.html

Chapter 2

Power Transformers

2.1 TRANSFORMER OPERATION AND FARADAY'S LAW

A transformer is an electromagnetic device having two or more stationary coils coupled through a mutual flux. A two-winding *ideal transformer* is shown in Fig. 2-1. An ideal transformer is assumed to have (i) an infinitely permeable core with no losses, (ii) lossless electrical windings, and (iii) no leakage fluxes.

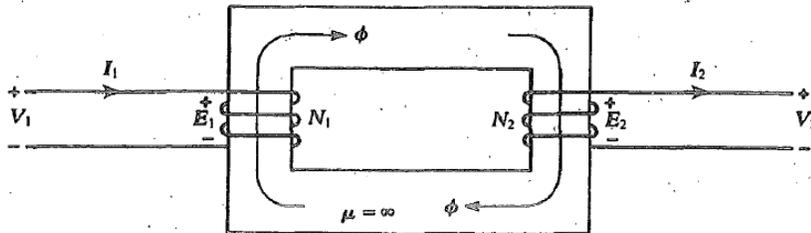


Fig. 2-1

The basic components of a transformer are the core, the *primary winding* N_1 , and the *secondary winding* N_2 . The action of a transformer is based on *Faraday's law of electromagnetic induction*, according to which a time-varying flux linking a coil induces an emf (voltage) in it. Thus, referring to Fig. 2-1, if ϕ is the flux linking the N_1 -turn winding, then its induced voltage, e_1 , is given by

$$e_1 = N_1 \frac{d\phi}{dt} \quad (\text{V}) \quad (2.1)$$

The direction of e_1 is such as to produce a current that gives rise to a flux which opposes the flux change $d\phi/dt$ (*Lenz's law*). The transformer being ideal, $e_1 = v_1$; that is, the instantaneous values of the induced voltage and the terminal voltage are equal. Hence, from (2.1),

$$\phi = \frac{1}{N_1} \int v_1 dt \quad (\text{Wb}) \quad (2.2)$$

Because only the time-variation of ϕ is important, we ignore the constant of integration in (2.2).

If

$$\phi = \phi_m \sin \omega t \quad (2.3)$$

then, from (2.1),

$$e_1 = \omega N_1 \phi_m \cos \omega t \quad (2.4)$$

Similarly, the voltage, e_2 , induced in the secondary is given by

$$e_2 = \omega N_2 \phi_m \cos \omega t \quad (2.5)$$

From (2.4) and (2.5),

$$\frac{e_1}{e_2} = \frac{N_1}{N_2}$$

which may also be written in terms of rms values as

$$\frac{E_1}{E_2} = \frac{N_1}{N_2} \equiv a \quad (2.6)$$

where a is known as the *turns ratio*. In the case that $N_2 > N_1$, one conventionally writes $1/a$ instead of a in (2.6); thus, the turns ratio is always greater than 1.

Because the transformer is ideal, the net mmf around the magnetic circuit must be zero; that is, if I_1 and I_2 are the primary and secondary currents, respectively, the $N_1I_1 - N_2I_2 = 0$, or

$$\frac{I_2}{I_1} = \frac{N_1}{N_2} \equiv a \quad (2.7)$$

From (2.6) and (2.7) it can be shown that if an impedance Z_2 is connected to the secondary, the impedance Z_1 seen at the primary satisfies

$$\frac{Z_1}{Z_2} = \left(\frac{N_1}{N_2} \right)^2 \equiv a^2 \quad (2.8)$$

2.2 EMF EQUATION OF A TRANSFORMER

For a sinusoidal flux, the rms value of the induced emf in the primary is, from (2.4),

$$E_1 = \frac{\omega N_1 \phi_m}{\sqrt{2}} = 4.44 f N_1 \phi_m \quad (V) \quad (2.9)$$

where $f = \omega/2\pi$ is the frequency in Hz.

2.3 TRANSFORMER LOSSES

In Section 2.1 we have considered an ideal transformer, which was assumed to have no losses. Obviously, an actual transformer has the following losses:

1. Core losses, which include the hysteresis and eddy-current losses (see Section 1.4).
2. Resistive (I^2R) losses in the primary and secondary windings.

2.4 EQUIVALENT CIRCUITS OF NONIDEAL TRANSFORMERS

A nonideal transformer differs from an ideal transformer in that the former has hysteresis and eddy-current (or core) losses, and has resistive (I^2R) losses in its primary and secondary windings. Furthermore, the core of a nonideal transformer is not perfectly permeable, and the transformer core requires a finite mmf for its magnetization. Also, not all fluxes link with the primary and secondary windings simultaneously because of leakages. Referring to Fig. 2-2, we observe that R_1 and R_2 are the respective resistances of the primary and secondary windings. The flux ϕ_c , which replaces the flux ϕ of Fig. 2-1, is called the *core flux* or *mutual flux*, as it links both the primary and secondary windings. The primary and secondary leakage fluxes are shown as ϕ_{l1} and ϕ_{l2} , respectively. Thus in Fig. 2-2 we have accounted for all the imperfections listed above, except the core losses. We will include the core losses as well as the rest of the imperfections in the equivalent circuit of a nonideal transformer. This circuit is also known as the *exact equivalent circuit*,

as it differs from the idealized equivalent circuit and the various approximate equivalent circuits. We now proceed to derive these circuits.

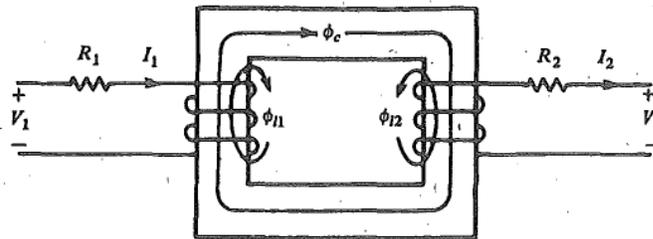


Fig. 2-2

An equivalent circuit of an ideal transformer is shown in Fig. 2-3(a). When the nonideal effects of winding resistances, leakage reactances, magnetizing reactance, and core losses are included, the circuit of Fig. 2-3(a) is modified to that of Fig. 2-3(b), where the primary and the secondary are coupled by an ideal transformer. By use of (2.6), (2.7), and (2.8), the ideal transformer may be removed from Fig. 2-3(b) and the entire equivalent circuit may be referred either to the primary, as shown in Fig. 2-4(a), or to the secondary, as shown in Fig. 2-4(b).

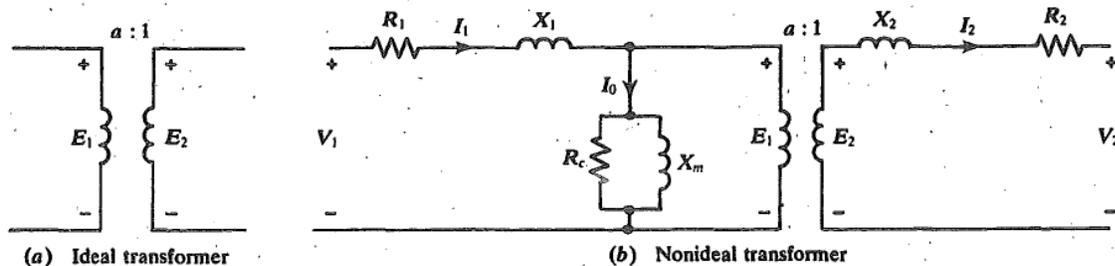


Fig. 2-3

A phasor diagram for the circuit Fig. 2-4(a), for lagging power factor, is shown in Fig. 2-5. In Figs. 2-3, 2-4, and 2-5 the various symbols are:

- a \equiv turns ratio (>1)
- E_1 \equiv primary induced voltage
- E_2 \equiv secondary induced voltage
- V_1 \equiv primary terminal voltage
- V_2 \equiv secondary terminal voltage
- I_1 \equiv primary current
- I_2 \equiv secondary current
- I_0 \equiv no-load (primary) current
- R_1 \equiv resistance of the primary winding
- R_2 \equiv resistance of the secondary winding
- X_1 \equiv primary leakage reactance
- X_2 \equiv secondary leakage reactance
- I_m, X_m \equiv magnetizing current and reactance
- I_c, R_c \equiv current and resistance accounting for the core losses

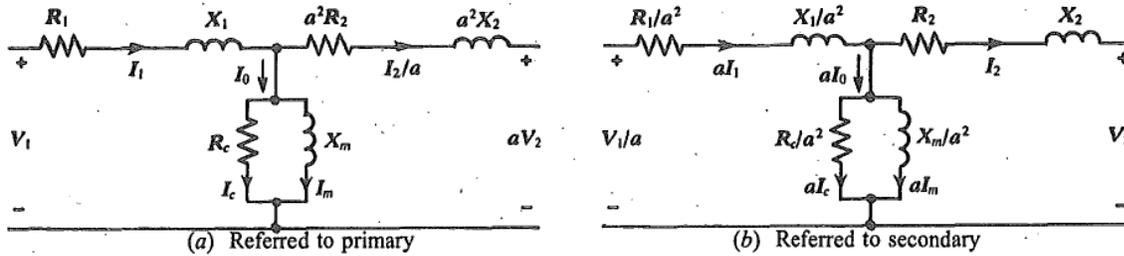


Fig. 2-4. Equivalent circuits of a nonideal transformer.

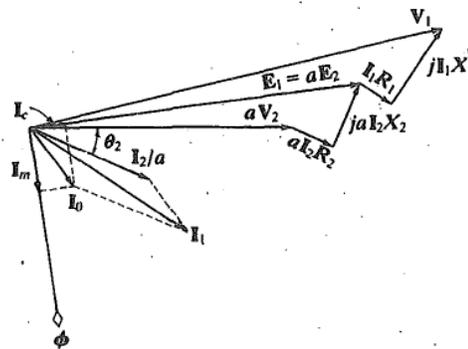


Fig. 2-5. Phasor diagram corresponding to Fig. 2-4(a).

2.5 TESTS ON TRANSFORMERS

Transformer performance characteristics can be obtained from the equivalent circuits of Section 2.4. The circuit parameters are determined either from design data or from test data. The two common tests are as follows.

Open-Circuit (or No-Load) Test

Here one winding is open-circuited and voltage—usually, rated voltage at rated frequency—is applied to the other winding. Voltage, current, and power at the terminals of this winding are measured. The open-circuit voltage of the second winding is also measured, and from this measurement a check on the turns ratio can be obtained. It is usually convenient to apply the test voltage to the winding that has a voltage rating equal to that of the available power source. In step-up voltage transformers, this means that the open-circuit voltage of the second winding will be higher than the applied voltage, sometimes much higher. Care must be exercised in guarding the terminals of this winding to ensure safety for test personnel and to prevent these terminals from getting close to other electrical circuits, instrumentation, grounds, and so forth.

In presenting the no-load parameters obtainable from test data, it is assumed that voltage is applied to the primary and the secondary is open-circuited. The no-load power loss is equal to the wattmeter reading in this test; core loss is found by subtracting the ohmic loss in the primary, which is usually small and may be neglected in some cases. Thus, if P_0 , I_0 , and V_0 are the input power, current, and voltage, then the core loss is given by

$$P_c = P_0 - I_0^2R_1 \tag{2.10}$$

The primary induced voltage is given in phasor form by

$$E_1 = V_0 \angle 0^\circ - (I_0 \angle \theta_0)(R_1 + jX_1) \quad (2.11)$$

where $\theta_0 \equiv$ no-load power-factor angle $= \cos^{-1}(P_0/V_0 I_0) < 0$. Other circuit quantities are found from:

$$R_c = \frac{E_1^2}{P_c} \quad (2.12)$$

$$I_c = \frac{P_c}{E_1} \quad (2.13)$$

$$I_m = \sqrt{I_0^2 - I_c^2} \quad (2.14)$$

$$X_m = \frac{E_1}{I_m} \quad (2.15)$$

$$a \approx \frac{V_0}{E_2} \quad (2.16)$$

Short-Circuit Test

In this test, one winding is short-circuited across its terminals, and a reduced voltage is applied to the other winding. This reduced voltage is of such a magnitude as to cause a specific value of current—usually, rated current—to flow in the short-circuited winding. Again, the choice of the winding to be short-circuited is usually determined by the measuring equipment available for use in the test. However, care must be taken to note which winding is short-circuited, for this determines the reference winding for expressing the impedance components obtained by this test. Let the secondary be short-circuited and the reduced voltage be applied to the primary.

With a very low voltage applied to the primary winding, the core-loss current and magnetizing current become very small, and the equivalent circuit reduces to that of Fig. 2-6. Thus, if P_s , I_s , and V_s are the input power, current, and voltage under short circuit, then, referred to the primary,

$$Z_s = \frac{V_s}{I_s} \quad (2.17)$$

$$R_1 + a^2 R_2 \equiv R_s = \frac{P_s}{I_s^2} \quad (2.18)$$

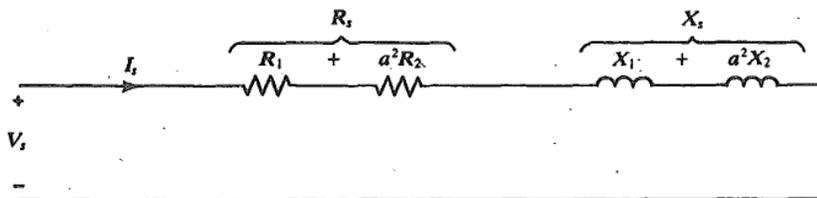


Fig. 2-6

$$X_1 + a^2 X_2 \equiv X_s = \sqrt{Z_s^2 - R_s^2} \tag{2.19}$$

Given R_1 and a , R_2 can be found from (2.18). In (2.19) it is usually assumed that the leakage reactance is divided equally between the primary and the secondary; that is,

$$X_1 = a^2 X_2 = \frac{1}{2} X_s \tag{2.20}$$

2.6 TRANSFORMER CONNECTIONS

Of the eight types of transformer connection shown in Table 2-1, the first six are for purposes of voltage transformation and the last two are for changing the number of phases. (Not included is the single-phase voltage transformer.) Each line segment in the diagrams corresponds to one winding of a two-winding transformer.

Table 2-1

Type of Connection	Primary	Secondary
Two-phase		
Three-phase, delta-delta		
Three-phase, delta-wye		
Three-phase, wye-wye		
Three-phase, open-delta		
Three-phase, tee		
Two-phase-to-three-phase (Scott)		
Three-phase-to-six-phase (diametrical)		

It is important to observe the polarity markings in polyphase transformer connections, and for the sake of illustration the connection of three identical transformers in delta-wye is shown in some detail in Fig. 2-7, which also shows the voltage phasor diagram. Notice the 30° phase shift between the line and phase voltages.

Basic Point-to-Point Calculation Procedure

Step 1. Determine the transformer full load amps (F.L.A.) from

$$3\phi \text{ Transformer } I_{F.L.A.} = \frac{kVA \times 1000}{E_{L-L} \times 1.732}$$

$$1\phi \text{ Transformer } I_{F.L.A.} = \frac{kVA \times 1000}{E_{L-L}}$$

either the nameplate, the following formulas or Table 1:

$$\text{Multiplier} = \frac{100}{\%Z_{\text{transformer}}}$$

Step 2. Find the transformer multiplier. See Notes 1 and 2

* **Note 1.** Get %Z from nameplate or Table 1. Transformer impedance (Z) helps to determine what the short circuit current will be at the transformer secondary. Transformer impedance is determined as follows: The transformer secondary is short circuited. Voltage is increased on the primary until full load current flows in the secondary. This applied voltage divided by the rated primary voltage (times 100) is the impedance of the transformer.

Example: For a 480 Volt rated primary, if 9.6 volts causes secondary full load current to flow through the shorted secondary, the transformer impedance is $9.6/480 = .02 = 2\%Z$.

* **Note 2.** In addition, UL 1561 listed transformers 25kVA and larger have a $\pm 10\%$ impedance tolerance. Short circuit amps can be affected by this tolerance. Therefore, for high end worst case, multiply %Z by .9. For low end of worst case, multiply %Z by 1.1. Transformers constructed to ANSI standards have a $\pm 7.5\%$ impedance tolerance (two-winding construction).

Step 3. Determine by formula or Table 1 the transformer let-through short-circuit current. See Notes 3 and 4.

$$I_{S.C.} = \text{Transformer}_{F.L.A.} \times \text{Multiplier}$$

Note 3. Utility voltages may vary $\pm 10\%$ for power and $\pm 5.8\%$ for 120 Volt lighting services. Therefore, for highest short circuit conditions, multiply values as calculated in step 3 by 1.1 or 1.058 respectively. To find the lower end worst case, multiply results in step 3 by .9 or .942 respectively.

Note 4. Motor short circuit contribution, if significant, may be added at all fault locations throughout the system. A practical estimate of motor short circuit contribution is to multiply the total motor current in amps by 4. Values of 4 to 6 are commonly accepted.

Step 4. Calculate the "f" factor.

$$3\phi \text{ Faults } f = \frac{1.732 \times L \times I_{3\phi}}{C \times n \times E_{L-L}}$$

$$1\phi \text{ Line-to-Line (L-L) Faults } f = \frac{2 \times L \times I_{L-L}}{C \times n \times E_{L-L}}$$

$$1\phi \text{ Line-to-Neutral (L-N) Faults } f = \frac{2 \times L \times I_{L-N}}{C \times n \times E_{L-N}}$$

Where:

L = length (feet) of conductor to the fault.

C = constant from Table 4 of "C" values for conductors and Table 5 of "C" values for busway.

n = Number of conductors per phase (adjusts C value for parallel runs)

I = Available short-circuit current in amperes at beginning of circuit.

E = Voltage of circuit.

† **Note 5.** The L-N fault current is higher than the L-L fault current at the secondary terminals of a single-phase center-tapped transformer. The short-circuit current available (I) for this case in Step 4 should be adjusted at the transformer terminals as follows: At L-N center tapped transformer terminals, $I_{L-N} = 1.5 \times I_{L-L}$ at Transformer Terminals.

At some distance from the terminals, depending upon wire size, the L-N fault current is lower than the L-L fault current. The 1.5 multiplier is an approximation and will theoretically vary from 1.33 to 1.67. These figures are based on change in turns ratio between primary and secondary, infinite source available, zero feet from terminals of transformer, and $1.2 \times \%X$ and $1.5 \times \%R$ for L-N vs. L-L resistance and reactance values. Begin L-N calculations at transformer secondary terminals, then proceed point-to-point.

Step 5. Calculate "M" (multiplier) or take from Table 2.

$$M = \frac{1}{1+f}$$

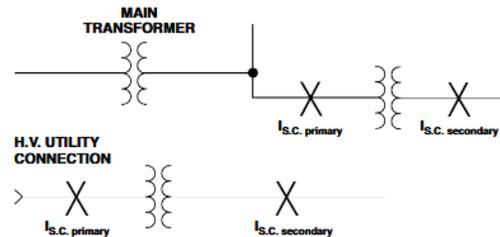
Step 6. Calculate the available short circuit symmetrical RMS current at the point of fault. Add motor contribution, if applicable.

$$I_{S.C. \text{ sym. RMS}} = I_{S.C.} \times M$$

Step 6A. Motor short circuit contribution, if significant, may be added at all fault locations throughout the system. A practical estimate of motor short circuit contribution is to multiply the total motor current in amps by 4. Values of 4 to 6 are commonly accepted.

Calculation of Short-Circuit Currents When Primary Available Short-Circuit Current is Known

Use the following procedure to calculate the level of fault current at the secondary of a second, downstream transformer in a system when the level of fault current at the transformer primary is known.



Step A. Calculate the "f" factor ($I_{S.C. \text{ primary}}$ known)

$$3\phi \text{ Transformer } (I_{S.C. \text{ primary and } I_{S.C. \text{ secondary are } 3\phi \text{ fault values)}) f = \frac{I_{S.C. \text{ primary}} \times V_{\text{primary}} \times 1.73 (\%Z)}{100,000 \times kVA_{\text{transformer}}}$$

$$1\phi \text{ Transformer } (I_{S.C. \text{ primary and } I_{S.C. \text{ secondary are } 1\phi \text{ fault values: } I_{S.C. \text{ secondary is L-L)}) f = \frac{I_{S.C. \text{ primary}} \times V_{\text{primary}} \times (\%Z)}{100,000 \times kVA_{\text{transformer}}}$$

Step B. Calculate "M" (multiplier).

$$M = \frac{1}{1+f}$$

Step C. Calculate the short-circuit current at the secondary of the transformer. (See Note under Step 3 of "Basic Point-to-Point Calculation Procedure".)

$$I_{S.C. \text{ secondary}} = \frac{V_{\text{primary}}}{V_{\text{secondary}}} \times M \times I_{S.C. \text{ primary}}$$

Three-Phase Short Circuits

System A

Available Utility
Infinite Assumption

1500 KVA Transformer
480V, 3Ø, 3.5%Z,
3.45% X, 0.56%R

$I_{LL} = 1804A$

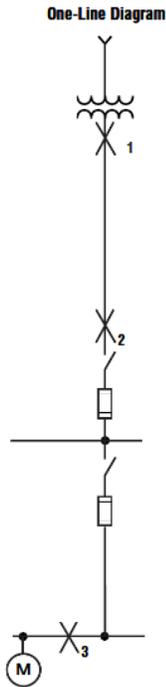
25' - 500kcmil Cu
3 Single Conductors
6 Per Phase
Magnetic Conduit

2000A Switch
KRP-C 2000SP Fuse

400A Switch
LPS-RK-400SP Fuse

50' - 500 kcmil Cu
3 Single Conductors
Magnetic Conduit

Motor Contribution*



Fault X₁

Step 1. $I_{LL} = \frac{1500 \times 1000}{480 \times 1.732} = 1804.3A$

Step 2. Multiplier $\frac{100}{3.5 \times 0.9^2} = 31.746$

Step 3. $I_{s.c.} = 1804.3 \times 31.746 = 57,279A$
 $I_{s.c. \text{ motor contribution}}^{**} = 4 \times 1804.3 = 7217A$
 $I_{total \text{ s.c. sym RMS}} = 57,279 + 7217 = 64,496A$

Fault X₂

Step 4. $f = \frac{1.732 \times 25 \times 57,279}{22,185 \times 6 \times 480} = 0.0388$

Step 5. $M = \frac{1}{1 + 0.0388} = 0.9626$

Step 6. $I_{s.c. \text{ sym RMS}} = 57,279 \times 0.9626 = 55,137A$
 $I_{s.c. \text{ motor contribution}}^{**} = 4 \times 1804.3 = 7217A$
 $I_{total \text{ s.c. sym RMS}} = 55,137 + 7217 = 62,354A$

Fault X₃

Step 4. $f = \frac{1.732 \times 50 \times 55,137}{22,185 \times 1 \times 480} = 0.4484$

Step 5. $M = \frac{1}{1 + 0.4483} = 0.6904$

Step 6. $I_{s.c. \text{ sym RMS}} = 55,137 \times 0.6904 = 38,067A$
 $I_{s.c. \text{ motor contribution}}^{**} = 4 \times 1804.3 = 7217A$
 $I_{total \text{ s.c. sym RMS (X}_3)} = 38,067 + 7217 = 45,284A$

*See note 4 on page 240.

**Assumes 100% motor load. If 50% of this load was from motors, $I_{LL \text{ motor contrib.}} = 4 \times 1804 \times 0.5 = 3,608A$

† See note 2 on page 240

System B

Available Utility
Infinite Assumption

1000 KVA Transformer
480V, 3Ø, 3.5%Z,

$I_{LL} = 1203A$

30' - 500kcmil Cu
3 Single Conductors
4 Per Phase
PVC Conduit

1600A Switch

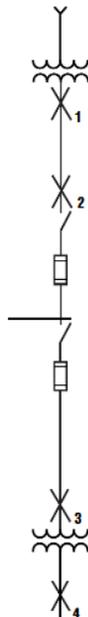
KRP-C 1500SP Fuse

400A Switch
LPS-RK-350SP Fuse

20' - 2/0 Cu
3 Single Conductors
2 Per Phase
PVC Conduit

225 KVA Transformer
208V, 3Ø
1.2%Z

One-Line Diagram



Fault X₁

Step 1. $I_{LL} = \frac{1000 \times 1000}{480 \times 1.732} = 1202.8A$

Step 2. Multiplier $= \frac{100}{3.5 \times 0.9^2} = 31.746$

Step 3. $I_{s.c.} = 1202.8 \times 31.746 = 38,184A$

Fault X₂

Step 4. $f = \frac{1.732 \times 30 \times 38,184}{26,706 \times 4 \times 480} = 0.0387$

Step 5. $M = \frac{1}{1 + 0.0387} = 0.9627$

Step 6. $I_{s.c. \text{ sym RMS}} = 38,184 \times 0.9627 = 36,761A$

Fault X₃

Step 4. $f = \frac{1.732 \times 20 \times 36,761}{2 \times 11,424 \times 480} = 0.1161$

Step 5. $M = \frac{1}{1 + 0.1161} = 0.8960$

Step 6. $I_{s.c. \text{ sym RMS}} = 36,761 \times 0.8960 = 32,937A$

Fault X₄

Step A. $f = \frac{32,937 \times 480 \times 1.732 \times (1.2 \times 0.9)}{100,000 \times 225} = 1.3144$

Step B. $M = \frac{1}{1 + 1.3144} = 0.4321$

Step C. $I_{s.c. \text{ sym RMS}} = \frac{480 \times 0.4321 \times 32,937}{208} = 32,842A$

This example assumes no motor contribution.

Single-Phase Short Circuits

Short circuit calculations on a single-phase center tapped transformer system require a slightly different procedure than 3Ø faults on 3Ø systems.

1. It is necessary that the proper impedance be used to represent the primary system. For 3Ø fault calculations, a single primary conductor impedance is used from the source to the transformer connection. This is compensated for in the 3Ø short circuit formula by multiplying the single conductor or single-phase impedance by 1.73.

However, for single-phase faults, a primary conductor impedance is considered from the source to the transformer and back to the source. This is compensated in the calculations by multiplying the 3Ø primary source impedance by two.

2. The impedance of the center-tapped transformer must be adjusted for the half-winding (generally line-to-neutral) fault condition.

The diagram at the right illustrates that during line-to-neutral faults, the full primary winding is involved but, only the half-winding on the secondary is involved. Therefore, the actual transformer reactance and resistance of the half-winding condition is different than the actual transformer reactance and resistance of the full winding condition. Thus, adjustment to the %X and %R must be made when considering line-to-neutral faults. The adjustment multipliers generally used for this condition are as follows:

- 1.5 times full winding %R on full winding basis.
- 1.2 times full winding %X on full winding basis.

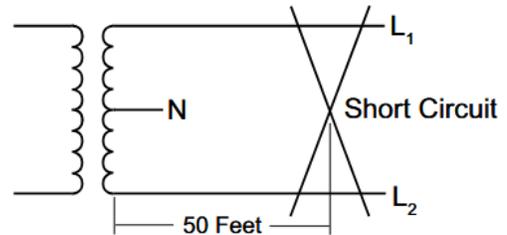
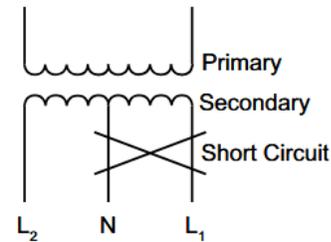
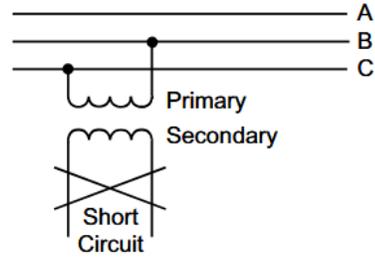
Note: %R and %X multipliers given in "Impedance Data for Single Phase Transformers" Table may be used, however, calculations must be adjusted to indicate transformer kVA/2.

3. The impedance of the cable and two-pole switches on the system must be considered "both-ways" since the current flows to the fault and then returns to the source. For instance, if a line-to-line fault occurs 50 feet from a transformer, then 100 feet of cable impedance must be included in the calculation.

The calculations on the following pages illustrate 1Ø fault calculations on a single-phase transformer system. Both line-to-line and line-to-neutral faults are considered.

Note in these examples:

- a. The multiplier of 2 for some electrical components to account for the single-phase fault current flow,
- b. The half-winding transformer %X and %R multipliers for the line-to-neutral fault situation, and



Single-Phase Short Circuits

System A

Available Utility
Infinite Assumption

75KVA, 1Ø Transformer.
1.22%X, 0.68%R
1.40%Z
120/240V

25' - 500kcmil Cu
Magnetic Conduit
3 Single Conductors

400A Switch
LPN-RK-400SP Fuse

50' - 3 AWG Cu
Magnetic Conduit
3 Single Conductors

One-Line Diagram



Line-to-Line (L-L) Fault

Fault X₁

$$\text{Step 1. } I_{L-L} = \frac{75 \times 1000}{240} = 312.5\text{A}$$

$$\text{Step 2. Multiplier} = \frac{100}{1.4 \times 0.9^\dagger} = 79.37$$

$$\text{Step 3. } I_{s.c. (L-L)} = 312.5 \times 79.37 = 24,802\text{A}$$

Fault X₂

$$\text{Step 4. } f = \frac{2 \times 25 \times 24,802}{22,185 \times 1 \times 240} = 0.2329$$

$$\text{Step 5. } M = \frac{1}{1 + 0.2329} = 0.8111$$

$$\text{Step 6. } I_{s.c. (L-L) (X_2)} = 24,802 \times 0.8111 = 20,116$$

Fault X₃

$$\text{Step 4. } f = \frac{2 \times 50 \times 20,116}{4774 \times 1 \times 240} = 1.7557$$

$$\text{Step 5. } M = \frac{1}{1 + 1.7557} = 0.3629$$

$$\text{Step 6. } I_{s.c. (L-L) (X_3)} = 20,116 \times 0.3629 = 7,300\text{A}$$

† In addition, UL 1561 listed transformers 25KVA and larger have a ± 10% impedance tolerance. Short circuit amps can be affected by this tolerance. Therefore, for high end worst case, multiply %Z by 0.9. For low end of worst case, multiply %Z by 1.1. Transformers constructed to ANSI standards have a ± 7.5% impedance tolerance (two-winding construction).

Line-to-Neutral (L-N) Fault

Fault X₁

$$\text{Step 1. } I_{L-L} = \frac{75 \times 1000}{240} = 312.5\text{A}$$

$$\text{Step 2. Multiplier} = \frac{100}{1.4 \times 0.9^\dagger} = 79.37$$

$$\text{Step 3* . } I_{s.c. (L-N)} = 24,802 \times 1.5 = 37,202\text{A}$$

Fault X₂

$$\text{Step 4. } f = \frac{2 \times 25 \times 37,202}{22,185 \times 1 \times 120} = 0.6987$$

$$\text{Step 5. } M = \frac{1}{1 + 0.6987} = 0.5887$$

$$\text{Step 6* . } I_{s.c. (L-N) (X_2)} = 37,202 \times 0.5887 = 21,900\text{A}$$

Fault X₃

$$\text{Step 4. } f = \frac{2 \times 50 \times 21,900^{**}}{4774 \times 1 \times 120} = 3.8323$$

$$\text{Step 5. } M = \frac{1}{1 + 3.823} = 0.2073$$

$$\text{Step 6* . } I_{s.c. (L-N) (X_3)} = 21,900 \times 0.2073 = 4,540\text{A}$$

* Note 5. The L-N fault current is higher than the L-L fault current at the secondary terminals of a single-phase center-tapped transformer. The short-circuit current available (I) for this case in Step 4 should be adjusted at the transformer terminals as follows: At L-N center tapped transformer terminals, $I_{L-N} = 1.5 \times I_{L-L}$ at Transformer Terminals.

**Assumes the neutral conductor and the line conductor are the same size.

Impedance & Reactance Data

Transformers

Table 1. Short-Circuit Currents Available from Various Size Transformers

(Based upon actual field nameplate data or from utility transformer worst case impedance)

Voltage and Phase	kVA	Full Load Amps	% Impedance†† (Nameplate)	Short Circuit Amps‡
120/240 1 ph.*	25	104	1.5	12175
	37.5	156	1.5	18018
	50	208	1.5	23706
	75	313	1.5	34639
	100	417	1.6	42472
	167	696	1.6	66644
	45	125	1.0	13879
120/208 3 ph.**	75	208	1.0	23132
	112.5	312	1.11	31259
	150	416	1.07	43237
	225	625	1.12	61960
	300	833	1.11	83357
	500	1388	1.24	124364
	750	2082	3.50	66091
	1000	2776	3.50	88121
	1500	4164	3.50	132181
	2000	5552	4.00	154211
	2500	6940	4.00	192764
277/480 3 ph.**	75	90	1.00	10035
	112.5	135	1.00	15053
	150	181	1.20	16726
	225	271	1.20	25088
	300	361	1.20	33451
	500	602	1.30	51463
	750	903	3.50	28672
	1000	1204	3.50	38230
	1500	1806	3.50	57345
	2000	2408	4.00	66902
	2500	3011	4.00	83628

*Single-phase values are L-N values at transformer terminals. These figures are based on change in turns ratio between primary and secondary, 100,000 KVA primary, zero feet from terminals of transformer, 1.2 (%X) and 1.5 (%R) multipliers for L-N vs. L-L reactance and resistance values and transformer X/R ratio = 3.

**Three-phase short-circuit currents based on "infinite" primary.

† UL listed transformers 25 kVA or greater have a ±10% impedance tolerance. Short-circuit amps shown in Table 1 reflect -10% condition. Transformers constructed to ANSI standards have a ±7.5% impedance tolerance (two-winding construction).

‡ Fluctuations in system voltage will affect the available short-circuit current. For example, a 10% increase in system voltage will result in a 10% greater available short-circuit currents than as shown in Table 1.

Table 2. "M" (Multiplier)

$$M = \frac{1}{1 + f}$$

f	M	f	M	f	M
0.01	0.99	0.50	0.67	7.00	0.13
0.02	0.98	0.60	0.63	8.00	0.11
0.03	0.97	0.70	0.59	9.00	0.10
0.04	0.96	0.80	0.55	10.00	0.09
0.05	0.95	0.90	0.53	15.00	0.06
0.06	0.94	1.00	0.50	20.00	0.05
0.07	0.93	1.20	0.45	30.00	0.03
0.08	0.93	1.50	0.40	40.00	0.02
0.09	0.92	1.75	0.36	50.00	0.02
0.10	0.91	2.00	0.33	60.00	0.02
0.15	0.87	2.50	0.29	70.00	0.01
0.20	0.83	3.00	0.25	80.00	0.01
0.25	0.80	3.50	0.22	90.00	0.01
0.30	0.77	4.00	0.20	100.00	0.01
0.35	0.74	5.00	0.17		
0.40	0.71	6.00	0.14		

Impedance Data for Single-Phase Transformers

kVA	Suggested X/R Ratio for Calculation	Normal Range of Percent Impedance (%Z)*	Impedance Multipliers** For Line-to-Neutral Faults	
			for %X	for %R
25.0	1.1	1.2-6.0	0.6	0.75
37.5	1.4	1.2-6.5	0.6	0.75
50.0	1.6	1.2-6.4	0.6	0.75
75.0	1.8	1.2-6.6	0.6	0.75
100.0	2.0	1.3-5.7	0.6	0.75
167.0	2.5	1.4-6.1	1.0	0.75
250.0	3.6	1.9-6.8	1.0	0.75
333.0	4.7	2.4-6.0	1.0	0.75
500.0	5.5	2.2-5.4	1.0	0.75

* National standards do not specify %Z for single-phase transformers. Consult manufacturer for values to use in calculation.

** Based on rated current of the winding (one-half nameplate kVA divided by secondary line-to-neutral voltage).

Note: UL Listed transformers 25 kVA and greater have a ± 10% tolerance on their impedance nameplate.

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Impedance Data for Single-Phase and Three-Phase Transformers-Supplement†

kVA	3Ø	%Z	Suggested X/R Ratio for Calculation
10	—	1.2	1.1
15	—	1.3	1.1
	75	1.11	1.5
	150	1.07	1.5
	225	1.12	1.5
	300	1.11	1.5
333	—	1.9	4.7
	500	1.24	1.5
500	—	2.1	5.5

† These represent actual transformer nameplate ratings taken from field installations.

Note: UL Listed transformers 25kVA and greater have a ±10% tolerance on their impedance nameplate.

Table 3. Various Types of Short -Circuit Currents as a Percent of Three Phase Bolted Faults (Typical).

Three Phase Bolted Fault	100%
Line-to-Line Bolted Fault	87%
Line-to-Ground Bolted Fault	25-125%* (Use 100% near transformer, 50% otherwise)
Line-to-Neutral Bolted Fault	25-125% (Use 100% near transformer, 50% otherwise)
Three Phase Arcing Fault	89% (maximum)
Line-to-Line Arcing Fault	74% (maximum)
Line to Ground Arcing Fault (minimum)	38% (minimum)

*Typically much lower but can actually exceed the Three Phase Bolted Fault if it is near the transformer terminals. Will normally be between 25% to 125% of three phase bolted fault value.

Conductors & Busways "C" Values

Table 4. "C" Values for Conductors

Copper												
AWG or kcmil	Three Single Conductors Conduit						Three-Conductor Cable Conduit					
	Steel			Nonmagnetic			Steel			Nonmagnetic		
	600V	5kV	15kV	600V	5kV	15kV	600V	5kV	15kV	600V	5kV	15kV
14	389	-	-	389	-	-	389	-	-	389	-	-
12	617	-	-	617	-	-	617	-	-	617	-	-
10	981	-	-	982	-	-	982	-	-	982	-	-
8	1557	1551	-	1559	1555	-	1559	1557	-	1560	1558	-
6	2425	2406	2389	2430	2418	2407	2431	2425	2415	2433	2428	2421
4	3806	3751	3696	3826	3789	3753	3830	3812	3779	3838	3823	3798
3	4774	4674	4577	4811	4745	4679	4820	4785	4726	4833	4803	4762
2	5907	5736	5574	6044	5926	5809	5989	5930	5828	6087	6023	5958
1	7293	7029	6759	7493	7307	7109	7454	7365	7189	7579	7507	7364
1/0	8925	8544	7973	9317	9034	8590	9210	9086	8708	9473	9373	9053
2/0	10755	10062	9390	11424	10878	10319	11245	11045	10500	11703	11529	11053
3/0	12844	11804	11022	13923	13048	12360	13656	13333	12613	14410	14119	13462
4/0	15082	13606	12543	16673	15351	14347	16392	15890	14813	17483	17020	16013
250	16483	14925	13644	18594	17121	15866	18311	17851	16466	19779	19352	18001
300	18177	16293	14769	20868	18975	17409	20617	20052	18319	22525	21938	20163
350	19704	17385	15678	22737	20526	18672	22646	21914	19821	24904	24126	21982
400	20566	18235	16366	24297	21786	19731	24253	23372	21042	26916	26044	23518
500	22185	19172	17492	26706	23277	21330	26980	25449	23126	30096	28712	25916
600	22965	20567	17962	28033	25204	22097	28752	27975	24897	32154	31258	27766
750	24137	21387	18889	29735	26453	23408	31051	30024	26933	34605	33315	29735
1,000	25278	22539	19923	31491	28083	24887	33864	32689	29320	37197	35749	31959
Aluminum												
14	237	-	-	237	-	-	237	-	-	237	-	-
12	376	-	-	376	-	-	376	-	-	376	-	-
10	599	-	-	599	-	-	599	-	-	599	-	-
8	951	950	-	952	951	-	952	951	-	952	952	-
6	1481	1476	1472	1482	1479	1476	1482	1480	1478	1482	1481	1479
4	2346	2333	2319	2350	2342	2333	2351	2347	2339	2353	2350	2344
3	2952	2928	2904	2961	2945	2929	2963	2955	2941	2966	2959	2949
2	3713	3670	3626	3730	3702	3673	3734	3719	3693	3740	3725	3709
1	4645	4575	4498	4678	4632	4580	4686	4664	4618	4699	4682	4646
1/0	5777	5670	5493	5838	5766	5646	5852	5820	5717	5876	5852	5771
2/0	7187	6968	6733	7301	7153	6986	7327	7271	7109	7373	7329	7202
3/0	8826	8467	8163	9110	8851	8627	9077	8981	8751	9243	9164	8977
4/0	10741	10167	9700	11174	10749	10387	11185	11022	10642	11409	11277	10969
250	12122	11460	10849	12862	12343	11847	12797	12636	12115	13236	13106	12661
300	13910	13009	12193	14923	14183	13492	14917	14698	13973	15495	15300	14659
350	15484	14280	13288	16813	15858	14955	16795	16490	15541	17635	17352	16501
400	16671	15355	14188	18506	17321	16234	18462	18064	16921	19588	19244	18154
500	18756	16828	15657	21391	19503	18315	21395	20607	19314	23018	22381	20978
600	20093	18428	16484	23451	21718	19635	23633	23196	21349	25708	25244	23295
750	21766	19685	17686	25976	23702	21437	26432	25790	23750	29036	28262	25976
1,000	23478	21235	19006	28779	26109	23482	29865	29049	26608	32938	31920	29135

Note: These values are equal to one over the impedance per foot and based upon resistance and reactance values found in IEEE Std 241-1990 (Gray Book), IEEE Recommended Practice for Electric Power Systems in Commercial Buildings & IEEE Std 242-1986 (Buff Book), IEEE Recommended Practice for Protection and Coordination of Industrial and Commercial Power Systems. Where resistance and reactance values differ or are not available, the Buff Book values have been used. The values for reactance in determining the C Value at 5 kV & 15 kV are from the Gray Book only (Values for 14-10 AWG at 5 kV and 14-8 AWG at 15 kV are not available and values for 3 AWG have been approximated).

Table 5. "C" Values for Busway

Ampacity	Busway				
	Plug-In		Feeder		High Impedance
	Copper	Aluminum	Copper	Aluminum	Copper
225	28700	23000	18700	12000	—
400	38900	34700	23900	21300	—
600	41000	38300	36500	31300	—
800	46100	57500	49300	44100	—
1000	69400	89300	62900	56200	15600
1200	94300	97100	76900	69900	16100
1350	119000	104200	90100	84000	17500
1600	129900	120500	101000	90900	19200
2000	142900	135100	134200	125000	20400
2500	143800	156300	180500	166700	21700
3000	144900	175400	204100	188700	23800
4000	—	—	277800	256400	—

Note: These values are equal to one over the impedance per foot for impedance in a survey of industry.

Ratings of Conductors and Tables to Determine Volt Loss

With larger loads on new installations, it is extremely important to consider volt loss, otherwise some very unsatisfactory problems are likely to be encountered.

The actual conductor used must also meet the other sizing requirements such as full-load current, ambient temperature, number in a raceway, etc.

How to Figure Volt Loss

Multiply *distance* (length in feet of one wire) by the *current* (expressed in amps) by the *figure* shown in table for the kind of current and the size of wire to be used, by one over the number of conductors per phase.

Then, put a decimal point in front of the last 6 digits—you have the *volt loss* to be

Example – 6 AWG copper wire, one per phase, in 180 feet of steel conduit—3 phase, 40 amp load at 80% power factor.

Multiply feet by amperes: $180 \times 40 = 7200$

Multiply this number by number from table for 6 AWG wire three-phase at 80% power factor: $7200 \times .745^{\dagger} = 5364000$

Multiply by $\frac{1}{\text{\#/phase}} \times 5364000 = \frac{1}{1} \times 5364000 = 5364000$

Place decimal point 6 places to left:

This gives volt loss to be expected: 5.364V

(For a 240V circuit the % voltage drop is $\frac{5.364}{240} \times 100$ or 2.23%.)

Table A and B take into consideration *reactance on AC circuits* as well as resistance of the wire.

Remember on short runs to check to see that the size and type of wire indicated has sufficient ampacity.

expected on that circuit.

How to Select Size of Wire

Multiply *distance* (length in feet of one wire) by the *current* (expressed in amps), by one over the number of conductors per phase.

Divide that figure into the permissible *volt loss* multiplied by 1,000,000.

Example – Copper in 180 feet of steel conduit—3 phase, 40 amp load at 80% power factor—maximum volt loss permitted from local code equals 5.5 volts.

Multiply feet by amperes by $\frac{1}{\text{\#/phase}} \quad 180 \times 40 \times \frac{1}{1} = 7200.$

Divide permissible volt loss multiplied by 1,000,000 by this number: $\frac{5.5 \times 1,000,000}{7200} = 764.$

Look under the column in Table A and B applying to the type of current and power factor for the value nearest, but not above your result – you have the size of wire needed.

Select number from Table A, three-phase at 80% power factor, that is nearest but not greater than 764. This number is 745 which indicates the size of wire needed: 6 AWG.

Line-to-Neutral

For line to neutral voltage drop on a 3 phase system, divide the three phase value by 1.73. For line to neutral voltage drop on a single phase system, divide single phase value by 2.

Open Wiring

The volt loss for open wiring installations depends on the separation between conductors. The volt loss is approximately equal to that for conductors in non-magnetic conduit.

Installation in Conduit, Cable or Raceway

NEC® Tables 310.15(B)(16) through 310.15(B)(19) give allowable ampacities (current-carrying capacities) for not more than three current carrying conductors in a conduit, cable, or raceway. Where the number of current carrying conductors exceeds three the allowable ampacity of each conductor must be reduced as shown in the following tables:

Installation in Conduit, Cable or Raceway per 310.15(B)(2)(a)

The Number of Conductors In One Conduit, Raceway Or Cable	Percentage of Values In Tables 310.16 And 310.18
4 to 6	80%
7 to 9	70%
10 to 20	50%
21 to 30	45%
31 to 40	40%
41 and over	35%

Conditions Causing Higher Volt Loss

The voltage loss is increased when a conductor is operated at a higher temperature because the resistance increases.

Room Temperature Affects Ratings

The ampacities (carrying capacities) of conductors are based on a room temperature of either 30°C or 40°C. For derating based upon 30°C ambient, if room temperature is higher, the ampacities are reduced by using the following multipliers; (for 0-2000 volt, insulated conductors not more than 3 conductors in raceway or direct buried, Table 310.15(B)(2)(a)). For room temperatures based upon a 40°C ambient, see Table 310.15(B)(2)(b).

Room Temperature Affects Ratings

Room Temperature °C	TW °F	Ampacity Multiplier		
		THW, THWN (60°C Wire)	THHN, XHHW* (75°C Wire)	(90°C Wire)
31-35	87-95	.91	.94	.96
36-40	96-104	.82	.88	.91
41-45	105-113	.71	.82	.87
46-50	114-122	.58	.75	.82
51-55	123-131	.41	.67	.76
56-60	132-140	∅	.58	.71
61-70	141-158	∅	.33	.58
71-80	159-176	∅	∅	.41

Table A – Copper Conductors – Ratings & Volt Loss†

Conduit	Wire Size	Ampacity			Direct Current	Volt Loss (See explanation prior page.)									
		Type T, TW (60°C Wire)	Type RH, THWN, RHW, THW (75°C Wire)	Type RHH, THHN, XHHW (90°C Wire)		Three-Phase (60 Cycle, Lagging Power Factor.)					Single-Phase (60 Cycle, Lagging Power Factor.)				
						100%	90%	80%	70%	60%	100%	90%	80%	70%	60%
Steel Conduit	14	20*	20*	25*	6140	5369	4887	4371	3848	3322	6200	5643	5047	4444	3836
	12	25*	25*	30*	3860	3464	3169	2841	2508	2172	4000	3659	3281	2897	2508
	10	30	35*	40*	2420	2078	1918	1728	1532	1334	2400	2214	1995	1769	1540
	8	40	50	55	1528	1350	1264	1148	1026	900	1560	1460	1326	1184	1040
	6	55	65	75	982	848	812	745	673	597	980	937	860	777	690
	4	70	85	95	616	536	528	491	450	405	620	610	568	519	468
	3	85	100	110	490	433	434	407	376	341	500	501	470	434	394
	2	95	115	130	388	346	354	336	312	286	400	409	388	361	331
	1	110	130	150	308	277	292	280	264	245	320	337	324	305	283
	0	125	150	170	244	207	228	223	213	200	240	263	258	246	232
	00	145	175	195	193	173	196	194	188	178	200	227	224	217	206
	000	165	200	225	153	136	162	163	160	154	158	187	188	184	178
	0000	195	230	260	122	109	136	140	139	136	126	157	162	161	157
	250	215	255	290	103	93	123	128	129	128	108	142	148	149	148
	300	240	285	320	86	77	108	115	117	117	90	125	133	135	135
	350	260	310	350	73	67	98	106	109	109	78	113	122	126	126
	400	280	335	380	64	60	91	99	103	104	70	105	114	118	120
	500	320	380	430	52	50	81	90	94	96	58	94	104	109	111
	600	335	420	475	43	43	75	84	89	92	50	86	97	103	106
	750	400	475	535	34	36	68	78	84	88	42	79	91	97	102
1000	455	545	615	26	31	62	72	78	82	36	72	84	90	95	
Non- Magnetic Conduit (Lead Covered Cables or Installation in Fibre or Other Non- Magnetic Conduit, Etc.)	14	20*	20*	25*	6140	5369	4876	4355	3830	3301	6200	5630	5029	4422	3812
	12	25*	25*	30*	3464	3464	3158	2827	2491	2153	4000	3647	3264	2877	2486
	10	30	35*	40*	2420	2078	1908	1714	1516	1316	2400	2203	1980	1751	1520
	8	40	50	55	1528	1350	1255	1134	1010	882	1560	1449	1310	1166	1019
	6	55	65	75	982	848	802	731	657	579	980	926	845	758	669
	4	70	85	95	616	536	519	479	435	388	620	599	553	502	448
	3	85	100	110	470	433	425	395	361	324	500	490	456	417	375
	2	95	115	130	388	329	330	310	286	259	380	381	358	330	300
	1	110	130	150	308	259	268	255	238	219	300	310	295	275	253
	0	125	150	170	244	207	220	212	199	185	240	254	244	230	214
	00	145	175	195	193	173	188	183	174	163	200	217	211	201	188
	000	165	200	225	153	133	151	150	145	138	154	175	173	167	159
	0000	195	230	260	122	107	127	128	125	121	124	147	148	145	140
	250	215	255	290	103	90	112	114	113	110	104	129	132	131	128
	300	240	285	320	86	76	99	103	104	102	88	114	119	120	118
	350	260	310	350	73	65	89	94	95	94	76	103	108	110	109
	400	280	335	380	64	57	81	87	89	89	66	94	100	103	103
	500	320	380	430	52	46	71	77	80	82	54	82	90	93	94
	600	335	420	475	43	39	65	72	76	77	46	75	83	87	90
	750	400	475	535	34	32	58	65	70	72	38	67	76	80	83
1000	455	545	615	26	25	51	59	63	66	30	59	68	73	77	

* The overcurrent protection for conductor types marked with an (*) shall not exceed 15 amperes for 14 AWG, 20 amperes for 12 AWG, and 30 amperes for 10 AWG copper; or 15 amperes for 12 AWG and 25 amperes for 10 AWG aluminum and copper-clad aluminum after any correction factors for ambient temperature and number of conductors have been applied.

† Figures are L-L for both single-phase and three-phase. Three-phase figures are average for the three-phase.

Table B – Aluminum Conductors – Ratings & Volt Loss†

Conduit	Wire Size	Ampacity			Direct Current	Volt Loss (See explanation two pages prior.)					Single-Phase				
		Type	Type	Type		Three-Phase					(60 Cycle, Lagging Power Factor.)				
		T, TW (60°C Wire)	RH, THWN, RHW, THW (75°C Wire)	RHH, THHN, XHHW (90°C Wire)		100%	90%	80%	70%	60%	100%	90%	80%	70%	60%
Steel Conduit	12	20*	20*	25*	6360	5542	5039	4504	3963	3419	6400	5819	5201	4577	3948
	10	25	30*	35*	4000	3464	3165	2836	2502	2165	4000	3654	3275	2889	2500
	8	30	40	45	2520	2251	2075	1868	1656	1441	2600	2396	2158	1912	1663
	6	40	50	60	1616	1402	1310	1188	1061	930	1620	1513	1372	1225	1074
	4	55	65	75	1016	883	840	769	692	613	1020	970	888	799	708
	3	65	75	85	796	692	668	615	557	497	800	771	710	644	574
	2	75	90	100	638	554	541	502	458	411	640	625	580	529	475
	1	85	100	115	506	433	432	405	373	338	500	499	468	431	391
	0	100	120	135	402	346	353	334	310	284	400	407	386	358	328
	00	115	135	150	318	277	290	277	260	241	320	335	320	301	278
	000	130	155	175	259	225	241	234	221	207	260	279	270	256	239
	0000	150	180	205	200	173	194	191	184	174	200	224	221	212	201
	250	170	205	230	169	148	173	173	168	161	172	200	200	194	186
	300	190	230	255	141	124	150	152	150	145	144	174	176	173	168
350	210	250	280	121	109	135	139	138	134	126	156	160	159	155	
400	225	270	305	106	95	122	127	127	125	110	141	146	146	144	
500	260	310	350	85	77	106	112	113	113	90	122	129	131	130	
600	285	340	385	71	65	95	102	105	106	76	110	118	121	122	
750	320	385	435	56	53	84	92	96	98	62	97	107	111	114	
1000	375	445	500	42	43	73	82	87	89	50	85	95	100	103	
Non- Magnetic Conduit (Lead Covered Cables or Installation in Fibre or Other Non- Magnetic Conduit, Etc.)	12	20*	20*	25*	6360	5542	5029	4490	3946	3400	6400	5807	5184	4557	3926
	10	25	30*	35*	4000	3464	3155	2823	2486	2147	4000	3643	3260	2871	2480
	8	30	40	45	2520	2251	2065	1855	1640	1423	2600	2385	2142	1894	1643
	6	40	50	60	1616	1402	1301	1175	1045	912	1620	1502	1357	1206	1053
	4	55	65	75	1016	883	831	756	677	596	1020	959	873	782	668
	3	65	75	85	796	692	659	603	543	480	800	760	696	627	555
	2	75	90	100	638	554	532	490	443	394	640	615	566	512	456
	1	85	100	115	506	433	424	394	360	323	500	490	455	415	373
	0	100	120	135	402	346	344	322	296	268	400	398	372	342	310
	00	115	135	150	318	277	281	266	247	225	320	325	307	285	260
	000	130	155	175	252	225	234	223	209	193	260	270	258	241	223
	0000	150	180	205	200	173	186	181	171	160	200	215	209	198	185
	250	170	205	230	169	147	163	160	153	145	170	188	185	177	167
	300	190	230	255	141	122	141	140	136	130	142	163	162	157	150
350	210	250	280	121	105	125	125	123	118	122	144	145	142	137	
400	225	270	305	106	93	114	116	114	111	108	132	134	132	128	
500	260	310	350	85	74	96	100	100	98	86	111	115	115	114	
600	285	340	385	71	62	85	90	91	91	72	98	104	106	105	
750	320	385	435	56	50	73	79	82	82	58	85	92	94	95	
1000	375	445	500	42	39	63	70	73	75	46	73	81	85	86	

* The overcurrent protection for conductor types marked with an (*) shall not exceed 15 amperes for 14 AWG, 20 amperes for 12 AWG, and 30 amperes for 10 AWG copper; or 15 amperes for 12 AWG and 25 amperes for 10 AWG aluminum and copper-clad aluminum after any correction factors for ambient temperature and number of conductors have been applied.

† Figures are L-L for both single-phase and three-phase. Three-phase figures are average for the three-phase.

Common Electrical Terminology

Ohm

The unit of measure for electric resistance. An ohm is the amount of resistance that will allow one amp to flow under a pressure of one volt.

Ohm's Law

The relationship between voltage, current, and resistance, expressed by the equation $E = IR$, where E is the voltage in volts, I is the current in amps, and R is the resistance in ohms.

One Time Fuses

Generic term used to describe a Class H nonrenewable cartridge fuse, with a single element.

Overcurrent

A condition which exists on an electrical circuit when the normal load current is exceeded. Overcurrents take on two separate characteristics – overloads and short-circuits.

Overload

Can be classified as an overcurrent which exceeds the normal full load current of a circuit. Also characteristic of this type of overcurrent is that it does not leave the normal current carrying path of the circuit – that is, it flows from the source, through the conductors, through the load, back through the conductors, to the source again.

Peak Let-Through Current, I_p

The instantaneous value of peak current let-through by a current-limiting fuse, when it operates in its current-limiting range.

Renewable Fuse (600V & below)

A fuse in which the element, typically a zinc link, may be replaced after the fuse has opened, and then reused. Renewable fuses are made to Class H standards.

Resistive Load

An electrical load which is characteristic of not having any significant inrush current. When a resistive load is energized, the current rises instantly to its steady-state value, without first rising to a higher value.

RMS Current

The RMS (root-mean-square) value of any periodic current is equal to the value of the direct current which, flowing through a resistance, produces the same heating effect in the resistance as the periodic current does.

Semiconductor Fuses

Fuses used to protect solid-state devices. See "High Speed Fuses."

Short-Circuit

Can be classified as an overcurrent which exceeds the normal full load current of a circuit by a factor many times (tens, hundreds or thousands greater). Also characteristic of this type of overcurrent is that it leaves the normal current carrying path of the circuit – it takes a "short cut" around the load and back to the source.

Short-Circuit Current Rating

The maximum short-circuit current an electrical component can sustain without the occurrence of excessive damage when protected with an overcurrent protective device.

Single-Phasing

That condition which occurs when one phase of a three-phase system opens, either in a low voltage (secondary) or high voltage (primary) distribution system. Primary or secondary single-phasing can be caused by any number of events. This condition results in unbalanced currents in polyphase motors and unless protective measures are taken, causes overheating and failure.

Threshold Current

The symmetrical RMS available current at the threshold of the current-limiting range, where the fuse becomes current-limiting when tested to the industry standard. This value can be read off of a peak let-through chart where the fuse curve intersects the A - B line. A threshold ratio is the relationship of the threshold current to the fuse's continuous current rating.

Time-Delay Fuse

A fuse with a built-in delay that allows temporary and harmless inrush currents to pass without opening, but is so designed to open on sustained overloads and short-circuits.

Voltage Rating

The maximum open circuit voltage in which a fuse can be used, yet safely interrupt an overcurrent. Exceeding the voltage rating of a fuse impairs its ability to clear an overload or short-circuit safely.

Withstand Rating

The maximum current that an unprotected electrical component can sustain for a specified period of time without the occurrence of extensive damage.

Electrical Formulas

To Find	Single-Phase	Two-Phase	Three-Phase	Direct Current
Amperes when kVA is known	$\frac{kVA \ 1000}{E}$	$\frac{kVA \ 1000}{E \ 2}$	$\frac{kVA \ 1000}{E \ 1.73}$	Not Applicable
Amperes when horsepower is known	$\frac{HP \ 746}{E \ \% \ eff. \ pf}$	$\frac{HP \ 746}{E \ 2 \ \% \ eff. \ pf}$	$\frac{HP \ 746}{E \ 1.73 \ \% \ eff. \ pf}$	$\frac{HP \ 746}{E \ \% \ eff.}$
Amperes when kilowatts are known	$\frac{kW \ 1000}{E \ pf}$	$\frac{kW \ 1000}{E \ 2 \ pf}$	$\frac{kW \ 1000}{E \ 1.73 \ pf}$	$\frac{kW \ 1000}{E}$
Kilowatts	$\frac{I \ E \ pf}{1000}$	$\frac{I \ E \ 2 \ pf}{1000}$	$\frac{I \ E \ 1.73 \ pf}{1000}$	$\frac{I \ E}{1000}$
Kilovolt-Amperes	$\frac{I \ E}{1000}$	$\frac{I \ E \ 2}{1000}$	$\frac{I \ E \ 1.73}{1000}$	Not Applicable
Horsepower	$\frac{I \ E \ \% \ eff. \ pf}{746}$	$\frac{I \ E \ 2 \ \% \ eff. \ pf}{746}$	$\frac{I \ E \ 1.73 \ \% \ eff. \ pf}{746}$	$\frac{I \ E \ \% \ eff.}{746}$
Watts	$E \ I \ pf$	$I \ E \ 2 \ pf$	$I \ E \ 1.73 \ pf$	$E \ I$
Energy Efficiency = $\frac{\text{Load Horsepower} \ 746}{\text{Load Input kVA} \ 1000}$				
Power Factor = $pf = \frac{\text{Power Consumed}}{\text{Apparent Power}} = \frac{W}{VA}$ or $\frac{kW}{kVA} = \cos\theta$				
I = Amperes	E = Volts	kW = Kilowatts	kVA = Kilovolt-Amperes	
HP = Horsepower	% eff. = Percent Efficiency	pf = Power Factor		

Transformer calculations



- Three-phase transformers
 - Dry-type
 - Standard transformers
 - Isolation transformers
 - Pole-mounted transformers
 - Single phase
 - Three phase connections
 - Pad-mounted transformers
 - Substation and unit substation transformers (multiple kVA ratings)
- Calculate:
 - FLA (primary and secondary for wire sizing)
 - Fault current (secondary – why not primary?)
 - Breaker/fuse size and kA rating

$I_{FLP} = \frac{1000 \text{ kVA}}{\sqrt{3} (13.8)}$
 $= 42 \text{ A}$
 $I_{FLS} = \frac{1000 \text{ kVA}}{\sqrt{3} (480)}$
 $= 1204 \text{ A}$

$kVA_{3\phi} = \sqrt{3} kV_{LL} I_L$
 $I_L = \frac{kVA_{3\phi}}{\sqrt{3} (kV_{L-L})}$

1000 kVA
 13.8 kV / 480V
 Z = 5%

I_{FLP}
 I_{FLS}
 $I_{F3\phi}$

$I_{3\phi FLA} = \frac{I_{FLS}}{Z_{pu}} = \frac{1204}{.05} = 24,080 \text{ A}$

$I_{FLS} = 1204 \text{ A}$ ✓

$kVA_{3\phi} = \sqrt{3} kV_{LL} I_L$
 $I_L = \frac{kVA_{3\phi}}{\sqrt{3} (kV_{L-L})}$

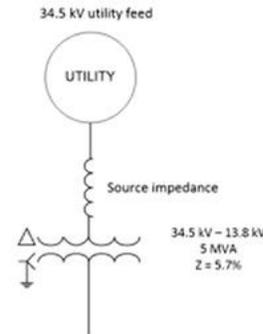
1000 kVA
 13.8 kV / 480V
 Z = 5%

I_{FLS}
 $I_{F3\phi}$

Infinite bus – what does that mean?



- Zero impedance from source – ignore everything from the transformer back into the utility system
- Really? Of course not, but the assumption is that the impedance is so low that it doesn't change your fault calculations much and it is more conservative
- How much of a difference is it? Usually less than 5%
- Voltage at the incoming service is assumed to be 1.0 p.u. – may or may not be a good assumption



Transformer calculations

Dry-type transformers



- 75 kVA, 480V/208-120V, delta-wye
 - What if it was 300 kVA?
 - K-rated, HMT, TP-1 – does it matter?



I_{FL}
 I_{FS}
 I_F

K-13
 75 kVA
 480V / 208-120V
 $Z = 4.62\%$
 ΔY

$\frac{75 \text{ kVA}}{\sqrt{3} (1.48)} = 90 \text{ A}$

$= 208 \text{ A} = \frac{75 \text{ kVA}}{\sqrt{3} (208 \text{ V})} = 208 \text{ A}$

$(75 \text{ kVA} = 208 \text{ A}) \checkmark$

$\text{VA}_{SC} = \frac{\text{kVA}_{FL}}{Z_{pu}} = \frac{75}{.0462} = 1623 \text{ kVA}$

$I_{F_{3\phi}} = \frac{1623 \text{ kVA}}{\sqrt{3} (208 \text{ V})} = 4511 \text{ A}$

$I_{FL} = \frac{75 \text{ kVA}}{\sqrt{3} (480)} = 90 \text{ A}$
 $I_F = 208 \text{ A} = \frac{75 \text{ kVA}}{\sqrt{3} (208 \text{ V})} = 208 \text{ A}$
 $(750 \text{ kVA} = 2080 \text{ A}) \checkmark$
 $I_{F_{3\phi}} = \frac{1623 \text{ k}}{\sqrt{3} (208)}$

Transformer calculations

Dry-type transformers



- 550 kVA, 460V/460V, delta-wye
 - What is an isolation transformer?
 - Why 460V and not 480V?
 - What if it is high resistance grounded (HRG) vs. solidly grounded?

TYPE: MOTOR DRIVE	THREE PHASE TRANSFORMER	CLASS: AA 60 HZ
TIPO: MOTOR DRIVE	TRANSFORMADOR TRIFASICO	CLASE: AA 60 HZ
PRI. 460 DELTA	SEC. 460Y/266	IMPEDANCE: 5.12 @ 170 °C
VOLTS: 460	VOLTS: 460	IMPEDANCIA: 5.12 @ 170 °C
KVA: 550	TEMP. RISE: 150 °C	WEIGHT: 2900 LBS
FRAME: GABINETE 920	ELEVACION DE TEMP.: 150 °C	PESO: 1315 KGS.
REFERENCE No./ No. DE REFERENCIA D56104	CLASS 220 INSULATION SYSTEM	MADE IN MEXICO
	SISTEMA DE AISLAMIENTO 220	HECHO EN MEXICO Example

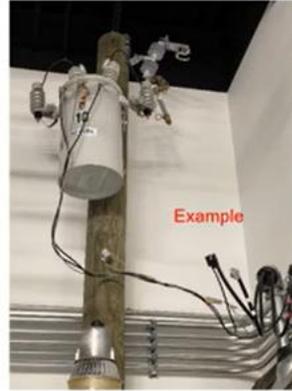
$I_{FL} = \frac{550 \text{ kVA}}{\sqrt{3} (460)} = 691 \text{ A}$
 $I_F = \frac{550 \text{ kVA}}{\sqrt{3} (480)} = 662 \text{ A}$
 $I_F = \frac{KVA_{sc}}{\sqrt{3} (Z)} = \frac{550}{\sqrt{3} (0.0512)} = 13,496 \text{ A}$

Transformer calculations

Pole-mounted transformers



- 10 kVA, single-phase
 - Example for your home (FLA, main load center)
 - How do you get the data? Climb the pole?



15

13.8KV
10KVA
240-120 split-phase
Z = ?

$\frac{13.8KV}{\sqrt{3}} = 7.97KV$

$KVA_{3\phi} = \sqrt{3} KV_{LL} I_L$
 $KVA = KV I_L$

$10KVA = 13.8KV \cdot I_{PL}$
 $I = \frac{10}{13.8}$
 $= .725A$

$I_{SFL} = \frac{10}{.240}$
 $= 41.67A$
 $= \frac{42A}{.024}$
 $= 1736A$

Transformer calculations

Pole-mounted transformers



- 25 kVA X 3 (75 kVA)
 - Example for strip mall
 - Normally delta/wye-grounded



17

17.8 kV
 13.8 kV
 480 V
 480 V
 480 V

$3 \times 25\text{ kVA}$
 $\Delta Y \pm$
 $13.8\text{ kV} / 277\text{ V}$
 $Z = 2.4\%$
 $I_{FLP} = \frac{75\text{ kVA}}{\sqrt{3} (13.8\text{ kV})}$
 $= 3.14\text{ A}$
 $I_{FLS} = 75 \times 1.2 = 90\text{ A} \checkmark$
 $I_{F\frac{2}{3}} = \frac{90\text{ A}}{0.024}$
 $= 3750\text{ A}$

Transformer calculations

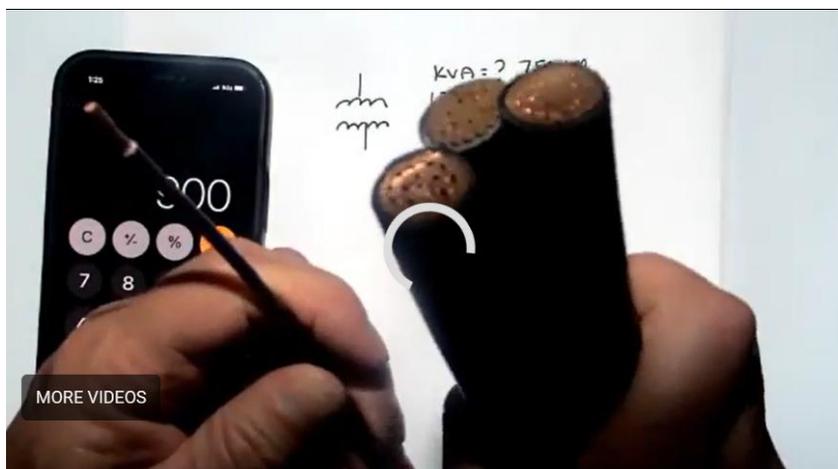
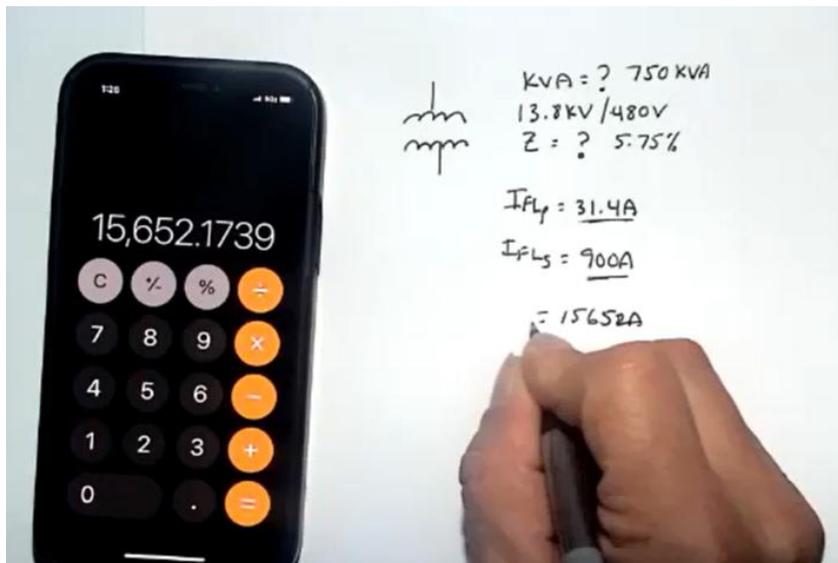
Pad-mounted transformers



- 750 kVA, 13.8 kV/480V, delta-wye
 - What would the wire size look like for primary/secondary?
 - How can you determine the nameplate info if you can't open the door?



Talk to Utility Company for impedance. Use 5.75% if guessing.



MORE VIDEOS

Transformer calculations

Substation transformers



- Substation transformer
 - 5000/6250 kVA
 - 34.5 kV/13.8 kV
 - Delta-wye
 - W/cooling



Handwritten calculations on a whiteboard:

34.5kV / 13.8kV
5000 / 6250 kVA
 $Z = 6.1\%$
 $\Delta Y7$

$I_{FLP} = \frac{6250 \text{ kVA}}{\sqrt{3}(34.5)}$
 $= 104.7 \text{ A}$

$I_{FLS} = \frac{6250 \text{ kVA}}{\sqrt{3}(13.8 \text{ kV})}$
 $= 261.8 \text{ A}$

$I_{F_{3\phi}} = \frac{\text{kVA}_{sc}}{Z_{pu}} = \frac{5000}{\sqrt{3}(13.8) \cdot .061}$
 $= 3433 \text{ A}$

Two transformers in series



- Which transformer has more actual “impedance” in ohms?
 - 30 kVA, $Z=5.75\%$, 480V/208V
 - 3 MVA, $Z=5.75\%$, 480V/208V
- What happens if you have to add the impedance of two transformers to calculate fault current?
 - 1000 kVA, $Z = 5\%$
 - 500 kVA, $Z = 5\%$

1000 kVA
 5%

500 kVA
 5%

Fault

Z_1

Z_2

$KVA_B = 500$
 $Z_B = \frac{500}{1000} \times 5\%$
 $= 2.5\%$

$Z_2 = 5\%$

$I_{F3\phi} = \frac{KVA_{sc}}{Z_{pu}} = \frac{KVA_B}{\frac{\sqrt{3} \cdot (480)}{Z_{pu \text{ total}}}}$

$= \frac{500}{\frac{\sqrt{3} \cdot (480)}{0.075}}$

$= \frac{1000}{\frac{\sqrt{3} \cdot (480)}{0.15}} = \frac{1200}{0.15}$

$= 8028$

$= 8000 \text{ A}$

Motor starting analysis



- Simplified motor starting analysis
 - Hand calculations
 - Thresholds of concern
- Formal motor starting computer simulations
- Demand charges?

Handwritten calculations on a whiteboard:

Transformer: 550 kVA, 480V, Z = 5.12%

Motor: 30 HP \approx 30 kVA, I_{FLA}

6x I_{FLA} = I_{inrush}

6x 30 = 180 kVA I_{inrush}

$\frac{\text{kVA inrush}}{\text{kVA transf}} \times Z = \frac{180}{550} \times 5.12\% = 1.67\%$

Motor starting calculations

Quick check – 30 HP



- Motor = 30 hp, assume 1 hp = 1 kVA
- Assume motor inrush = 600% of FLA
- During starting, motor kVA = 6x30 = 180 kVA
- Transformer = 550 kVA, Z=5.12%
- Motor starting kVA is approx. 1/3 of transformer kVA (180 kVA/ 550 kVA)
- Therefore, voltage drop across transformer may be about 1/3 of the transformer impedance (1/3 x 5.12% = 1.67%)
- No formal analysis should be necessary

550 kVA
 480 V
 $Z = 5.12\%$

$6x \rightarrow \text{M}$

$30 \text{ HP} \approx \text{kVA} = 30 \text{ kVA}$
 I_{FLA}
 $6x I_{FLA} = I_{\text{Inrush}}$
 $6x 30 = 180 \text{ kVA Inrush}$

$= \frac{\text{kVA inrush}}{\text{kVA transf}} \times Z = \frac{180}{550} \times 5.12\% = 1.67\%$

$75 \text{ HP} \Rightarrow \frac{450 \text{ kVA}}{550 \text{ kVA}} \times 5.12\% = 4.2\%$

550 kVA
 480 V
 $Z = 5.12\%$

$6x \rightarrow \text{M}$

$30 \text{ HP} \approx \text{kVA} = 30 \text{ kVA}$
 I_{FLA}
 $6x I_{FLA} = I_{\text{Inrush}}$
 $6x 30 = 180 \text{ kVA Inrush}$

$= \frac{\text{kVA inrush}}{\text{kVA transf}} \times Z = \frac{180}{550} \times 5.12\% = 1.67\%$

$75 \text{ HP} \Rightarrow \frac{450 \text{ kVA}}{550 \text{ kVA}} \times 5.12\% = 4.2\%$

$300 \text{ HP} \Rightarrow \frac{300 \times 6 = 1800 \text{ kVA}}{550 \text{ kVA}} \times 5.12\% = 16.5\%$

Motor starting calculations

Quick check – 300 HP

- Motor = 300 hp, assume 1 hp = 1 kVA
- Assume motor inrush = 600% of FLA
- During starting, motor kVA = $6 \times 300 = 1800 \text{ kVA}$
- Transformer = 550 kVA, $Z = 5.12\%$
- Motor starting kVA is approx. 325% of transformer kVA ($1800 \text{ kVA} / 550 \text{ kVA}$)
- Therefore, voltage drop across transformer may be about 325% of the transformer impedance ($325\% \times 5.12\% = 16.5\%$)
- Formal analysis should be considered, especially if system impedance is significant, or if other loads may be affected

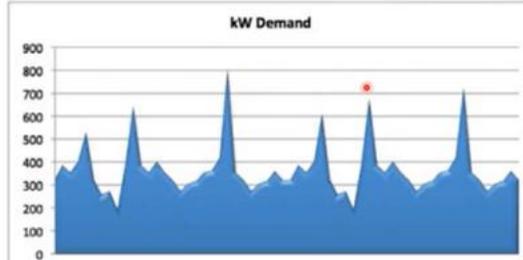


Motor starting and demand charges



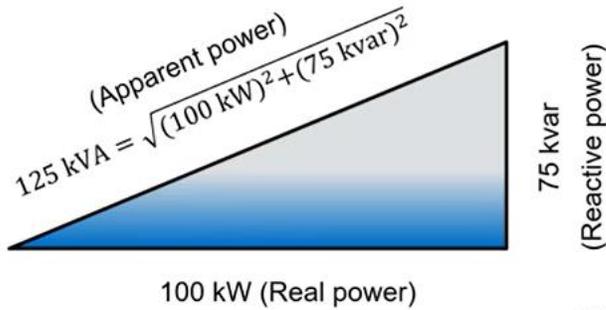
If you start a large motor (500 HP), is that going to increase your demand charge from the utility?

- kWh – 5-15 cents/kWh
- kW demand – \$5-\$15/kW

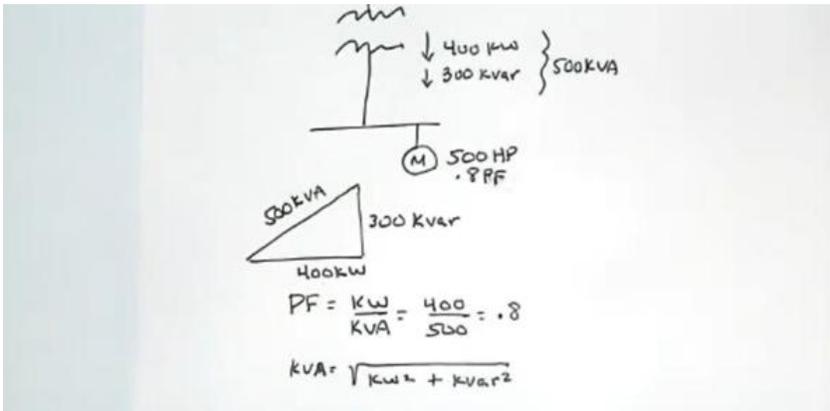


32

Power factor correction Power triangle



$$PF = \frac{kW}{kVA} = \frac{100}{125} = 0.8$$



Handwritten calculations on a whiteboard:

- Initial load: 500 kVA, 400 kW, 300 kVAR, PF = .97
- Target PF = .95
- Required kVA: $kVA_N = \frac{400 \text{ kW}}{.95} = 421 \text{ kVA}$
- Required kVAR: $kVAR = \sqrt{421^2 - 400^2} = 131 \text{ kVAR}$
- Final kVA: $kVA = \sqrt{400^2 + 100^2} = 412 \text{ kVA}$
- Final PF: $PF_N = \frac{400}{412} = .97$

Calculator display: 0.970873786407767

Basic rules of thumb



- 480V – 1.2 ratio of kVA to Amps
- 208V, 75 kVA – 208A (then ratio up for kVA)
- HP = kVA
- Normally assume infinite bus if ratio of transformers is 10:1 (kVA size) – conservative for fault calculations

$kVA_{3P} = 3 * kVA_{1P}$ <p>Pause (k)</p> $I_{FLA} = \sqrt{3} * kV_{L-L} * I_L$	$I_{FLA} = \frac{kVA_{3P}}{\sqrt{3} * kV_{L-L}}$ $I_{F(3P)} = \frac{I_{FLA-S}}{Z}$	$PF = \frac{kW}{kVA}$ $kVA = \sqrt{kW^2 + kvar^2}$
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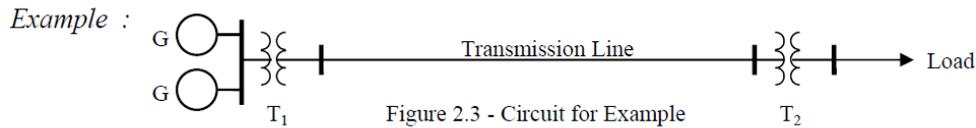
Eaton Transformers 101

Bussman SCCR Part 1 – Codes and Applications

Part 2 – Determining SCCR with UL 508, Supplement SB

Part 3 – Strategies for increasing Control Panel SCCR

Fault Analysis of Multiple Source Faults:



In the single line diagram shown in figure 2.3, each three phase generator G is rated at 200 MVA, 13.8 kV and has reactances of 0.85 pu and are generating 1.15 pu. Transformer T₁ is rated at 500 MVA, 13.5 kV/220 kV and has a reactance of 8%. The transmission line has a reactance of 7.8 Ω. Transformer T₂ has a rating of 400 MVA, 220 kV/33 kV and a reactance of 11%. The load is 250 MVA at a power factor of 0.85 lag. Convert all quantities to a common base of 500 MVA, and 220 kV on the line and draw the circuit diagram with values expressed in pu.

Solution:

The base voltage at the generator is $(220 \times 13.5 / 220)$ 13.5 kV, and on the load side is $(220 \times 33 / 220)$ 33 kV. [Since we have selected the voltage base as that corresponding to the voltage on that side of the transformer, we automatically get the voltage on the other side of the transformer as the base on that side of the transformer and the above calculation is in fact unnecessary.]

Generators G

Reactance of 0.85 pu corresponds 0.355 pu on 500 MVA, 13.5 kV base (see earlier example)

Generator voltage of 1.15 corresponds to 1.176 on 500 MVA, 13.5 kV base

Transformer T₁

Reactance of 8% (or 0.08 pu) remains unchanged as the given base is the same as the new chosen base.

Transmission Line

Reactance of 7.8 Ω corresponds to $7.8 \times 500 / 220^2 = 0.081$ pu

Transformer T₂

Reactance of 11% (0.11 pu) corresponds to $0.11 \times 500 / 400 = 0.1375$ pu

(voltage base is unchanged and does not come into the calculations)

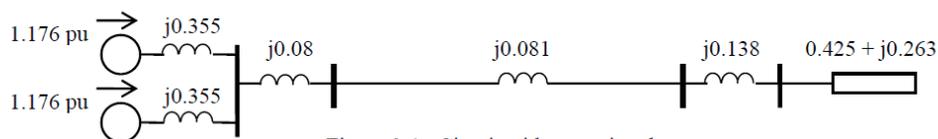
Load

Load of 250 MVA at a power factor of 0.85 corresponds to $250 / 500 = 0.5$ pu at a power factor of 0.85 lag (power factor angle = 31.79°)

∴ resistance of load = $0.5 \times 0.85 = 0.425$ pu

and reactance of load = $0.5 \times \sin 31.79^\circ = 0.263$ pu

The circuit may be expressed in per unit as shown in figure 2.4.



The Short circuit capacity (SCC) of a busbar is the fault level of the busbar. The strength of a busbar (or the ability to maintain its voltage) is directly proportional to its SCC. An infinitely strong bus (or Infinite bus bar) has an infinite SCC, with a zero equivalent impedance and will maintain its voltage under all conditions.

Magnitude of short circuit current is time dependant due to synchronous generators. It is initially at its largest value and decreasing to steady value. These higher fault levels tax Circuit Breakers adversely so that current limiting reactors are sometimes used.

The Short circuit MVA is a better indicator of the stress on CBs than the short circuit current as CB has to withstand recovery voltage across breaker following arc interruption.

The currents flowing during a fault is determined by the internal emfs of machines in the network, by the impedances of the machines, and by the impedances between the machines and the fault.

Figure 2.6 shows a part of a power system, where the rest of the system at two points of coupling have been represented by their Thevenin's equivalent circuit (or by a voltage source of 1 pu together its fault level which corresponds to the per unit value of the effective Thevenin's impedance).

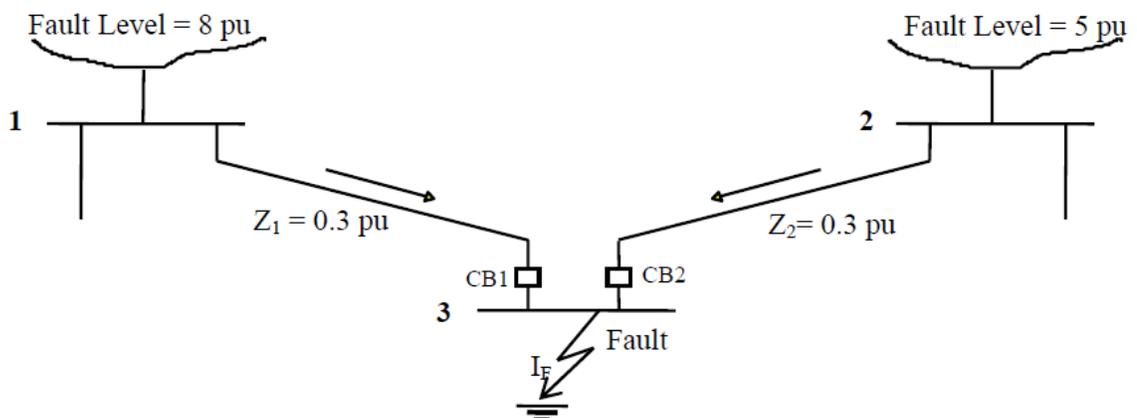


Figure 2.6 – Circuit for Fault Level Calculation

With CB1 and CB2 open, short circuit capacities are

$$\text{SCC at bus 1} = 8 \text{ p.u. gives } Z_{g1} = 1/8 = 0.125 \text{ pu}$$

$$\text{SCC at bus 2} = 5 \text{ p.u. gives } Z_{g2} = 1/5 = 0.20 \text{ pu}$$

Each of the lines are given to have a per unit impedance of 0.3 pu.

$$Z_1 = Z_2 = 0.3 \text{ p.u.}$$

With CB1 and CB2 closed, what would be the SCCs (or Fault Levels) of the busbars in the system ?

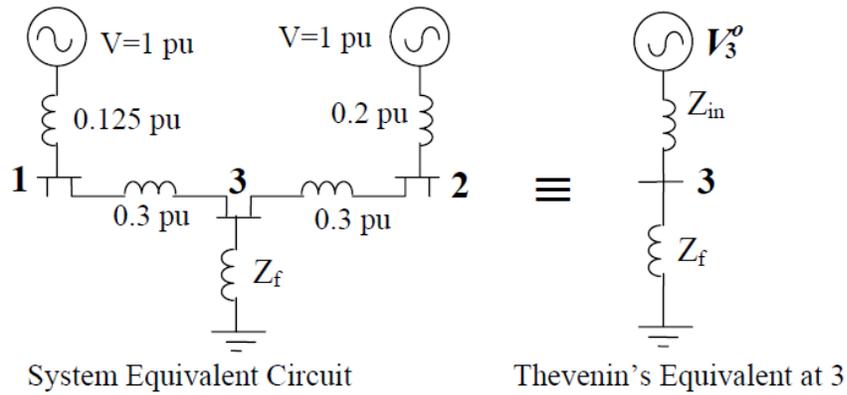


Figure 2.7a Determination of Short circuit capacities

This circuit can be reduced and analysed as in figure 2.7b.

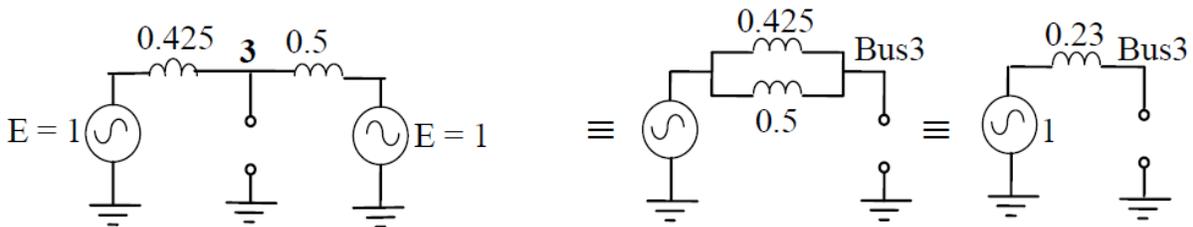


Figure 2.7b Determination of Short circuit capacity at Bus 3

Thus, the equivalent input impedance is given by to give Z_{in} as 0.23 pu at bus 3, so that the short circuit capacity at busbar 3 is given as

$$|SCC_3| = 1/0.23 = 4.35 \text{ p.u}$$

The network may also be reduced keeping the identity of Bus 1 as in figure 2.7c.

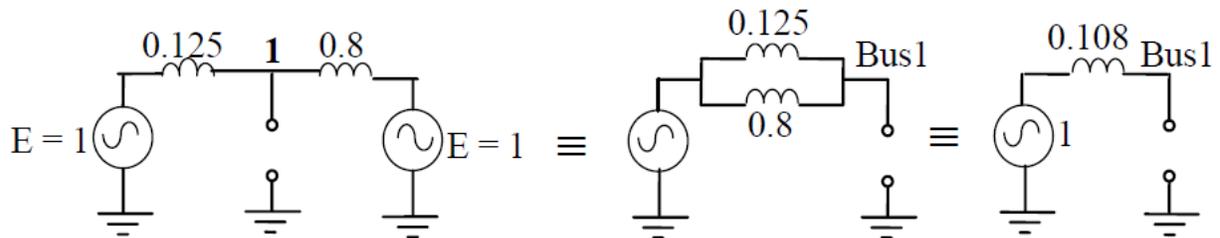


Figure 2.7c Determination of Short circuit capacity at Bus 1

Thus, the equivalent input impedance is given by to give Z_{in} as 0.108 pu at bus 1, so that the short circuit capacity at busbar 1 is given as

$$|SCC1| = 1/0.108 = 9.25 \text{ p.u}$$

This is a 16% increase on the short circuit capacity of bus 1 with the circuit breakers open.

The network may also be reduced keeping the identity of Bus 2. This would yield a value of Z_{in} as 0.157 pu, giving the short circuit capacity at busbar 2 as

$$|SCC2| = 1/0.157 = 6.37 \text{ p.u}$$

This is a 28% increase on the short circuit capacity of bus 2 with the circuit breakers open.

Typical maximum values of short circuit capacities at substations in Sri Lanka in 2000 are shown in table 2.1. Actual fault currents are lower than these values due to the presence of fault impedance in the circuit.

Symmetrical Three Phase Fault Analysis

Asymmetrical Three Phase Fault Analysis

ECAP

SKM

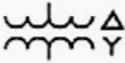
Eaton Youtube – Circuit Breakers Explained

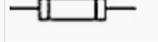
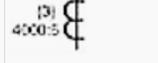
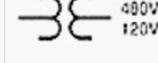
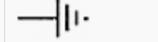
Rockwell Youtube – Centerline 2100 MCC Features

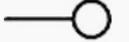
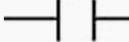
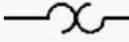
<https://www.eaton.com/content/dam/eaton/products/design-guides---consultant-audience/eaton-low-voltage-mcc-design-guide-dg043001en.pdf>

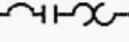
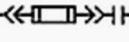
Reading One Line Drawings – Eaton – Youtube

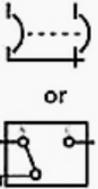
Individual electrical symbols

Symbol	Identification	Explanation
	Transformer	Represents a variety of transformers from liquid filled to dry types. Additional information is normally printed next to symbol indicating winding connections, primary /secondary voltages and KVA or MVA ratings.
	Removable or drawout circuit breaker	Normally represents a MV drawout circuit breaker 5kV and above.
	Future removable or drawout circuit breaker position	Represents a structure equipped to accept circuit breaker in the future, commonly known as provisions.
	Non-drawout circuit breaker	Represents a fixed mounted low voltage circuit breaker.

	Removable or drawout circuit breaker	Represents a drawout low voltage circuit breaker.
	Disconnect switch	Represents a switch in low or medium/high voltage applications (open position shown)
	Fuse	Represents low voltage and power fuses.
	Bus duct	Represents low and medium/high voltage bus duct.
	Current transformer	Represents current transformers mounted in assembled equipment. A ratio of 4000A to 5A shown.
	Potential or voltage transformer	Represents potential transformers usually mounted in assembled equipment. A ratio of 480V to 120V shown.
	Ground (earth)	Represents a grounding (earthing) point

	Battery	Represents a battery in an equipment package
	Motor	Represents a motor and is also shown with an “M” inside the circle. Additional motor information is commonly printed next to symbol, such as horsepower, RPM and voltage.
	Normally open (NO) contact	Can represent a single contact or single pole switch in the open position for motor control
	Normally closed (NC) contact	Can represent a single contact or single pole switch in the closed position for motor control
	Indicating light	The letter inside circle indicates the color. The color red is indicated.
	Overload relay	Protects a motor should an overload condition develop.
	Capacitor	Represents a variety of capacitors.

	Ammeter	A letter is usually shown to designate the meter type (A = ammeter, V = voltmeter, etc.)
	Instantaneous overcurrent protective relay	The device number designates the relay type (50 = instantaneous overcurrent, 59 = overvoltage, 86 = lockout, etc.)
	Emergency generator	The symbol is frequently shown in conjunction with a transfer switch.
	Fused disconnect switch	The symbol is a combination of a fuse and disconnect switch with the switch in the open position.
	Low voltage motor control	The symbol is a combination of a normally open contact (switch), overload relay, motor and disconnect device.
	Medium voltage motor starter	The symbol is a combination of a drawout fuse, normally open contact (switch) and motor.
	Meter center	A series of circle symbols representing meters usually mounted in a common enclosure.

	Load center or panelboard	One circuit breaker representing a main device and other circuit breakers representing feeder circuits usually in a common enclosure.
	Transfer switch	<ul style="list-style-type: none"> • Circuit breaker type transfer switch • Non-circuit breaker type transfer switch
	Current transformer with connected ammeter	The instrument connected could be a different instrument or several different instruments identified by the letter.
	Protective relays connected to current transformer	Device numbers indicate types of relays connected, such as: <ul style="list-style-type: none"> • 67 – Directional overcurrent • 51 – Time overcurrent

Arc Flash Training -

Youtube – OSHA’s New Arc Flash Guidance: Major Implications for De-Energized and Low Voltage Work – Tyndale

Chapter 2

Power System Representation

The basic components of a power system are generators, transformers, transmission lines, and loads. The interconnections among these components in the power system may be shown in a so-called one-line diagram. For analysis, the equivalent circuits of the components are shown in a reactance diagram or an impedance diagram.

2.1 ONE-LINE DIAGRAMS

Figure 2-1 shows the symbols used to represent the typical components of a power system. Figure 2-2 is a *one-line diagram* for a power system consisting of two generating stations connected by a transmission line; note the use of the symbols of Fig. 2-1. The advantage of such a one-line representation is its simplicity: One phase represents all three phases of the balanced system; the equivalent circuits of the components are replaced by their standard symbols; and the completion of the circuit through the neutral is omitted.

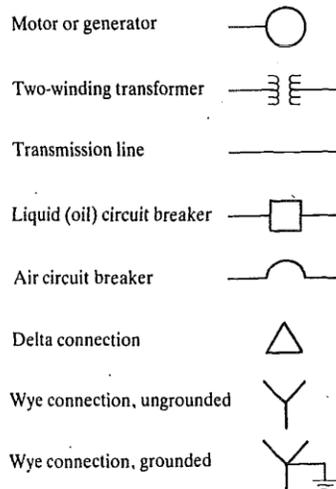


Fig. 2-1.

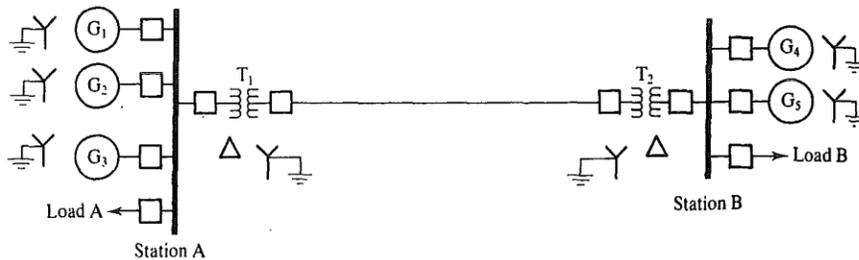


Fig. 2-2.

2.2 IMPEDANCE AND REACTANCE DIAGRAMS

The one-line diagram may serve as the basis for a circuit representation that includes the equivalent circuits of the components of the power system. Such a representation is called an *impedance diagram*, or a *reactance diagram* if resistances are neglected. The impedance and reactance diagrams corresponding to Fig. 2-2 are shown in Fig. 2-3(a) and (b), respectively. Note that only a single phase is shown.

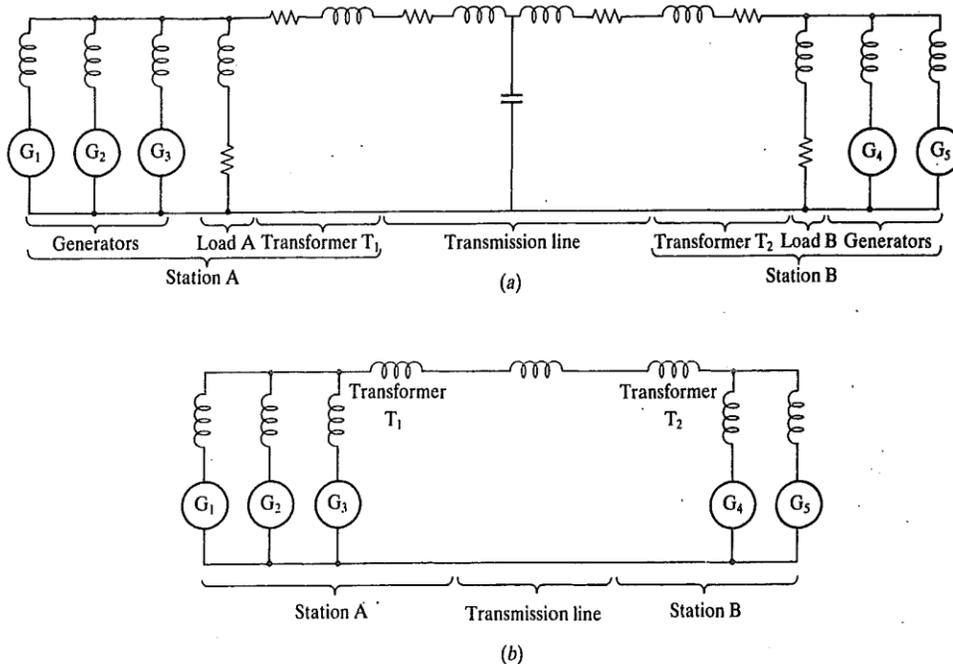


Fig. 2-3.

The following assumptions have been incorporated into Fig. 2-3(a):

1. A generator can be represented by a voltage source in series with an inductive reactance. The internal resistance of the generator is negligible compared to the reactance.
2. The loads are inductive.
3. The transformer core is ideal, and the transformer may be represented by a reactance.
4. The transmission line is a medium-length line and can be denoted by a T circuit. An alternative representation, such as a π circuit, is equally applicable.
5. The delta-wye-connected transformer T_1 may be replaced by an equivalent wye-wye-connected transformer (via a delta-to-wye transformation) so that the impedance diagram may be drawn on a per-phase basis.

(The exact nature and values of the impedances or reactances are determined by methods discussed in later chapters.)

The reactance diagram, Fig. 2-3(b), is drawn by neglecting all resistances, the static loads, and the capacitance of the transmission line.

2.3 PER-UNIT REPRESENTATION

Computations for a power system having two or more voltage levels become very cumbersome when it is necessary to convert currents to a different voltage level wherever they flow through a transformer (the change in current being inversely proportional to the transformer turns ratio). In an alternative and simpler system, a set of base values, or *base quantities*, is assumed for each voltage class, and each parameter is expressed as a decimal fraction of its respective base. For instance, suppose a base voltage of 345 kV has been chosen, and under certain operating conditions the actual system voltage is 334 kV; then the ratio of actual to base voltage is 0.97. The actual voltage may then be expressed as 0.97 *per-unit*. In an equally common practice, per-unit quantities are multiplied by 100 to obtain percent quantities; our example voltage would then be expressed as 97 *percent*.

Per-unit and percent quantities and their bases exhibit the same relationships and obey the same laws (such as Ohm's law and Kirchhoff's laws) as do quantities in other systems of units.

A minimum of four base quantities is required to completely define a per-unit system; these are voltage, current, power, and impedance (or admittance). If two of them are set arbitrarily, then the other two become fixed. The following relationships hold on a per-phase basis:

$$\text{Base current} = \frac{\text{base voltamperes}}{\text{base voltage}} \quad (\text{in amperes}) \quad (2.1)$$

$$\text{Base impedance} = \frac{\text{base voltage}}{\text{base current}} \quad (\text{in ohms}) \quad (2.2)$$

$$\text{Per-unit voltage} = \frac{\text{actual voltage}}{\text{base voltage}} \quad (\text{per unit, or pu}) \quad (2.3)$$

$$\text{Per-unit current} = \frac{\text{actual current}}{\text{base current}} \quad (\text{per unit, or pu}) \quad (2.4)$$

$$\text{Per-unit impedance} = \frac{\text{actual impedance}}{\text{base impedance}} \quad (\text{per unit, or pu}) \quad (2.5)$$

In a three-phase system, the base kVA may be chosen as the three-phase kVA, and the base voltage as the line-to-line voltage; or, the base values may be taken as the phase quantities. In either case, the per-unit three-phase kVA and voltage on the three-phase kVA base and the per-unit per-phase kVA and voltage on the kVA-per-phase base remain the same.

2.4 CHANGE OF BASE

The per-unit (pu) impedance of a generator or transformer, as supplied by the manufacturer, is generally based on the rating of the generator or transformer itself. However, such a per-unit impedance can be referred to a new voltampere base with the equation

$$(\text{Per-unit impedance})_{\text{new base}} = \frac{(\text{VA})_{\text{new base}} (\text{kV})_{\text{old base}}^2}{(\text{VA})_{\text{old base}} (\text{kV})_{\text{new base}}^2} (\text{per-unit impedance})_{\text{old base}} \quad (2.6)$$

If the old base voltage and new base voltage are the same, then (2.6) simplifies to

$$(\text{Per-unit impedance})_{\text{new base}} = \frac{(\text{VA})_{\text{new base}}}{(\text{VA})_{\text{old base}}} (\text{per-unit impedance})_{\text{old base}} \quad (2.7)$$

The impedances of transmission lines are expressed in ohms, but can be easily converted to pu values on a given voltampere base using (2.1) to (2.5).

2.5 SUMMARY OF THREE-PHASE CIRCUIT RELATIONSHIPS

A three-phase circuit may be connected either in wye or in delta. In a balanced three-phase circuit the phase and the line values of the current, power, and voltage are related as follows (the subscripts p and l refer to phase and line values, respectively):

Wye connection:

$$\begin{aligned} I_p &= I_l \\ V_p &= V_l/\sqrt{3} \\ P &= \sqrt{3}V_l I_l \cos \theta_p \end{aligned}$$

Delta connection:

$$\begin{aligned} I_p &= I_l/\sqrt{3} \\ V_p &= V_l \\ P &= \sqrt{3}V_l I_l \cos \theta_p \end{aligned}$$

The delta and wye impedances are related by

$$Z_{\text{wye}} = \frac{1}{3}Z_{\text{delta}}$$

For both types of connections, the apparent and reactive powers are, respectively,

$$VA = \sqrt{3}V_l I_l$$

and

$$Q = \sqrt{3}V_l I_l \sin \theta_p$$

From the above, it is clear that the phase angle may be obtained as

$$\tan \theta_p = \frac{Q}{P}$$

Solved Problems

- 2.1 The base impedance and base voltage for a given power system are 10Ω and 400 V , respectively. Calculate the base kVA and the base current.

From Ohm's law,

$$\text{Base current} = \frac{400}{10} = 40 \text{ A}$$

$$\text{Base kVA} = \frac{40 \times 400}{1000} = 16 \text{ kVA}$$

- 2.2 The base current and base voltage of a 345-kV system are chosen to be 3000 A and 300 kV , respectively. Determine the per-unit voltage and the base impedance for the system.

From (2.2),

$$\text{Base impedance} = \frac{300 \times 10^3}{3000} = 100 \Omega$$

From (2.3),

$$\text{Per-unit voltage} = \frac{345}{300} = 1.15 \text{ pu}$$

- 2.3** If the rating of the system of Problem 2.2 is 1380 MVA, calculate the per-unit current referred to the base of Problem 2.2.

We need the actual current in the system:

$$\text{Actual current} = \frac{1380 \times 10^6}{345 \times 10^3} = 4000 \text{ A}$$

Then, from (2.4),

$$\text{Per-unit current} = \frac{4000}{3000} = 1.33 \text{ pu}$$

- 2.4** Express a 100- Ω impedance, a 60-A current, and a 220-V voltage as per-unit quantities referred to the base values of Problem 2.1.

From (2.5),

$$\text{Per-unit impedance} = \frac{100}{10} = 10 \text{ pu}$$

From (2.4),

$$\text{Per-unit current} = \frac{60}{40} = 1.5 \text{ pu}$$

From (2.3),

$$\text{Per-unit voltage} = \frac{220}{400} = 0.55 \text{ pu}$$

- 2.5** A single-phase, 10-kVA, 200-V generator has an internal impedance Z_g of 2 Ω . Using the ratings of the generator as base values, determine the generated per-unit voltage that is required to produce full-load current under short-circuit conditions.

In per-unit terms, we have

$$\text{Base voltage} = 200 \text{ V} = 1 \text{ pu}$$

$$\text{Base kVA} = 10 \text{ kVA} = 1 \text{ pu}$$

Then, by (2.1),

$$\text{Base current} = \frac{10,000}{200} = 50 \text{ A} = 1 \text{ pu}$$

The generated voltage required to produce the rated current under short circuit is $IZ_g = 50 \times 2 = 100 \text{ V}$; or, in per-unit terms, $100/200 = 0.5 \text{ pu}$.

- 2.6** Let a 5-kVA, 400/200-V transformer be approximately represented by a 2- Ω reactance referred to the low-voltage side. Considering the rated values as base quantities, express the transformer reactance as a per-unit quantity.

We have

$$\text{Base voltamperes} = 5000 \text{ VA} \quad \text{and} \quad \text{base voltage} = 200 \text{ V}$$

so that, by (2.1) and (2.2),

$$\text{Base current} = \frac{5000}{200} = 25 \text{ A}$$

$$\text{Base impedance} = \frac{200}{25} = 8 \Omega$$

Then the per-unit reactance referred to the low-voltage side is

$$\text{Per-unit reactance} = \frac{2}{8} = 0.25 \text{ pu}$$

2.7 Repeat Problem 2.6, expressing all quantities in terms of the high-voltage side.

Here we have

$$\text{Base voltamperes} = 5000 \text{ VA} \quad \text{and} \quad \text{base voltage} = 400 \text{ V}$$

$$\text{Hence,} \quad \text{Base current} = \frac{5000}{400} = 12.5 \text{ A}$$

$$\text{and} \quad \text{Base impedance} = \frac{400}{12.5} = 32 \Omega$$

The transformer reactance referred to the high-voltage side is

$$\text{High-side reactance} = 2 \left(\frac{400}{200} \right)^2 = 8 \Omega$$

and the per-unit high-side reactance is $8/32 = 0.25 \text{ pu}$.

2.8 Express the per-unit impedance Z_{pu} and per-unit admittance Y_{pu} of a power system in terms of the base voltage V_{base} and the base voltamperes $(\text{VA})_{\text{base}}$.

From (2.2), the base impedance is

$$Z_{\text{base}} = \frac{\text{base voltage}}{\text{base current}} = \frac{V_{\text{base}}}{(\text{VA})_{\text{base}}/V_{\text{base}}} = \frac{V_{\text{base}}^2}{(\text{VA})_{\text{base}}}$$

Then, from (2.5), the per-unit impedance is

$$Z_{\text{pu}} = \frac{\text{actual impedance}}{\text{base impedance}} = \frac{Z}{Z_{\text{base}}} = \frac{Z(\text{VA})_{\text{base}}}{V_{\text{base}}^2} \quad \text{pu}$$

The per-unit admittance is

$$Y_{\text{pu}} = \frac{1}{Z_{\text{pu}}} = \frac{YV_{\text{base}}^2}{(\text{VA})_{\text{base}}} \quad \text{pu}$$

2.9 A 345-kV transmission line has a series impedance of $(4 + j60) \Omega$ and a shunt admittance of $j2 \times 10^{-3} \text{ S}$. Using 100 MVA and the line voltage as base values, calculate the per-unit impedance and per-unit admittance of the line.

From the results of Problem 2.8, we have

$$Z_{\text{pu}} = (4 + j60) \frac{100 \times 10^6}{(345 \times 10^3)^2} = (3.36 + j50.4) \times 10^{-3} \text{ pu}$$

$$Y_{\text{pu}} = (j2 \times 10^{-3}) \frac{(345 \times 10^3)^2}{100 \times 10^6} = j2.38 \text{ pu}$$

2.10 A three-phase, wye-connected system is rated at 50 MVA and 120 kV. Express 40,000 kVA of three-phase apparent power as a per-unit value referred to (a) the three-phase system kVA as base and (b) the per-phase system kVA as base.

(a) For the three-phase base,

$$\text{Base kVA} = 50,000 \text{ kVA} = 1 \text{ pu}$$

and $\text{Base kV} = 120 \text{ kV (line to line)} = 1 \text{ pu}$
 so $\text{Per-unit kVA} = \frac{40,000}{50,000} = 0.8 \text{ pu}$

(b) For the per-phase base,

$\text{Base kVA} = \frac{1}{3} \times 50,000 = 16,667 = 1 \text{ pu}$
 and $\text{Base kV} = \frac{120}{\sqrt{3}} = 69.28 \text{ kV} = 1 \text{ pu}$
 so $\text{Per-unit kVA} = \frac{1}{3} \times \frac{40,000}{16,667} = 0.8 \text{ pu}$

- 2.11** A three-phase, wye-connected, 6.25-kVA, 220-V synchronous generator has a reactance of 8.4Ω per phase. Using the rated kVA and voltage as base values, determine the per-unit reactance. Then refer this per-unit value to a 230-V, 7.5-kVA base.

For the first base, we have

$$\text{Base voltamperes} = 6250 = 1 \text{ pu} \quad \text{and} \quad \text{base voltage} = 220 = 1 \text{ pu}$$

Then $\text{Base current} = \frac{6250}{\sqrt{3} \times 220} = 16.4 = 1 \text{ pu}$

and $\text{Base reactance} = \frac{220}{16.4} = 13.4 = 1 \text{ pu}$

so that $\text{Per-unit reactance} = \frac{8.4}{13.4} = 0.627 \text{ pu}$

For the 230-V, 7.5-kVA base we obtain, from (2.6),

$$\text{Per-unit reactance} = 0.627 \left(\frac{220}{230} \right)^2 \frac{7500}{6250} = 0.688 \text{ pu}$$

- 2.12** A three-phase, 13-kV transmission line delivers 8 MVA of load. The per-phase impedance of the line is $(0.01 + j0.05) \text{ pu}$, referred to a 13-kV, 8-MVA base. What is the voltage drop across the line?

The given base quantities yield

$$\text{Base kVA} = 8000 = 1 \text{ pu}$$

and $\text{Base kV} = 13 = 1 \text{ pu}$

Then the other base quantities are

$$\text{Base current} = \frac{8000}{13\sqrt{3}} = 355.3 = 1 \text{ pu}$$

and $\text{Base impedance} = \frac{13,000}{355.3} = 36.6 = 1 \text{ pu}$

From these, we find the actual values as

$$\text{Impedance} = 36.6(0.01 + j0.05) = (0.366 + j1.83) \Omega$$

and $\text{Voltage drop} = 355.3(0.366 + j1.83) = 130 + j650 = 663.1 \text{ V}$

- 2.13** A portion of a power system consists of two generators in parallel, connected to a step-up transformer that links them with a 230-kV transmission line. The ratings of these components

are

Generator G_1 : 10 MVA, 12 percent reactance

Generator G_2 : 5 MVA, 8 percent reactance

Transformer: 15 MVA, 6 percent reactance

Transmission line: $(4 + j60) \Omega$, 230 kV

where the percent reactances are computed on the basis of the individual component ratings. Express the reactances and the impedance in percent with 15 MVA as the base value.

Equation (2.7) gives, for generator G_1 ,

$$\text{Percent reactance} = 12 \left(\frac{15}{10} \right) = 18 \text{ percent}$$

For generator G_2 ,

$$\text{Percent reactance} = 8 \left(\frac{15}{5} \right) = 24 \text{ percent}$$

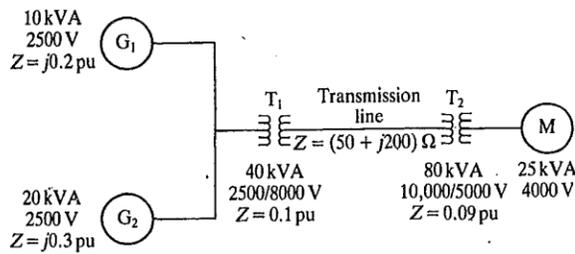
For the transformer,

$$\text{Percent reactance} = 6 \left(\frac{15}{15} \right) = 6 \text{ percent}$$

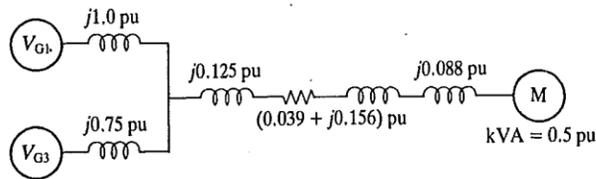
And for the transmission line, from (2.2) and (2.7),

$$\text{Percent impedance} = (4 + j60) \frac{15 \times 10^6}{(230 \times 10^3)^2} \times 100 = (0.113 + j1.7) \text{ percent}$$

2.14 Draw an impedance diagram for the system shown in Fig. 2-4(a), expressing all values as per-unit values.



(a)



(b)

Fig. 2-4.

We arbitrarily choose 50 kVA to be the base kVA. Then, from (2.6), for generator G_1 ,

$$Z_{pu} = j0.2 \frac{(2500)^2(50)}{(2500)^2(10)} = j1.0 \text{ pu}$$

For generator G_2 ,

$$Z_{pu} = j0.3 \frac{(2500)^2(50)}{(2500)^2(20)} = j0.75 \text{ pu}$$

For transformer T_1 ,

$$Z_{pu} = j0.1 \frac{(2500)^2(50)}{(2500)^2(40)} = j0.125 \text{ pu}$$

For the transmission line,

$$Z_{pu} = (50 + j200) \frac{50,000}{8000^2} = 0.039 + j0.156 \text{ pu}$$

For transformer T_2 ,

$$Z_{pu} = j0.09 \frac{(10,000)^2(50)}{(8000)^2(80)} = j0.088 \text{ pu}$$

And finally for motor M ,

$$\text{kVA}_{pu} = \frac{25}{50} = 0.5 \text{ pu}$$

These values produce the reactance diagram in Fig. 2-4(b).

- 2.15 Draw an impedance diagram for the system shown in Fig. 2-5(a), expressing all values as percent values.

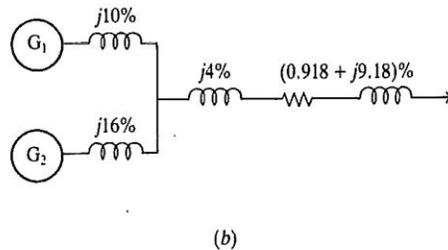
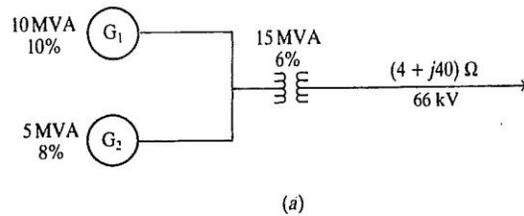


Fig. 2-5.

Let us arbitrarily choose 10 MVA to be the base MVA. Then, for generator G_1

$$\text{Percent impedance} = 10 \left(\frac{10}{10} \right) = 10 \text{ percent}$$

For generator G_2 ,

$$\text{Percent impedance} = 8 \left(\frac{10}{5} \right) = 16 \text{ percent}$$

For the transformer,

$$\text{Percent impedance} = 6 \left(\frac{10}{15} \right) = 4 \text{ percent}$$

And for the transmission line,

$$\text{Percent impedance} = (4 + j40) \frac{10 \times 10^6}{(66 \times 10^3)^2} \times 100 = (0.918 + j9.18) \text{ percent}$$

These values produce Fig. 2-5(b).

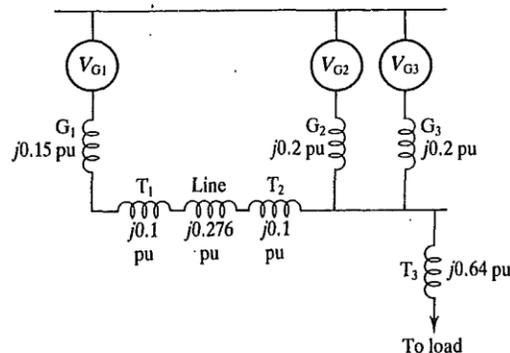
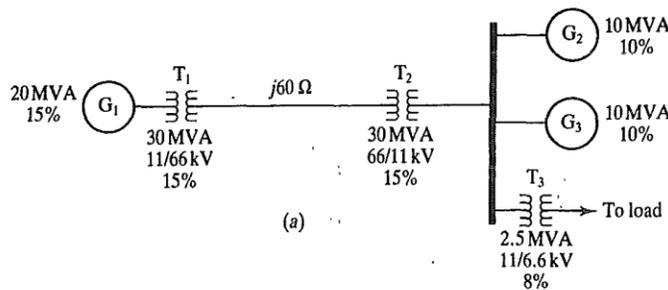
2.16 Draw a per-unit reactance diagram for the system shown in Fig. 2-6(a).

We arbitrarily choose 20 MVA and 66 kV as base values. The per-unit reactance diagram is that shown as Fig. 2-6(b), where, for G_1 , $X_{pu} = j0.15$ pu because its percent reactance is 15 percent with the same kVA base. Also for G_2 and G_3 ,

$$X_{pu} = \frac{20}{10} j0.1 = j0.2 \text{ pu}$$

For T_1 and T_2 ,

$$X_{pu} = \frac{20}{30} j0.15 = j0.1 \text{ pu}$$



(b)

Fig. 2-6.

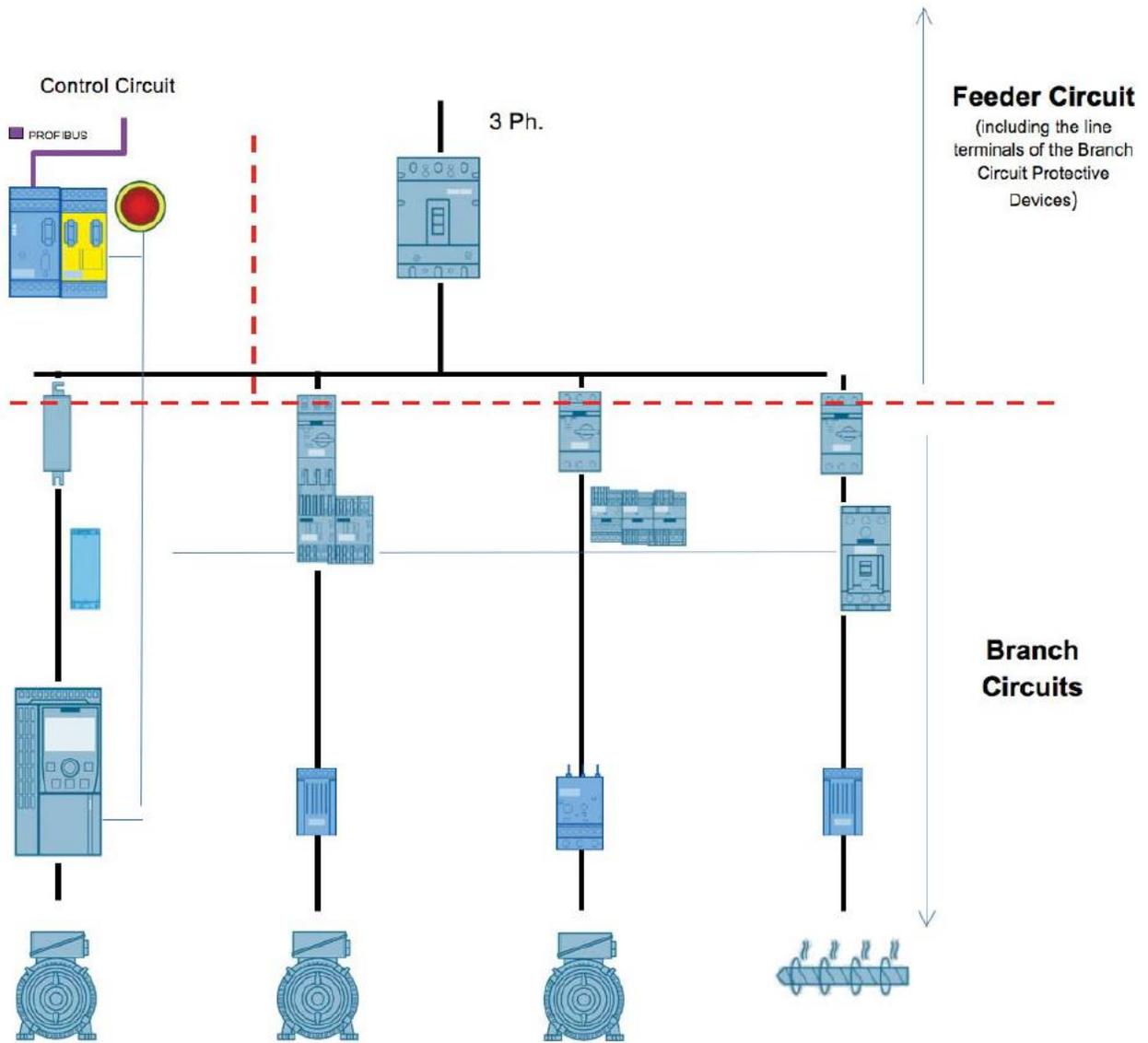
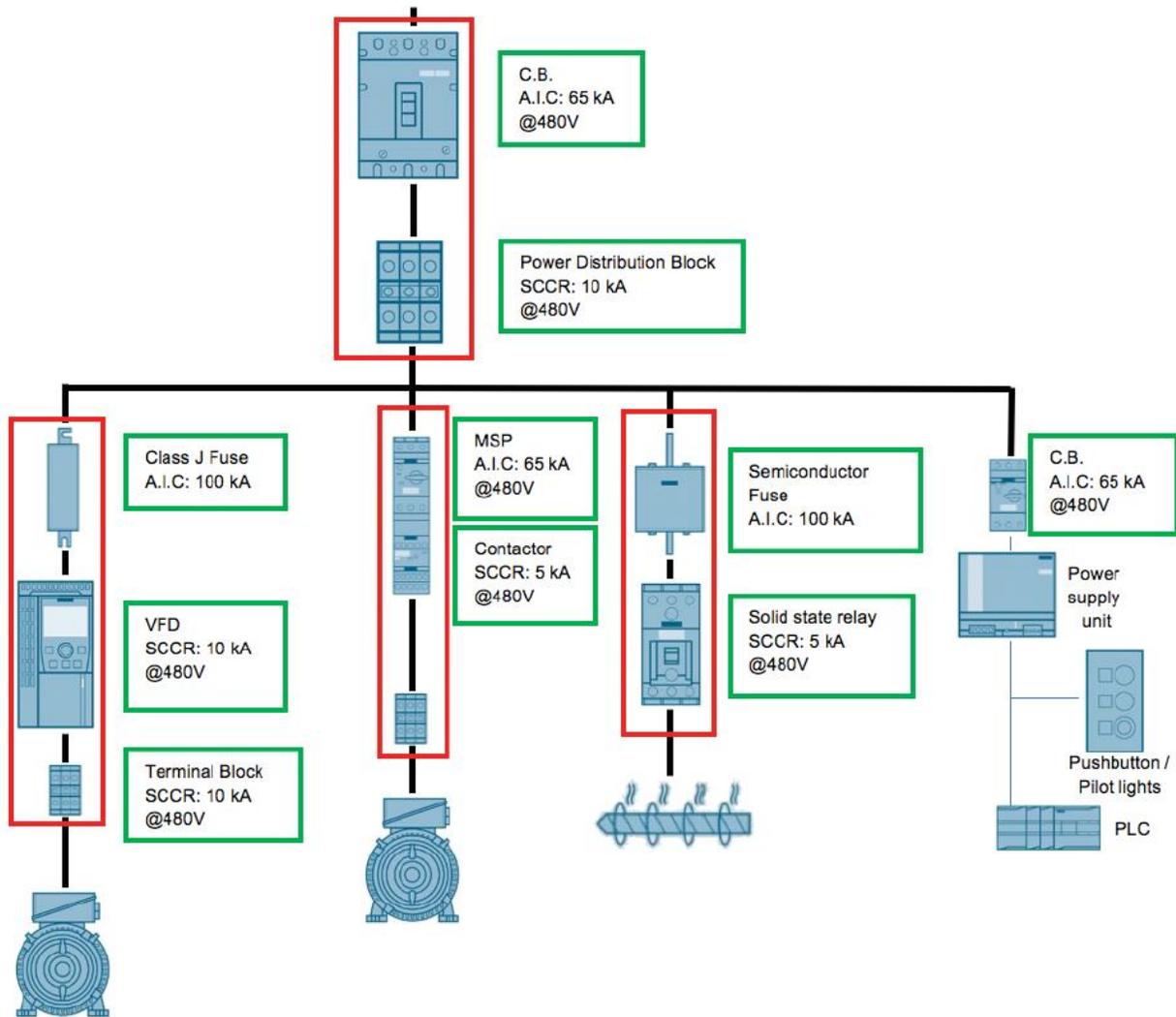


Figure 2: Graphic explanation of Step 1: Determination of the SCCR for individual power circuit components



SCCR of the individual components.
 Ratings are either from the nameplate or for unmarked components from Table SB4.1 UL508A

High Fault/Capacity Short Circuit Current
 Ratings are based on Combination Tests, certified by UL and documented in a UL Certificate of Compliance.
www.usa.siemens.com/sccr

Figure 3.1: Graphic explanation / example of Step 2: Use of current limiting transformers in the feeder circuit

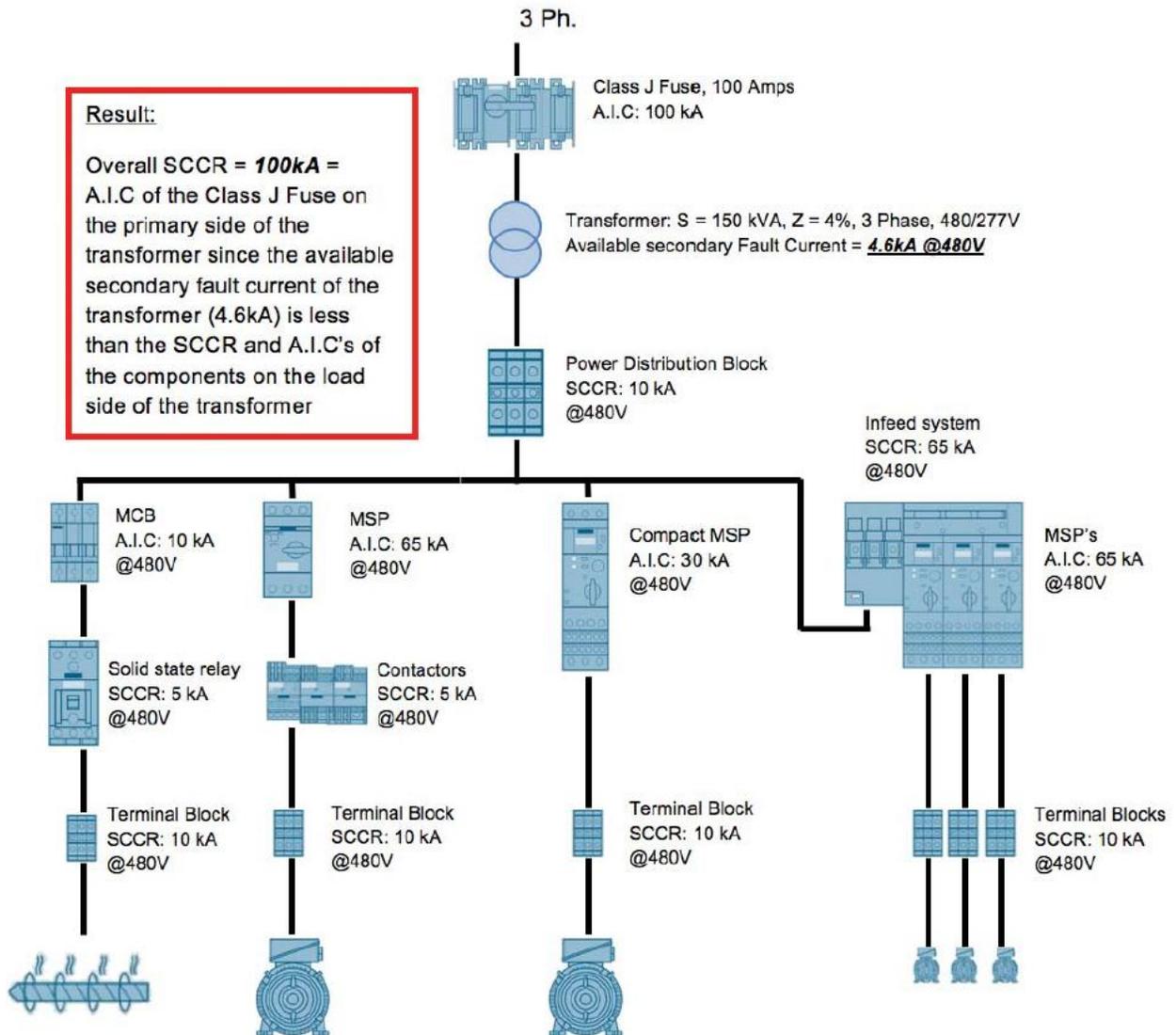


Figure 3.2: Graphic explanation / example of Step 2: Use of current limiting fuses in the feeder circuit

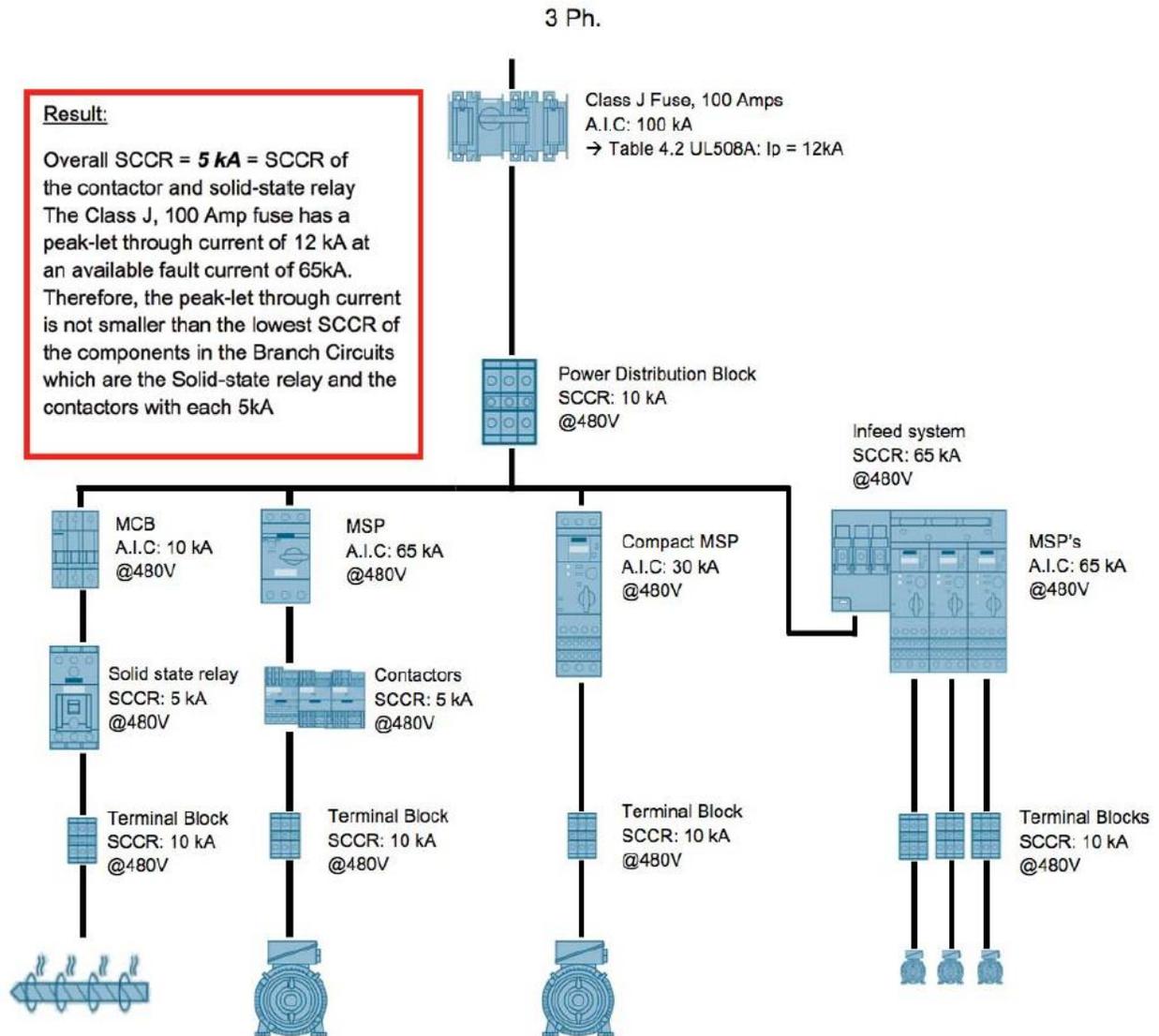
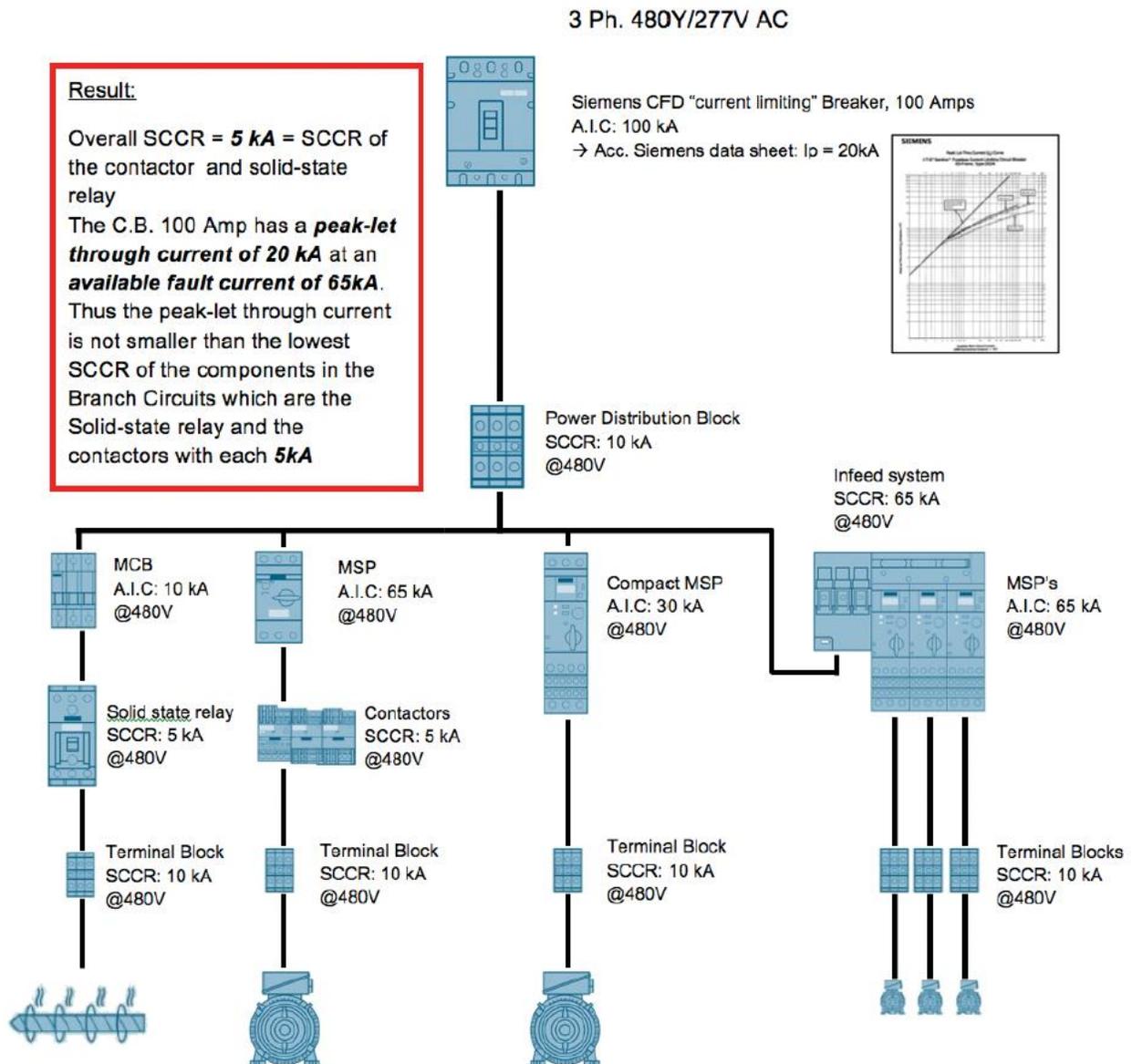


Figure 3.3: Graphic explanation / example of Step 2: Use of circuit breaker marked as “current limiting” in the feeder circuit



Fault Calculations

The operation of a power system departs from normal after the occurrence of a fault.

Faults give rise to abnormal operating conditions-usually excessive currents and voltages at certain points on the system-which are guarded against with various types of protective equipment.

6.1 TYPES OF FAULTS

Various types of short-circuit faults that can occur on a transmission line are depicted in Fig. 6-1; the frequency of occurrence decreases from part (a) to part (f). Although the balanced three-phase short circuit in Fig. 6-1(d) is relatively uncommon, it is the most severe fault and therefore determines the rating of the line-protecting circuit breaker. A fault study includes the following:

1. Determination of the maximum and minimum three-phase short-circuit currents
2. Determination of unsymmetrical fault currents, as in single ground, line-to-line, and open-circuit faults
3. Determination of the ratings of required circuit breakers
4. Investigation of schemes of protective relaying
5. Determination of voltage levels at strategic points during a fault

The short-circuit faults depicted in Fig. 6-1 are called shunt faults; open circuits, which may be caused by broken conductors, for instance, are categorized as series faults.

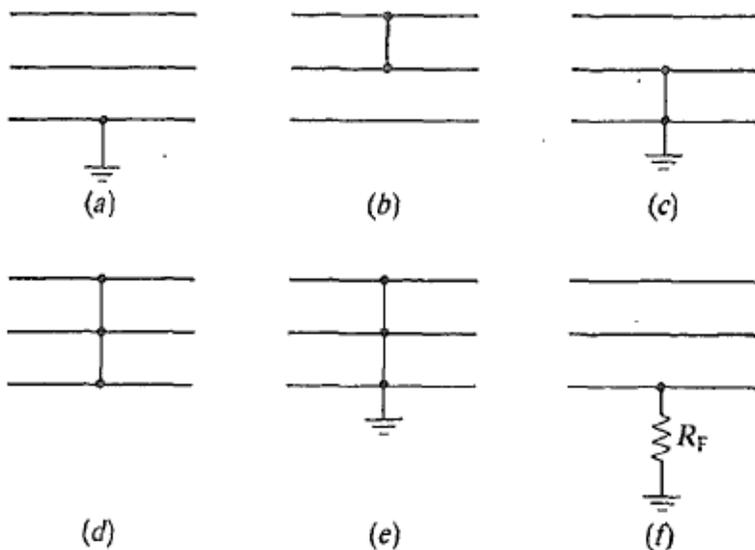


Fig. 6-1.

6.2 SYMMETRICAL FAULTS

A balanced three-phase short circuit [Fig. 6-1(d)] is an example of a symmetrical fault. Balanced three-phase fault calculations can be carried out on a per-phase basis, so that only single-phase equivalent circuits need be used in the analysis. Invariably, the circuit constants are expressed in per-unit terms, and all calculations are made on a per-unit basis. In short-circuit calculations, we often evaluate the short-circuit MVA (megavolt-amperes), which is equal to

$$\sqrt{3} V_l I_f$$

where V_l is the nominal line voltage in kilovolts, and I_f is the fault current in kiloamperes.

Short Circuit Fault Calculations

A power system functions normally until after the occurrence of a fault in the system. The good news is fault events can be minimized or avoided through diligent electrical design, accurate record keeping information on equipment/devices/motors, proper installation, and use of agency-certified equipment.

There are three major sources of fault current: an electric utility power system, a generator, and a motor. Short circuit faults are called shunt faults. An open-circuit condition is known as a series fault. Any phase/circuit to ground condition is called a ground fault. Among all faults, a balanced 3-phase short circuit is the most critical and serious. However, it is one of the least likely of faults to occur. The elements in a power distribution system that limit or impede the fault current value include: cables, transformers, and reactors.

The NEC requires protection to personnel and electrical systems against damage during short circuit conditions. Generally, circuit breaker ratings are determined for the worst-case fault situation.

You typically perform short circuit calculations when working with complex and interactive power distribution systems; however, manual calculations can be used for more simplified systems.

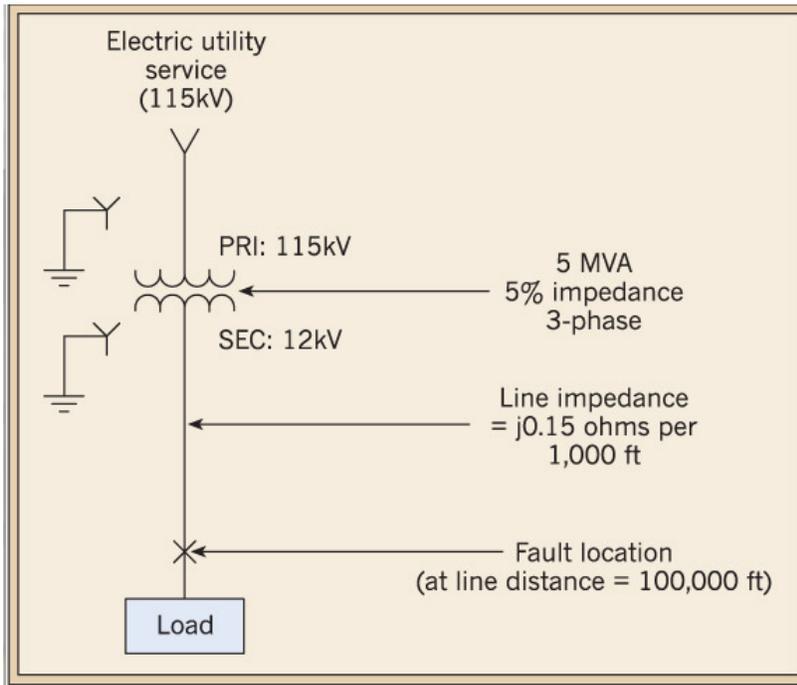
A short circuit calculation determines the amount of current that can flow at certain points in the distribution system.

An electrical device or piece of equipment can then be selected for appropriate rating (withstand or interrupting rating) based on these calculations.

A few simple examples to show how you can quickly and easily calculate fault currents.

We'll first use the concept of admittance to calculate the fault current in the system shown on the next slide. Admittance is a measure of how easily a circuit or device will allow a current to flow. It is the inverse of impedance, which is defined as a measure of opposition to current.

On the next slide, we assume the grid is infinite and can supply an infinite amount of power to the fault



Step 1: Because the electric utility can't provide us with fault information at this particular site, we assume it to be infinite. We calculate the maximum amount of power that the transformer (XFMR) will allow to flow to its secondary (i.e., load side) using this formula.

$$\text{XFMR let through power} = \text{XFMR kVA rating} \div [\% \text{ Impedance} \div 100]$$

$$= 5,000\text{kVA} \div [5 \div 100]$$

$$= 100,000\text{kVA}$$

Step 2: Now we can calculate the cable let-through power. This is defined as the amount of power that the cable would let through from an infinite source to the load side of the XFMR. The formula we use for this step is as follows:

$$\text{Cable let through power} = [(\text{kV phase-phase})^2] \div [\text{cable impedance (ohms) per phase, per 1,000 ft} \times \text{total distance (ft)}]$$

$$= [(12\text{kV})^2] \div \{0.15 \div 1,000 \text{ ft}\} \times 100,000 \text{ ft}$$

$$= 9,600\text{kVA let through}$$

Step 3: Next, we calculate the total let-through fault power by using the following formula:

$$\text{Net fault power} = 1 \div [(1 \div \text{XFMR let through power}) + (1 \div \text{cable let through power})]$$

$$= 1 \div [(1 \div 100,000\text{kVA}) + (1 \div 9,600\text{kVA})]$$

$$= 8,759\text{kVA}$$

Step 4: Now we can find the fault current using the following formula:

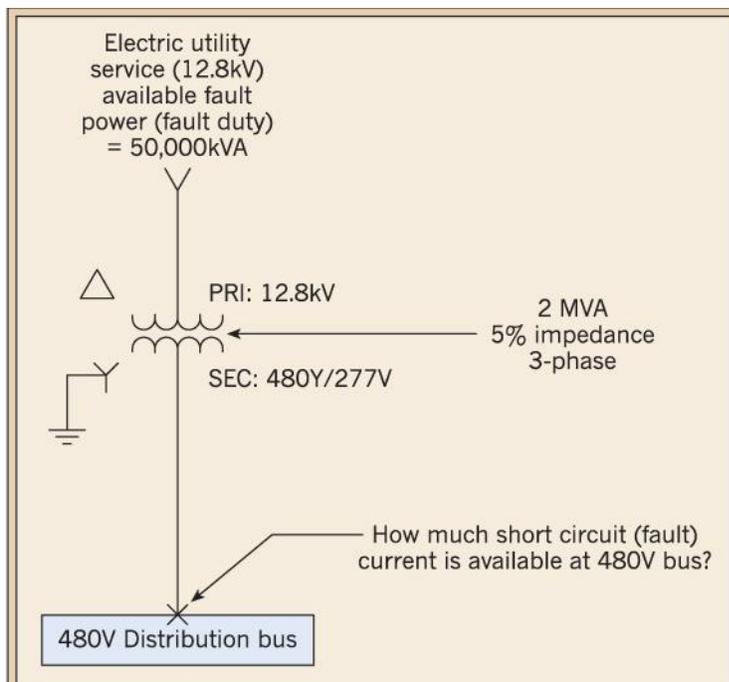
$$\text{Fault current} = \text{net fault power} \div (\text{secondary XFMR voltage rating} \times \sqrt{3})$$

$$= 8,759\text{kVA} \div (12\text{kV} \times \sqrt{3})$$

$$= 421\text{A}$$

Let's work through another sample calculation where the electric utility fault power level is known.

The given value is 50,000kVA



In this situation, we assume cable length to be minimal and therefore neglect its effect since the impedance is minimal.

$$\text{Step 1: XFMR let through power} = 2,000\text{kVA} \div [5 \div 100]$$

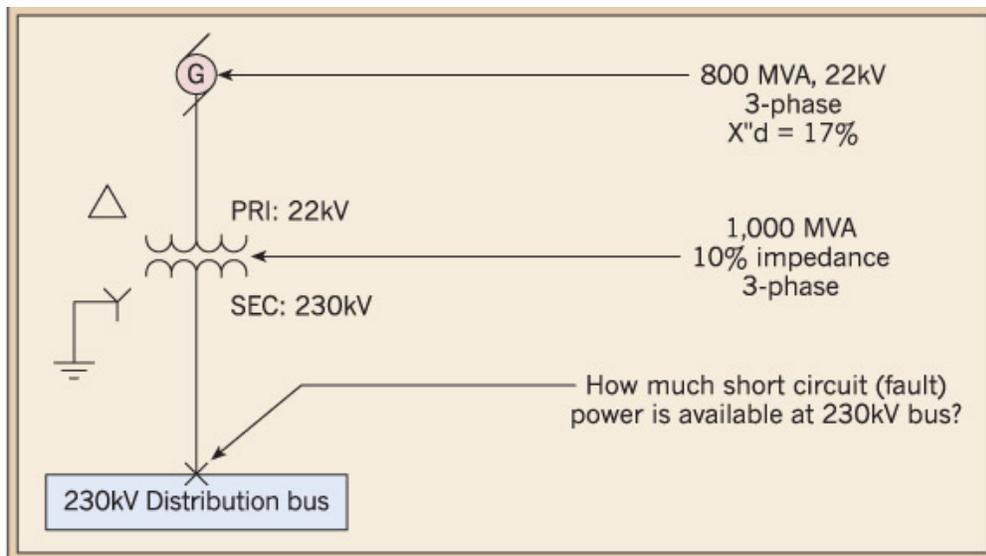
$$= 40,000\text{kVA}$$

Step 2: Net fault power = $1 \div [(1 \div 40,000\text{kVA}) + (1 \div 50,000\text{kVA})]$
 = 22,222kVA

Step 3: Fault current = $22,222\text{kVA} \div (0.48\text{kV} \times \sqrt{3})$
 = 26,729A

For our third and final example calculation, we'll work a problem where generator data is available to us.

When working with generators, we introduce a new value known as sub-transient reactance, which is referred to as X''_d . This value is typically shown on the generator nameplate or can be obtained directly from the manufacturer.



Step 1: The short circuit kVA available at the generator is calculated using the following formula:

Generator fault power (MVA) = generator MVA rating $\div X''_d$
 = $800 \text{ MVA} \div 0.17$
 = 4,706 MVA

Step 2: XFMR let through power = $1,000 \text{ MVA} \div [10 \div 100]$
 = 10,000 MVA

Step 3: Net fault power = $1 \div [(1 \div 10,000 \text{ MVA}) + (1 \div 4,706 \text{ MVA})]$
 = 3,200 MVA

As you can see, simplified short circuit fault calculations can be performed using a basic understanding of sources of fault power and current and impedance values that impede the short circuit power flow.

General Methods for Network Calculations

Rearranging these equations and rewriting them in matrix form, we obtain

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} y_{10} + y_{12} + y_{13} & -y_{12} & -y_{13} & 0 \\ -y_{12} & y_{20} + y_{12} + y_{23} + y_{24} & -y_{23} & -y_{24} \\ -y_{13} & -y_{23} & y_{30} + y_{13} + y_{23} + y_{34} & -y_{34} \\ 0 & -y_{24} & -y_{34} & y_{40} + y_{24} + y_{34} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} \quad (7.3)$$

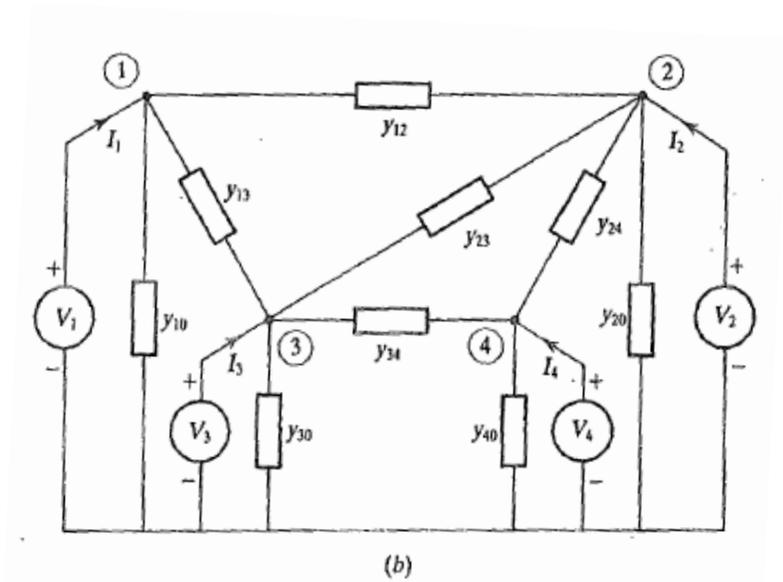
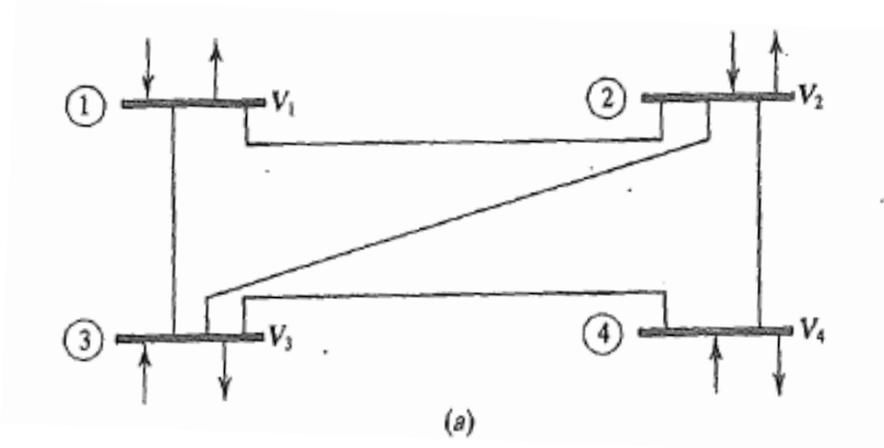


Fig. 7-2.

Equation (7.3) may be written as

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} \quad (7.4)$$

where

$$Y_{11} = y_{10} + y_{12} + y_{13}$$

$$Y_{22} = y_{20} + y_{12} + y_{23} + y_{24}$$

$$Y_{33} = y_{30} + y_{13} + y_{23} + y_{34}$$

$$Y_{44} = y_{40} + y_{24} + y_{34}$$

$$Y_{12} = Y_{21} = -y_{12}$$

$$Y_{13} = Y_{31} = -y_{13}$$

$$Y_{14} = Y_{41} = -y_{14} = 0$$

$$Y_{23} = Y_{32} = -y_{23}$$

$$Y_{24} = Y_{42} = -y_{24}$$

$$Y_{34} = Y_{43} = -y_{34}$$

Can you check 115 V live?

Yes, but **it is not considered safe or acceptable practice** under OSHA, NFPA 70E, and NEC guidelines.

Here's why:

- **115 VAC is still dangerous.**
 - Currents as low as 30 mA across the heart can be fatal.
 - Household 120 V circuits can deliver much higher fault current.
 - Contact can cause muscle lock, burns, or death.
- **"Working live" is generally prohibited.**
 - OSHA (29 CFR 1910.333) and NFPA 70E require that equipment be de-energized before work, unless:
 1. It's infeasible (like troubleshooting that requires voltage present), **and**
 2. Proper PPE and procedures are followed.
- **If troubleshooting is required live:**
 - Use insulated tools and appropriate PPE (arc-rated clothing, safety glasses, insulated gloves).
 - Only qualified persons should do it.
 - Barriers and safe approach boundaries should be set up.
- **Best practice:** Always de-energize the panel and verify absence of voltage with a tester before touching conductors. Lockout/tagout (LOTO) should be applied.

☞ So, even at 115 V, it's not considered "safe" to work live — you're only permitted to if there's no alternative (like diagnostic measurements) and you follow full electrical safety standards.

Would you like me to pull the **exact OSHA/NFPA rule language** on live work at 115 VAC so you can see what the regulations say?

You said:

do they need protective clothing

ChatGPT said:

Yes — protective clothing (PPE) is required if someone is working live, even at 115 VAC.

Here's the breakdown from **NFPA 70E** and **OSHA 1910.269 / 1910.333**:

1. **Shock protection PPE**

- **Insulated gloves** rated for 600 V (minimum) with leather protectors.
- **Insulated tools** rated for 1000 V.
- Safety glasses or face shield.

2. **Arc flash protection**

- Even 115 V circuits can produce an arc flash under certain conditions (high available fault current, close working distance, or in panelboards).
- NFPA 70E requires an **arc flash risk assessment**. If an arc flash hazard exists, arc-rated clothing (FR shirt/pants or coverall, face shield/hood) must be worn.
- In some cases (like control circuits or small branch circuits in a panel), the risk may be calculated as low, but **PPE can't be skipped without a formal risk assessment**.

3. **Exceptions**

- If the panel is de-energized and verified safe, no arc-rated PPE is required.
- For diagnostic/troubleshooting at ≤ 240 V, NFPA 70E sometimes allows a reduced PPE category — but gloves and eye protection are still mandatory.

⚡ **Key point:**

Yes, protective clothing is required if the work is live. The exact level depends on the arc flash risk analysis, but at a minimum, that means insulated gloves, safety glasses, and non-melting clothing.

Do you want me to give you the **minimum NFPA 70E PPE category** that usually applies to a standard 120 V panelboard?

You said:

yes

ChatGPT said:

Got it 👉 — here's what **NFPA 70E (2024 edition)** says about a **typical 120/240 V panelboard** (like you'd see in homes and light commercial buildings).

Minimum PPE for live work on 120 V panels

- **Safety glasses** (ANSI Z87.1 rated)
- **Arc-rated face shield** (with balaclava if needed)
- **Hearing protection** (ear canal inserts)
- **Insulated gloves** with leather protectors (Class 00 or 0, rated 500–1000 V)
- **Arc-rated shirt and pants or coverall** (Category 1 or 2, depending on fault current and clearing time)
- **Leather footwear**

Why Category 1 or 2?

- NFPA 70E Table 130.7(C)(15)(a) lists “**Panelboards or other equipment rated 240 V and below**”.
- If the available fault current is **<25 kA** and clearing time is **<0.03 seconds**, the hazard risk category is typically **Category 1**.
- That means **arc-rated clothing with minimum 4 cal/cm² protection** (flame-resistant shirt/pants).
- In many real-world 120 V panels, conditions put it at **Category 1**, but in higher-fault-current buildings, it can bump up to **Category 2** (8 cal/cm²).

Bottom line

Yes, even for 115–120 V panels, NFPA 70E usually requires **at least Category 1 PPE** if live work must be done. That means **arc-rated shirt/pants, gloves, face shield, and eye/hearing protection**.

☞ Do you want me to lay out a **sample PPE kit checklist** (like what a worker should actually put on before opening a 120 V panel live)?

You said:

what about 24 V only

ChatGPT said:

Great question. At **24 V**, the picture changes a lot.

Shock hazard

- OSHA and NFPA 70E define **50 volts AC or DC** as the threshold for requiring shock protection.
- Below 50 V, the shock risk is considered *not hazardous* for most people under normal conditions.
- So at 24 V AC or DC, **no shock-rated gloves or insulated tools are required by code**, because the voltage is too low to reliably penetrate human skin resistance.

Arc flash hazard

- Arc flash requires enough voltage to both **initiate** and **sustain** an arc.
- At 24 V, there isn't enough voltage to sustain an arc in air — so **NFPA 70E does not consider there to be an arc flash hazard** at this level.

What PPE is needed at 24 V?

- **No arc-rated clothing required.**
- **No shock PPE required** (since <50 V).
- **Basic safety glasses** are still good practice (mechanical protection from dropped tools, wire ends, etc.).

⚡ Bottom line:

If the only live conductors are **24 V**, a worker does **not need protective electrical PPE** under NFPA 70E. Just standard work clothes and safety glasses are fine.

Do you want me to give you the **exact NFPA 70E clause** that exempts <50 V circuits from shock/arc PPE?

You said:

yes

ChatGPT said:

Here's the **NFPA 70E language** that covers this:

Shock Protection (Threshold of 50 V)

NFPA 70E (2024), **Article 110.4 (A)(1)**:

*“Electrical conductors and circuit parts that have an operating voltage of **50 volts or more** shall be considered energized until proven otherwise.”*

☞ This means circuits **below 50 V** (like 24 V) are *not treated as shock hazards*.

Arc Flash Hazard

NFPA 70E (2024), **Article 130.2 (A)** requires an **arc flash risk assessment** for equipment that is “**50 volts or greater.**”

☞ So for <50 V, there’s no arc flash risk assessment or PPE requirement.

✓ Summary from NFPA 70E:

- <50 V → No shock PPE required.
- <50 V → No arc flash PPE required.
- **Basic eye protection** (ANSI Z87.1 safety glasses) is still recommended for mechanical safety, but that’s general shop practice, not an electrical PPE rule.

Would you like me to also pull the **OSHA regulation (1910.333)** that matches this same 50 V threshold, so you’ve got both NFPA and OSHA wording?

You said:

yes

ChatGPT said:

Here’s the **OSHA side of it** — it lines up with NFPA 70E:

OSHA 29 CFR 1910.333(a)(1):

*“Live parts to which an employee may be exposed shall be deenergized before the employee works on or near them, **unless the employer can demonstrate that deenergizing introduces additional or increased hazards or is infeasible ...**”*

But OSHA gives the **voltage threshold** in a note to this section:

OSHA 29 CFR 1910.303(g)(2)(i):

*“Exposed live parts operating at **50 volts or more** shall be guarded against accidental contact by approved cabinets or other forms of approved enclosures ...”*

☞ This establishes the **50 V threshold** for both guarding and PPE. Below that, OSHA does **not** require shock protection.

✓ **So combined with NFPA 70E:**

- **24 V circuits** → no shock PPE, no arc flash PPE.
- OSHA and NFPA both kick in at **50 V and above**.

Would you like me to put together a **voltage-to-PPE quick reference chart** (e.g., <50 V, 120 V, 480 V) so you have a simple cheat sheet?

You said:

yes

ChatGPT said:

Perfect 🎯 — here's a **quick reference PPE guide by voltage level**, combining **NFPA 70E (2024)** and **OSHA 1910 Subpart S** rules:

↳ Voltage-to-PPE Quick Reference

< 50 V (AC or DC)

- **Shock Hazard:** ✗ None (OSHA & NFPA both exempt below 50 V)
- **Arc Flash Hazard:** ✗ None (arc can't sustain at this voltage)
- **Required PPE:** Standard work clothes + **safety glasses** (mechanical protection only)

120/240 V (Single-phase & small panelboards)

- **Shock Hazard:** ✓ Yes (insulated gloves & tools required if live)
 - **Arc Flash Hazard:** ✓ Possible, but usually low (depends on fault current and breaker clearing time)
 - **Typical PPE Category:** **Category 1** (≥ 4 cal/cm² FR shirt & pants, face shield, gloves, glasses, hearing protection)
 - **When Needed:** Only if live work is required (e.g., troubleshooting). Otherwise, de-energize.
-

480 V (Three-phase panels, MCCs, switchgear)

- **Shock Hazard:** ✓ Significant (gloves, insulated tools mandatory)
 - **Arc Flash Hazard:** ✓ Higher risk (high fault current, longer clearing times)
 - **Typical PPE Category:** **Category 2–4** ($\geq 8\text{--}40$ cal/cm²; may require arc flash suit/hood)
 - **Risk Assessment:** Arc flash study or use NFPA 70E tables to determine exact PPE.
-

600 V+ (industrial systems)

- **Shock Hazard:** ✓ Very high (special gloves, insulated mats, approach boundaries)
 - **Arc Flash Hazard:** ✓ Severe (blast, molten copper, high incident energy)
 - **Typical PPE Category:** **Category 3–4** (25–40 cal/cm²; full arc flash suit with hood, gloves, balaclava, hearing protection, footwear)
 - **Risk Assessment:** Always required — no guessing.
-

✓ Rule of Thumb:

- **<50 V** → No electrical PPE required.
- **50–240 V** → Gloves + Cat 1 PPE minimum if live.
- **480–600 V** → Gloves + Cat 2–4 PPE depending on arc flash study.
- **>600 V** → Full arc flash suit, strict approach boundaries, detailed engineering analysis.

[Grounding and bonding - YouTube](#)

Grounding and bonding Two professional engineers (Dan **Carnovale** and Tom Domitrovich) with years of power systems experience will clarify this often misunderstood concept while making...

- Video Duration: 29 min

NEC Requirements for Grounding of Separately Derived Systems

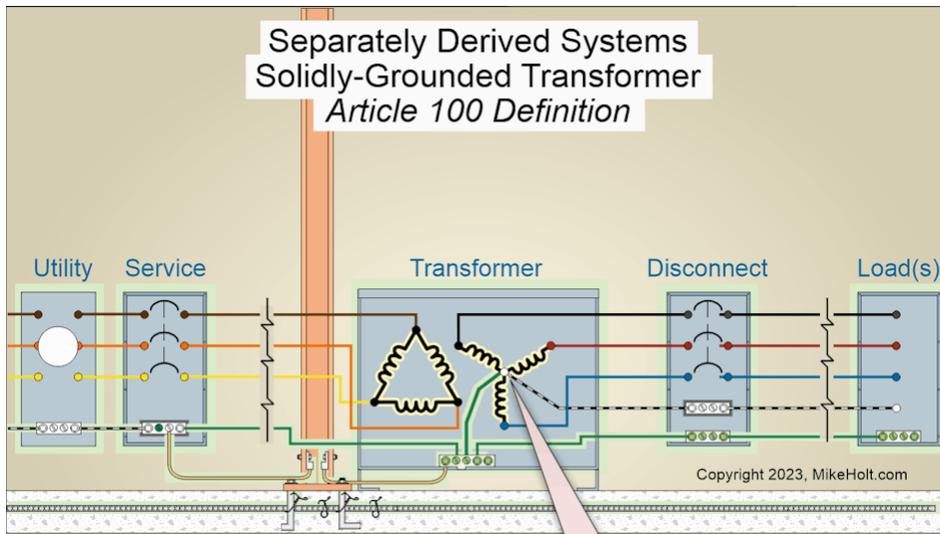
Do you understand how the grounding and bonding requirements for the separately derived systems?

[Mike Holt](#)

Oct. 21, 2025

Takeaways

- A separately derived system (SDS) has no direct connection to other sources except through grounding and bonding.
- Grounding and bonding must be performed at the source, not on the load side, to prevent potential differences and ensure safety.
- System bonding jumpers connect the secondary neutral to the equipment grounding conductor at specific points, complying with NEC Sec. 250.28 and Sec. 250.102(C).
- Multiple transformer SDSs can share a common grounding electrode conductor, but connections must be accessible and made with approved methods like exothermic welding.



A separately derived system (SDS) is an electrical power supply having no direct connection(s) to the circuit conductors of any other electrical source other than those established by grounding and bonding connections [Art. 100], as shown in **Fig. 1**.

Except for autotransformers, transformers are separately derived because the primary conductors have no direct electrical connection to the secondary conductors.

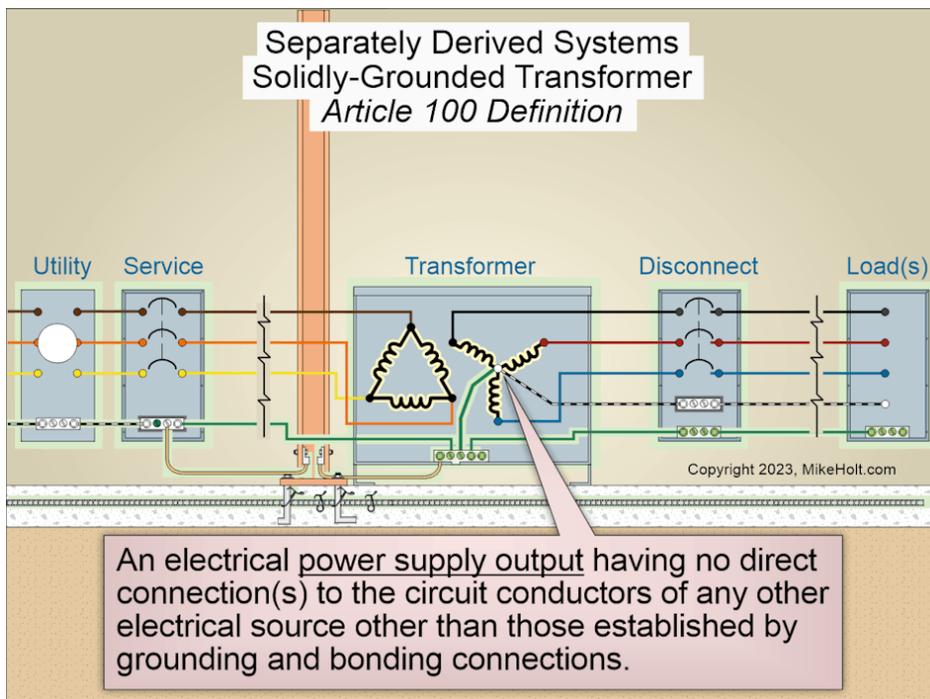


Fig. 1. A “separately derived system” (SDS) is an electrical power supply having no direct connection(s) to the circuit conductors of any other electrical source other than those established by grounding and bonding connections.

Transformer SDSs must be bonded and grounded per Sec. 250.30(A)(1) through (A)(8). On the load side of the bonding jumper, you cannot connect the neutral conductor to:

- The transformer case.
- Metal parts of equipment.
- Equipment grounding conductors.

System bonding jumper

A system bonding jumper is the connection between the neutral conductor (or grounded-phase conductor) and the equipment grounding conductor (EGC), supply-side bonding jumper, or both at a transformer SDS.

A system bonding jumper must be installed at the secondary neutral point or the secondary disconnect neutral terminal (not both). The system bonding jumper must comply with Sec. 250.28 and be sized per Sec. 250.102(C).

The grounding electrode conductor (GEC) must terminate at the point where the system bonding jumper has been installed [Sec. 250.30(A)(5)].

Exception No. 2: If a structure is supplied by a feeder from an outdoor transformer SDS, a system bonding jumper at both the source and the first disconnect is permitted if it does not establish a parallel path for the neutral current. The neutral conductor cannot be smaller than the size specified for the system bonding jumper, and it is not required to be larger than the phase conductor(s).

A system bonding jumper at the source connects the secondary neutral point of the system to the metal enclosure of the transformer SDS [Sec. 250.30(A)(1)(a)]. A system bonding jumper at the first disconnecting means connects the neutral conductor of the transformer secondary to the metal enclosure at the secondary disconnect [Sec. 250.30(A)(1)(b)].

Supply-side bonding jumper

A supply-side bonding jumper installed from the transformer enclosure to the secondary disconnect enclosure can be a non-flexible metal raceway or of the wire type.

The supply-side bonding jumper can be rigid metal conduit (RMC), intermediate metal conduit (IMC), or electrical metallic tubing (EMT) run between the transformer SDS enclosure and the secondary system disconnect enclosure. A nonmetallic or flexible raceway must have a supply-side bonding jumper of the wire type.

A supply-side bonding jumper of the wire type must be sized per Sec. 250.102(C) based on the size or area of the secondary phase conductors in the raceway or cable (**Fig. 2**).

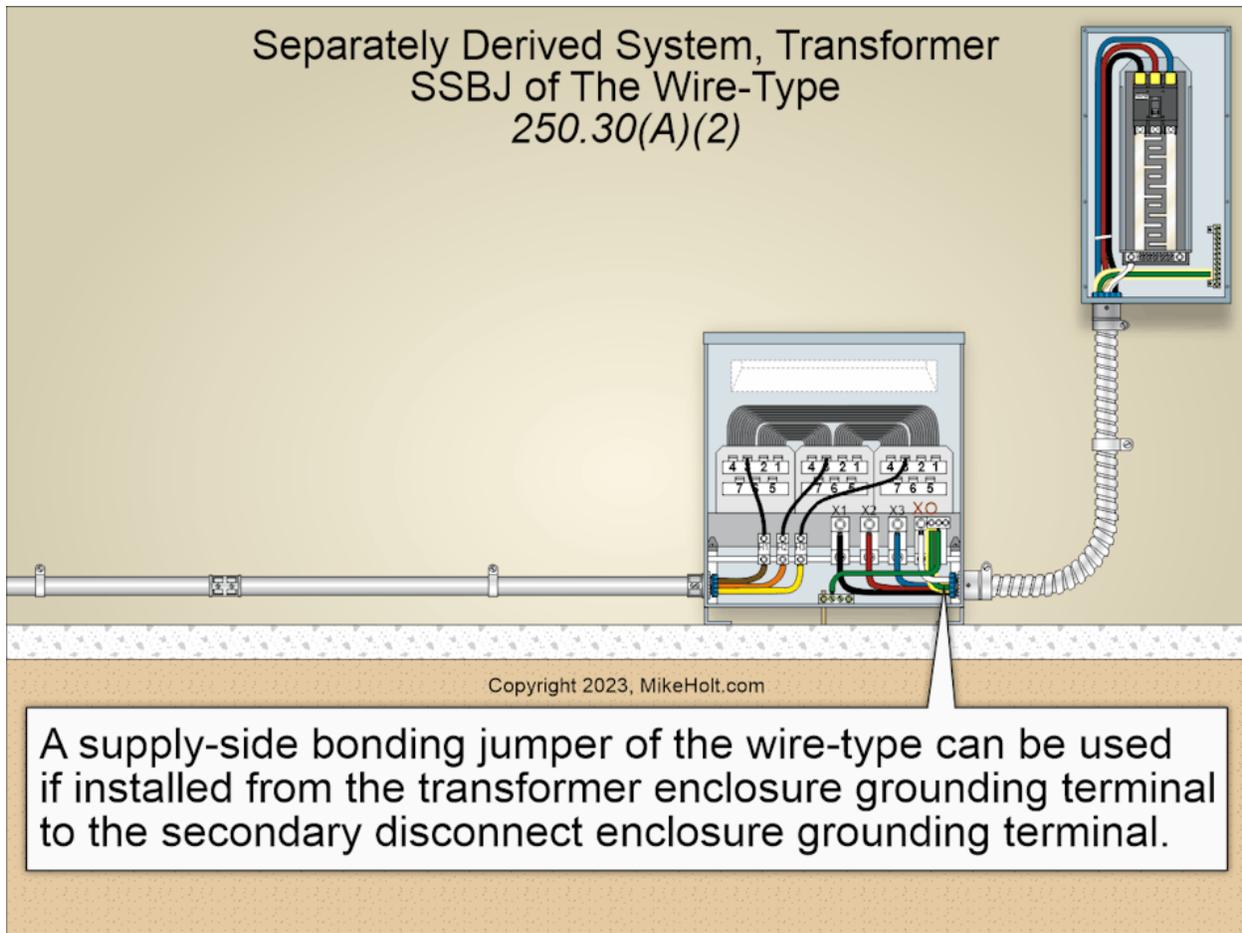


Fig. 2. A supply-side bonding jumper of the wire type must be sized per Sec. 250.102(C) based on the size or area of the secondary phase conductors in the raceway or cable.

System grounded conductor

The system neutral conductor is not required to be larger than the phase conductors [Sec. 250.30(A)(3)]. Oversizing the neutral conductor does not solve excessive unbalanced current returning in the neutral. It is merely a coping strategy that still leaves you with the energy waste and other problems related to reliability issues.

Suppose the system bonding jumper is installed at the secondary system disconnect instead of at the transformer SDS, and the neutral is run in a single raceway. In that case, the neutral must be run from the transformer SDS to the secondary system disconnect. It must be sized per Table 250.102(C)(1) based on the size or area of the secondary phase conductor.

What if you run that neutral paralleled in two or more raceways or cables? In that case, you size the neutral conductor(s) in each raceway or cable set connected in parallel based on the largest phase conductor in each raceway or cable using Table 250.102(C)(1). But don't go smaller than 1/0 AWG.

Grounding electrode conductor

A transformer SDS installed indoors must be grounded to the building grounding electrode system [Sec. 250.30(A)(4)].

The GEC for a transformer SDS must be sized per Sec. 250.66 [Sec. 250.30(A)(5)]. The GEC must terminate at the neutral conductor at the same point where the system bonding jumper is connected.

To prevent objectionable neutral current from flowing onto metal parts [Sec. 250.6], the GEC must originate at the same point on the transformer SDS as where the system bonding jumper is connected [Sec. 250.30(A)(1)].

Exception No. 1: If the system bonding jumper is a wire or busbar [Sec. 250.30(A)(1)], the GEC can terminate at the grounding terminal, bar, or bus where the system bonding jumper terminates, instead of on the neutral terminal.

Grounding electrode conductor example no. 1

Question: What size GEC is required for a 45kVA, 3-phase, 480V to 120V/208V transformer when the secondary conductors are sized at 1/0 AWG?

(a) 6 AWG (b) 4 AWG (c) 3 AWG (d) 2 AWG

Answer: (a) 6 AWG [Table 250.66]

Grounding electrode conductor example no. 2

Question: What size GEC is required for a 75kVA, 3-phase, 480V to 120V/208V transformer when the secondary conductors are sized at 4/0 AWG?

(a) 6 AWG (b) 4 AWG (c) 3 AWG (d) 2 AWG

Answer: (d) 2 AWG [Table 250.66]

Grounding electrode conductor example no. 3

Question: What size GEC is required for a 112.50kVA, 3-phase, 480V to 120V/208V transformer when the secondary conductors are sized at 600kcmil?

(a) 1/0 AWG (b) 2/0 AWG (c) 3/0 AWG (d) 4/0 AWG

Answer: (b) 2/0 AWG

Where there are multiple transformer SDSs, a GEC tap from each of them to a common GEC is permitted [Sec. 250.30(A)(6)]. This connection must be made at the same point on the

(2) Listed connections to aluminum or copper busbars at least ¼ in. thick × 2 in. wide, and of a length to accommodate the terminations necessary for the installation.

(3) Exothermic Welding. Tap GECs must remain without a splice or joint.

If located outdoors, the grounding electrode connection must be made at the transformer SDS [Sec. 250.30(C)].

The GEC must comply with Sec. 250.64(A), (B), (C), and (E) [Sec. 250.30(A)(7)]. For example, the GEC must be copper within 18 in. of the surface of the earth [Sec. 250.64(A)], securely fastened to the surface on which it is carried [Sec. 250.64(B)(1)], and adequately protected if exposed to physical damage [Sec. 250.64(B)(2) and (3)]. Also, ferrous metal enclosures enclosing a GEC must be made electrically continuous from the point of attachment to cabinets or equipment to the grounding electrode [Sec. 250.64(E)].

Ensuring success with SDS

While a source might be separately derived, the rules for grounding and bonding it are not derived from some separate set of physics. Keep two principles in mind when bonding and grounding any SDS.

First, we ground at the source but not on the load side. That is why there's a grounding connection at an SDS. Second, we bond so there is no difference in potential between non-current-carrying metallic objects.

The 10 Worst Grounding Mistakes You'll Ever Make

Aug 1, 2008 [David Herres](#) | *Electrical Construction and Maintenance*

Why common errors in residential, commercial, and industrial wiring can lead to fire and electric shock hazards.

Proper grounding and bonding prevent unwanted voltage on non-current-carrying metal objects, such as tool and appliance casings, raceways, and enclosures, as well as facilitate the correct operation of overcurrent devices. But beware of wiring everything to a ground rod and considering the job well done. There are certain subtleties you must follow to adhere to applicable NEC rules and provide safe installations to the public and working personnel. Although ground theory is a vast subject, on which whole volumes have been written, let's take a look at some of the most common grounding errors you may run into on a daily basis.

1. Improper replacement of non-grounding receptacles. Dwellings and non-dwellings often contain non-grounding receptacles (**Photo 1**). It's not the NEC's intent to immediately replace all noncompliant equipment with each new edition of the Code. In fact, it's perfectly fine to leave the old "two prongers" in place. But because an intact functioning equipment ground is such an obvious safety feature, most electricians tend to replace these old relics whenever possible.



Photo 1. This non-grounding receptacle is typical of those found in older homes across the country.

There are several ways you can complete this upgrade, many of which are erroneous and strictly against the Code. For example, never apply the following non-NEC-compliant solutions:

- Hook up a new grounding receptacle on the theory that this is a step in the right direction. This can lead future electricians and occupants to believe they are fully protected by a non-functioning ground receptacle.
- Connect the green grounding terminal of a grounded receptacle via a short jumper to the grounded neutral conductor. This practice is totally noncompliant and dangerous because when a load is connected, voltage will appear on both the neutral and ground wires. Therefore, any noncurrent-carrying appliance or tool case will become energized, causing shock to the user, who is typically partially or totally grounded.
- Run an individual ground conductor from the green grounding terminal of a grounded receptacle to the nearest water pipe or other grounded object. This “floating ground” presents various hazards. It is likely that this ground rod of convenience will have several ohms of ground resistance so that, in case of ground fault within a connected tool or appliance, the breaker will not trip — and exposed metal will remain energized.
- Run an individual ground conductor back to the entrance panel and connect it to the neutral bar or grounding strip. This solution is somewhat better, but still noncompliant. Any grounding conductor must be within the circuit cable or raceway. One objection is that an individual conductor could be damaged or removed in the course of work taking place in the future.

What are the correct ways to handle this type of situation, when you find yourself working with non-grounded receptacles?

- The best approach is to run a new branch circuit back to the panel, verifying presence of a valid ground. Because this procedure usually involves fishing cable behind walls or, in some cases, removing and then replacing wall finish, it's not always feasible unless a total rewiring job is being performed.
- Another possibility is to replace the two-prong receptacle with a GFCI. Hook up the two wires and leave the grounding terminal unattached. Included with the GFCI is a sticker that says, “No equipment ground.” This sticker must be in place so that future electricians and users are not misled. The thinking behind this strategy is that even though the tool or appliance case is not grounded, the GFCI will provide enhanced safety. It's important to note that a GFCI functions properly without the presence of a grounding conductor. The device compares current flowing through the hot and neutral conductors and trips if a difference of more than 5 milliamps is detected.
- Non-grounding receptacles are still manufactured. If replacement is necessary (and acquiring a ground is not feasible), installation of a new non-grounding receptacle is a way to go.

2. Installation of a satellite dish, telephone, CATV, or other low-voltage equipment without proper grounding. If you look at a number of satellite dish installations in your neighborhood, a certain percentage will inevitably not be grounded at all. Of those that are grounded, there is still a high probability many are not fully compliant. For example, the grounding electrode conductor could be too long, too small, have unlisted clamps at terminations, have excess bends, or be connected to a single ground rod but not be bonded to other system grounds.

For NEC purposes, a satellite dish is an antenna, and installation requirements are found in Chapter 8, Communications Systems. Article 810, Radio and Television Equipment, details the installation requirements. Part II deals with receiving Equipment — Antenna Systems. This type of equipment, which includes the satellite dish, must have a listed antenna discharge unit, which can be either outside the building or inside between the point of entrance of the lead-in conductors and the receiver — and as near as possible to the entrance of the conductors to the building. The antenna discharge unit is not to be located near combustible material and certainly not within a hazardous (classified) location.

The antenna discharge unit must be grounded. The grounding conductor is usually copper; however, you can use aluminum or copper-clad aluminum if it's not in contact with masonry or earth. Outside, aluminum or copper-clad aluminum cannot be within 18 inches of the earth.



Photo 2. Grounding means for a satellite dish must be located at the point of entrance to the building. In this particular installation, the grounding conductor is integral with the coax from the dish, but the installer did not bond it to other system grounds.

The grounding conductor can be bare or insulated, stranded or solid, and must be securely fastened in place and run in a straight line from the discharge unit to the grounding electrode (**Photo 2**). If the building has an intersystem bonding termination, the grounding conductor is to be connected to it or to one of the following:

- Grounding electrode system.

- Grounded interior metal water piping system within 5 feet of point of entrance to the building.
- Power service accessible grounding means external to the building.
- Metallic power service raceway.
- Service equipment enclosure.
- Grounding electrode conductor or its metal enclosure.

If this grounding conductor is installed within a metal raceway, you must bond the metal raceway to it at both ends. For this reason, if raceway is deemed necessary for extra protection, UL-listed PVC (rigid non-metallic conduit) is generally used. The grounding conductor must be no smaller than 10 AWG copper.

Where separate electrodes are used, you must connect the antenna discharge unit grounding means to the premises power system grounding system by a 6 AWG copper conductor. Needless to say, grounding a satellite dish goes well beyond simply driving a ground rod at the point of entrance.

Grounding for CATV is slightly different. Typically, CATV is brought into the building via coaxial cable, which has a center conductor, insulating spacer, and outer electrical shield. Because of the spacer, capacitive coupling is diminished so that the cable provides a high-quality signal for data, voice, and video transmission. Improper grounding of coaxial cable used for CATV is very common.

There is no antenna discharge unit as required for satellite dish installation. Instead, the shield of the coaxial cable is connected to an insulated grounding conductor that is limited to copper but may be stranded or solid. The grounding conductor is 14 AWG minimum so that it has current-carrying capacity approximately equal to the outer shield of the coaxial cable.

The major distinguishing characteristic is that for one- and two-family homes the grounding conductor cannot exceed 20 feet in length and should preferably be shorter. If a grounding electrode such as the Intersystem Bonding Termination is not within 20 feet, it is necessary to drive a ground rod for that purpose. However, even after this dedicated grounding means is established, in order to be NEC-compliant, the installation must have a bonding jumper not smaller than 6 AWG or equivalent, which is connected between the CATV system's grounding electrode and the power grounding electrode system for the building. Omitting this jumper is a serious Code violation, second only to no grounding at all. You must bond all system grounds, antenna, power, CATV, telephone, and so on with a heavy bonding jumper.

3. Non-installation of GFCIs where required. Recent Code editions have mandated increased use of GFCIs. In dwelling units, GFCIs are required on all 125V, single-phase, 15A and 20A receptacles in: bathrooms; garages; accessory buildings with a floor at or below grade level not intended as a habitable room, limited to storage, work and similar areas; outdoors; kitchens along countertops; within 6 feet of outside edge of laundry, utility, and wet bar sinks; and boathouses. In other than dwelling units, GFCIs are required on all 125V, single-phase, 15A and 20A receptacles in bathrooms, kitchens, rooftops, outdoors, and within 6 feet of the outside edge of sinks.

Other areas requiring the use of GFCIs include: boat hoists, aircraft hangars, drinking fountains, cord- and plug-connected vending machines, high-pressure spray washers, hydromassage bathtubs, carnivals, circuses, fairs (and the like), electrically operated pool covers, portable or mobile electric signs, electrified truck parking space supply equipment, elevators, dumbwaiters, escalators, moving walks, platform lifts/stairway chairlifts, fixed electric space heating cables, fountains, commercial garages, electrical equipment for naturally and artificially made bodies of water, pipeline heating, therapeutic pools and tubs, boathouses, construction sites, health-care facilities, marinas/boatyards, pools, recreational vehicles, sensitive electronic equipment, spas, and hot tubs.

4. Improperly connecting the equipment-grounding conductor to the system neutral. You must connect a grounded neutral conductor to normally noncurrent-carrying metal parts of equipment, raceways, and enclosures only through the main bonding jumper (or, in the case of a separately derived system, through a system bonding jumper). Make this connection at the service disconnecting means, not downstream. When you buy a new entrance panel, a screw or other main bonding jumper is usually included in the packaging. Attached to it are instructions stipulating that it is to be installed only when the panel is to be used as service equipment.

It's a major error to install a main bonding jumper in a box used as a subpanel fed by a 4-wire feeder. It's also wrong not to install it when the panel is used as service equipment. Improper redundant connection of grounded neutral to equipment-grounding conductors can result in objectionable circulating current and presence of voltage on metal tool or appliance casings. You should connect grounded neutral and equipment-grounding conductors at the service disconnect. Then separate them — never to rejoin again. Additional optional ground rods may be connected anywhere along the equipment-grounding conductor but never to the grounded neutral.

5. Improperly grounding frames of electric ranges and clothes dryers. Prior to the 1996 version of the NEC, it was common practice to use the neutral as an equipment ground. Now, however, all frames of electric ranges, wall-mounted ovens, counter-mounted cooking units, clothes

dryers, and outlet or junction boxes that are part of these circuits must be grounded by a fourth wire: the equipment-grounding conductor.

An exception permits retention of the pre-1996 arrangement for existing branch-circuit installations only where an equipment-grounding conductor is not present. Several other conditions must be met. If possible, the best course of action is to run a new 4-wire branch circuit from the panel. If you must keep an old appliance, be sure to remove the neutral to frame bonding jumper if an equipment-grounding conductor is to be connected.

6. Failure to ground submersible well pumps. At one time, submersible well pumps were not required to be grounded because they were not considered accessible. However, it was noted that workers would pull the pump, lay it on the ground, and energize it to see if it would spin. If, due to a wiring fault, the case became live, the overcurrent device would not function, causing a shock hazard. The 2008 NEC requires a fourth equipment-grounding conductor that you must now lug to the top of the well casing. Many people assume that in a 3-wire submersible pump system one wire is a "ground." In actuality, submersible pump cable consists of three wires (plus equipment-grounding conductor) twisted together and unjacketed. Yellow is a common 240V leg, black is run, and red is start, which the control box energizes for a short period of time. Prior to the new grounding requirement, everything was hot.

7. Failure to properly attach the ground wire to electrical devices. Wiring daisy-chained devices in such a way that removing one of them breaks the equipment grounding continuity is a common problem. The preferred way to ground a wiring device is to connect incoming and outgoing equipment-grounding conductors to a short bare or green jumper. The bare or green insulated jumper is then connected to the grounding terminal of the device.

8. Failure to install a second ground rod where required. A single ground rod that does not have a resistance to ground of 25 ohms or less must be augmented by a second ground rod. Once the second ground rod is installed, it's not necessary for the two to meet the resistance requirement. As a practical matter, few electricians do the resistance measurement.

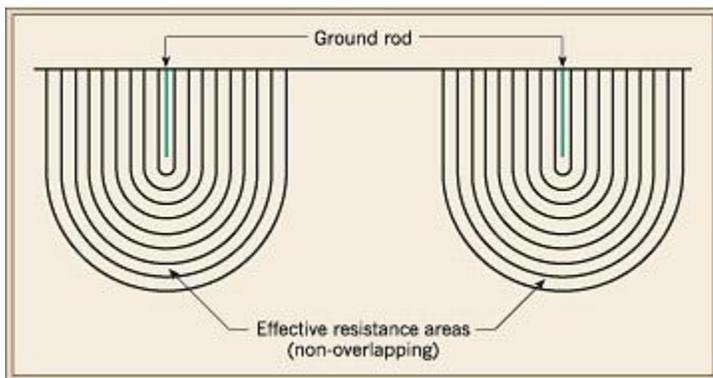


Figure. Non-overlapping effective resistance areas reduce net resistance.

You cannot use a simple ohmmeter because that would require a known perfect ground. Special equipment and procedures are needed, so it's common practice to simply drive a second ground rod. You must locate them at least 6 feet apart. Greater distance is even better (**Figure**). If both rods and the bare ground electrode conductor connecting them are directly under the drip line of the roof, ground resistance will be further diminished. This is because the soil along this line is more moist. Ground resistance greatly increases when soil becomes dry.

9. Failure to properly reattach metal raceway that is used as an equipment-grounding conductor. When equipment is relocated, replaced, or removed for repair, many times equipment ground paths are broken. If these connections are not fixed, there's an accident waiting to happen (**Photo 3**). Setscrews, locknuts, and threads should be fully engaged and continuity tests performed before equipment is put back into service. Dirt and corrosion can also compromise ground continuity.



Photo 3. Standard locknuts or bushings shall not be the sole connection for grounding purposes.

NEC Article 250.4 requires that electrical equipment, wiring, and other electrically conductive material likely to become energized shall be installed in a manner that creates a low-impedance circuit from any point on the wiring system to the electrical supply source to facilitate the operation of overcurrent devices.

10. Failure to bond equipment ground to water pipe. Improper connections are often seen in the field. Screw clamps and other improvised connections do not provide permanent low impedance bonding. The worst method would be to just wrap the wire around the pipe or to omit this bonding altogether.



Photo 4. Someone used a water pipe clamp to improperly connect a ground wire to this ground rod.

In a dwelling, a conductor must be run to metallic water pipe, if present, and connected with a UL-listed pipe grounding clamp (**Photo 4**). This bonding conductor is to be sized according to Table 250.66, based on the size of the largest ungrounded service entrance conductor or equivalent area for parallel conductors.

Harmonics:

25 - How do I know if my project will have a harmonics issue and how do I determine what the right solution is – Youtube – Dan Carnovale - Eaton

Are harmonics less today than in the past due to improved inverter design

ChatGPT said:

Good question — harmonics are still an issue, but **yes, in general they are less of a problem today than in the past** because of advances in inverter and power electronics design.

Here's why:

1. Switching Technology

- **Older inverters (1980s–90s):** Used thyristors or simple IGBTs with low switching frequencies (hundreds of Hz to a few kHz). These produced strong low-order harmonics (5th, 7th, 11th, etc.) that were hard to filter.
- **Modern inverters:** Use fast IGBTs or SiC/GaN transistors switching in the tens to hundreds of kHz. This pushes distortion into much higher frequencies, which are easier to filter and don't propagate as far in power systems.

2. Control Strategies

- Early designs relied on **square wave or six-step switching**, which generated high harmonic distortion.
- Today, **sinusoidal PWM (SPWM)**, **space vector PWM (SVPWM)**, and **multilevel topologies** (NPC, flying capacitor, cascaded H-bridge) allow output waveforms much closer to sinusoidal with lower THD.

3. Built-in Filtering

- Many older drives and inverters required **external harmonic filters** to meet IEEE 519 or IEC standards.
- Modern ones often have **DC-link chokes, input reactors, or built-in active filters** that reduce harmonic injection at both the input (current harmonics) and output (voltage harmonics).

4. Active Harmonic Compensation

- Some high-end systems now include **active front ends (AFE)**, which use bidirectional converters with controlled switching that can draw near-unity power factor and inject *almost no harmonics*.

5. Standards Pressure

- Utilities and standards (IEEE 519, IEC 61000 series) have forced manufacturers to **reduce THD** in inverter products.
-

✔ Summary:

Yes — harmonics are significantly reduced today compared to the past, thanks to higher switching speeds, advanced PWM control, multilevel topologies, and active filtering. That said, harmonics haven't gone away — especially in large variable frequency drives (VFDs), solar inverters, and data centers with dense nonlinear loads. Utilities still monitor and enforce limits.

Do you want me to break down how much **THD levels have changed (say, typical numbers from older drives vs. modern ones)?**

You said:

yes

ChatGPT said:

Great — here's a **numerical comparison of typical Total Harmonic Distortion (THD)** levels in older vs. modern inverter systems:

◆ Current THD at Input (Drawn from the Grid)

- **Older 6-pulse VFDs (no line reactor or DC choke):**
 - 40–100% current THD
 - Very strong 5th and 7th harmonics
- **With line reactor/DC link choke (still common in 1990s–2000s):**
 - 25–35% THD
- **12-pulse rectifiers (large drives, older technology):**
 - 10–15% THD (required phase-shifting transformers)
- **Modern active front end (AFE) or 18-pulse rectifier:**

- 2–5% THD, near unity power factor
-

◆ Voltage THD at Output (Supplied to the Motor/Load)

- **Older square-wave or six-step inverters (1970s–1980s):**
 - 20–30% voltage THD, very hard on motors
 - **Early PWM (1990s):**
 - 5–10% THD, depending on switching frequency and filter
 - **Modern PWM (SPWM, SVPWM with IGBT/SiC):**
 - <3–5% THD typically
 - **Multilevel inverters (NPC, cascaded H-bridge, flying capacitor):**
 - <2–3% THD, waveform almost sinusoidal
-

◆ Real-World Example

- **IEEE 519 (1992 version):** Utilities allowed up to 5% voltage THD at PCC.
 - **Modern solar inverters and AFEs:** Typically operate at <3% current THD and <2% voltage THD — far below the older levels.
-

✓ Bottom Line:

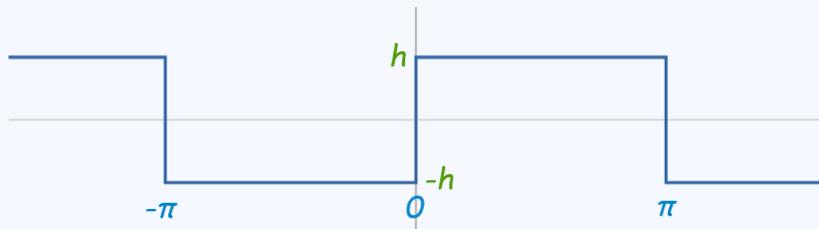
- **1980s–1990s:** A typical industrial drive could easily inject **30–50% current THD** and **>10% voltage THD** without filters.
- **Today (2020s):** With AFEs, multilevel topologies, and high-frequency PWM, most systems stay under **5% THD**, sometimes as low as **2%**.

How to calculate harmonics in a square wave:

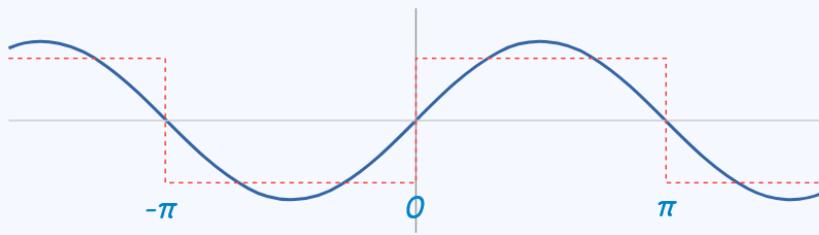
Square Wave

Can we use sine waves to make a **square wave**?

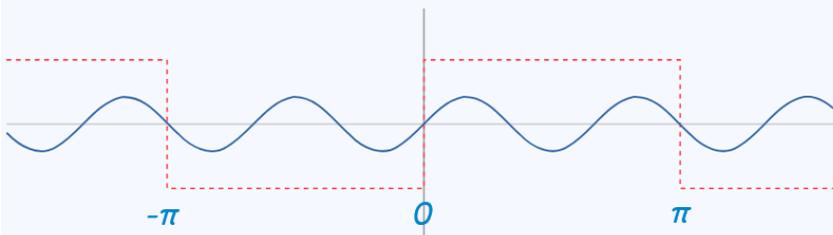
Our target is this square wave:



Start with **$\sin(x)$** :



Then take **$\sin(3x)/3$** :

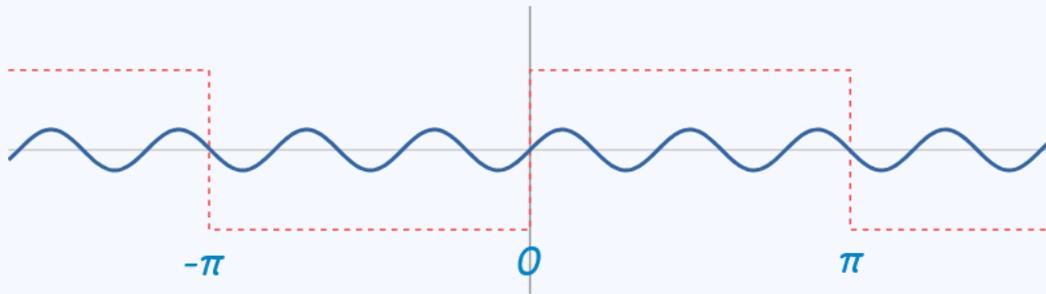


And add it to make **$\sin(x) + \sin(3x)/3$** :

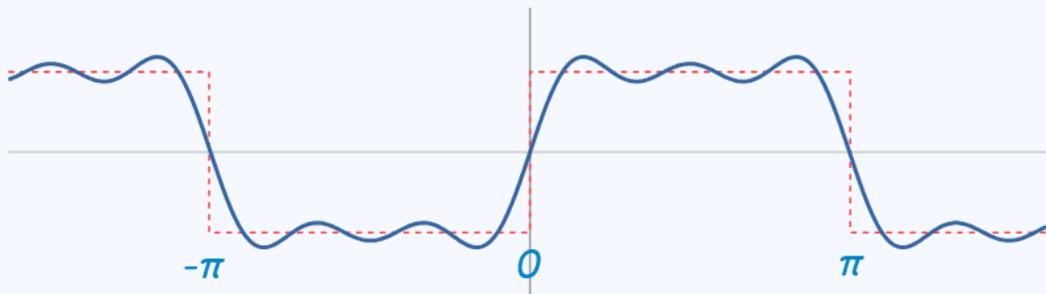


Can you see how it starts to look a little like a square wave?

Now take $\sin(5x)/5$:



Add it also, to make $\sin(x)+\sin(3x)/3+\sin(5x)/5$:

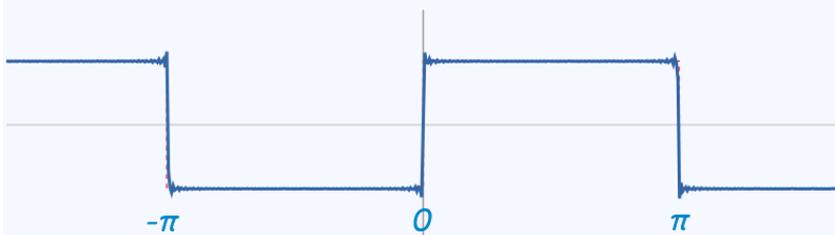


Getting better! Let's add a lot more sine waves.

Using 20 sine waves we get $\sin(x)+\sin(3x)/3+\sin(5x)/5 + \dots + \sin(39x)/39$:



Using 100 sine waves we get $\sin(x)+\sin(3x)/3+\sin(5x)/5 + \dots + \sin(199x)/199$:



And if we could add infinite sine waves in that pattern we would **have** a square wave!

So we can say that:

$$\text{a square wave} = \sin(x) + \frac{\sin(3x)}{3} + \frac{\sin(5x)}{5} + \dots \text{ (infinitely)}$$

That is the idea of a Fourier series.

By adding infinite sine (and or cosine) waves we can make other functions, even if they are a bit weird.

Finding the Coefficients

How did we know to use $\sin(3x)/3$, $\sin(5x)/5$, and so on?

There are formulas!

First let us write down a full series of sines and cosines, with a name for all coefficients:

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(nx \frac{\pi}{L}\right) + \sum_{n=1}^{\infty} b_n \sin\left(nx \frac{\pi}{L}\right)$$

Where:

- $f(x)$ is the function we want (such as a square wave)
- L is **half of the period** of the function
- a_0 , a_n and b_n are **coefficients** that we need to calculate!

To find the coefficients a_0 , a_n and b_n we use these formulas:

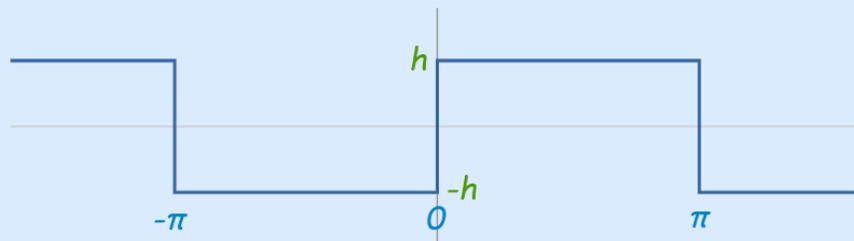
$$a_0 = \frac{1}{2L} \int_{-L}^L f(x) dx$$
$$a_n = \frac{1}{L} \int_{-L}^L f(x) \cos\left(nx \frac{\pi}{L}\right) dx$$
$$b_n = \frac{1}{L} \int_{-L}^L f(x) \sin\left(nx \frac{\pi}{L}\right) dx$$

So this is what we do:

- Take our **target function**, **multiply it by sine** (or cosine) and **integrate** (find the area)
- Do that for $n=0$, $n=1$, and so on to calculate each coefficient
- And after we calculate all coefficients, we put them into the series formula above

Let us see how to do each step and then assemble the result at the end!

Example: This Square Wave:

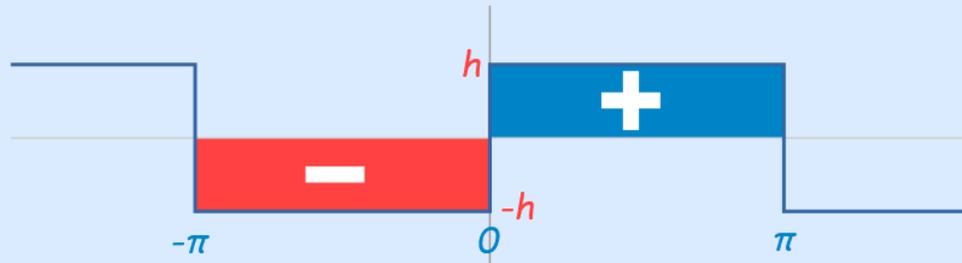


- $L = \pi$ (the Period is 2π)
- The square wave is from $-h$ to $+h$

Now our job is to calculate a_0 , a_n and b_n

a_0 is the net area between $-L$ and L , then divided by $2L$. It is basically an **average** of $f(x)$ in that range.

Looking at this sketch:



The net area of the square wave from $-L$ to L is **zero**.

So we know that:

$$a_0 = 0$$

For a_1 we know that $n=1$ and $L=\pi$, so:

$$a_1 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos\left(1x \frac{\pi}{\pi}\right) dx$$

Which simplifies to:

$$a_1 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(x) dx$$

Now, because the square wave changes abruptly at $x=0$ we need to break the calculation into **$-\pi$ to 0** and **0 to π** ,

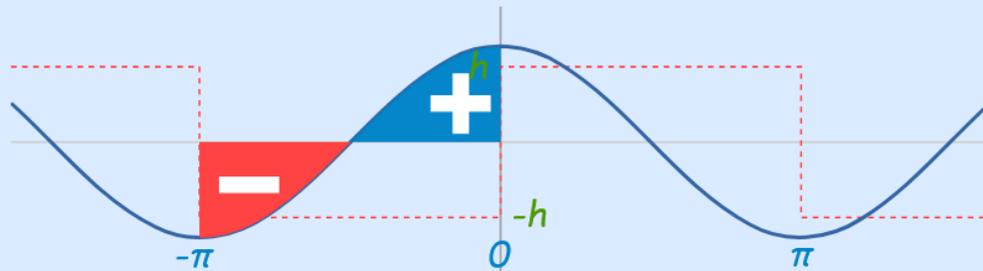
From $-\pi$ to 0 we know $f(x)$ is simply equal to $-h$:

$$\frac{1}{\pi} \int_{-\pi}^0 -h \cos(x) dx$$

We can move the constant $-h$ outside the integral:

$$\frac{-h}{\pi} \int_{-\pi}^0 \cos(x) dx$$

Let's sketch $\cos(x)$:

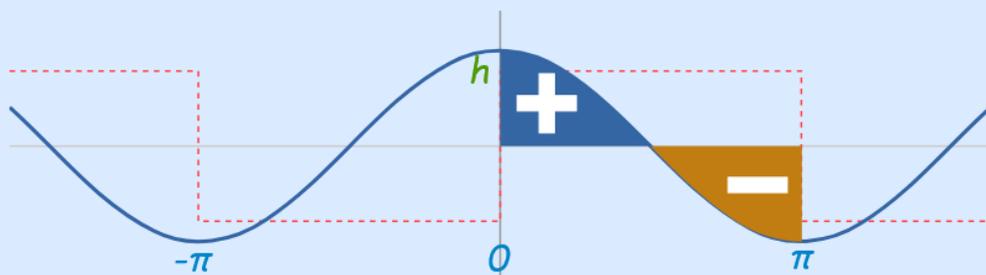


The net area of $\cos(x)$ from $-\pi$ to 0 is **zero**.

So the net area must be 0:

$$\frac{-h}{\pi} \int_{-\pi}^0 \cos(x) dx = 0$$

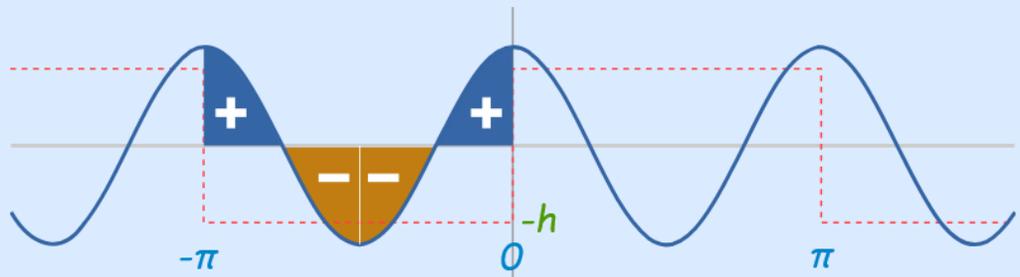
The same idea applies from 0 to π ,



The net area of $\cos(x)$ from 0 to π is **zero**.

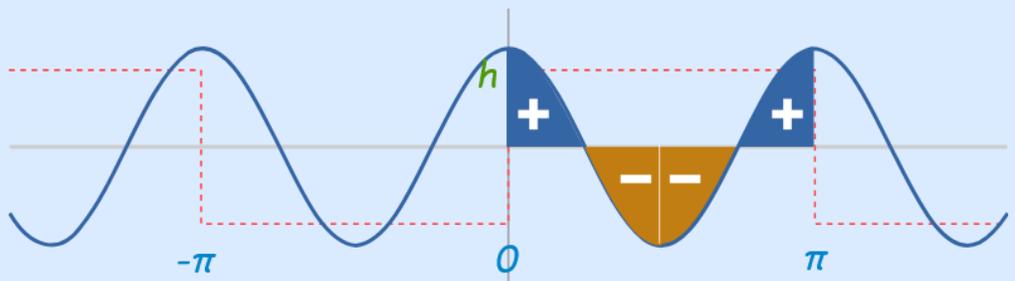
Now let us look at a_2

Aaaand ... the same thing happens!



The net area of $\cos(2x)$ from $-\pi$ to 0 is **zero**.

And:



The net area of $\cos(2x)$ from 0 to π is also **zero**.

So we know that:

$$a_2 = 0$$

In fact we can extend this idea to every value of \mathbf{a} and conclude that:

$$a_n = 0$$

So far there has been no need for any major calculations! A few sketches are thought to have been enough.

But now on to the **sine** function!

For **b_1** we know that $n=1$ and $L=\pi$, so:

$$b_1 = \frac{1}{\pi} \int_{-\pi}^{\pi} \sin\left(x \frac{\pi}{\pi}\right) dx$$

Which simplifies to:

$$b_1 = \frac{1}{\pi} \int_{-\pi}^{\pi} \sin(x) dx$$

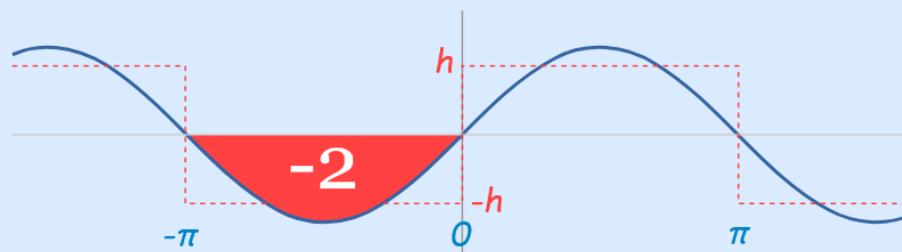
and as before, because of the abrupt change at $x=0$, we need to break the calculation into **$-\pi$ to 0** and **0 to π** ,

So, just looking at the integral **from $-\pi$ to 0** , we know $f(x) = -h$:

We can move the constant **$-h$** outside the integral:

$$\frac{-h}{\pi} \int_{-\pi}^0 \sin(x) dx$$

And **$\sin(x)$** looks like this:



How do we know the area is -2 ?

First we use [Integration Rules](#) to find the integral of $\sin(x)$ is $-\cos(x)$:

Then we calculate the [definite integral](#) between $-\pi$ and 0 by calculating the value of $-\cos(x)$ for 0 , and for $-\pi$, and then subtracting:

$$[-\cos(0)] - [-\cos(-\pi)] = -1 - 1 = -2$$

So, between $-\pi$ and 0 we get

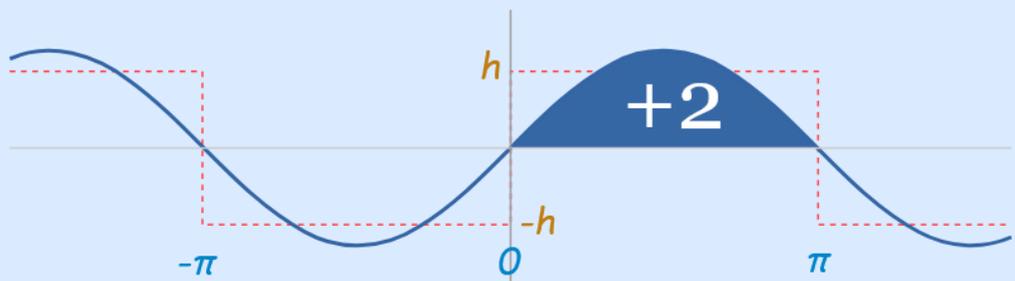
$$\frac{-h}{\pi}(-2)$$

Next we look at the integral from 0 to π :

$$\frac{h}{\pi} \int_0^{\pi} \sin(x) dx$$

And its integral is:

$$[-\cos(\pi)] - [-\cos(0)] = 1 - [-1] = 2$$



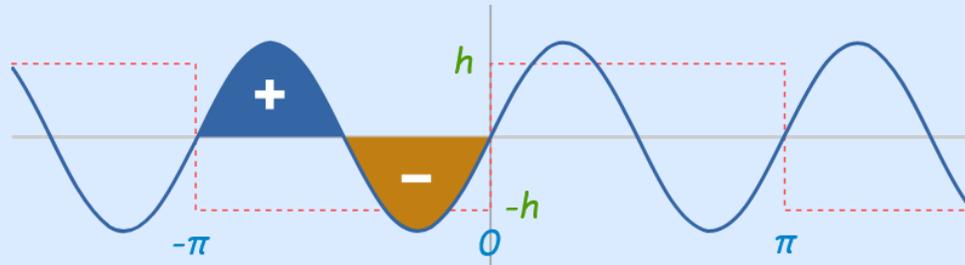
Now, combining both sides we get:

$$b_1 = \frac{1}{\pi} [(-h) \times (-2) + (h) \times (2)] = \frac{4h}{\pi}$$

For b_2 we have this integral:

$$\frac{-h}{\pi} \int_{-\pi}^{\pi} \sin(2x) dx$$

From $-\pi$ to 0 it looks like this:



The net area of $\sin(2x)$ from $-\pi$ to 0 is **zero**.

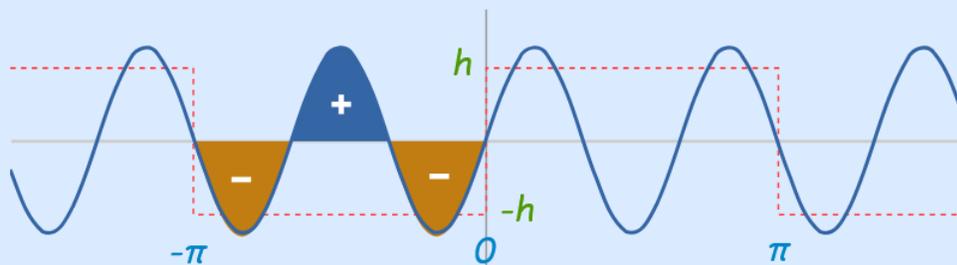
And we have seen this kind of thing before, so we conclude that:

$$b_2 = 0$$

For b_3 we have this integral:

$$\frac{-h}{\pi} \int_{-\pi}^{\pi} \sin(3x) dx$$

From $-\pi$ to 0 we get this interesting situation:

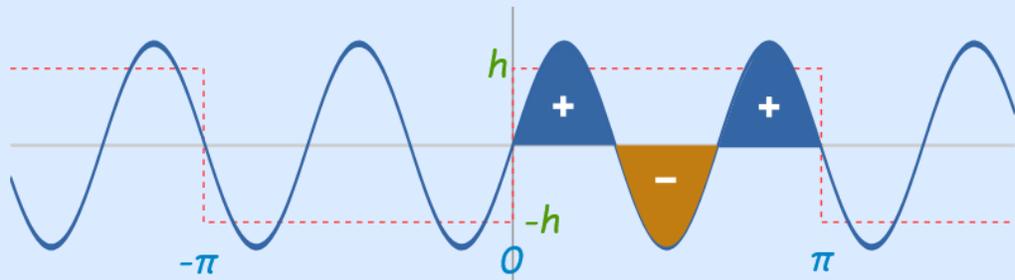


Two areas cancel, but the third one is important!

So it is like the b_1 integral, but with only one-third of the area.

So it is like the b_1 integral, but with only one-third of the area.

For **0 to π** we have:

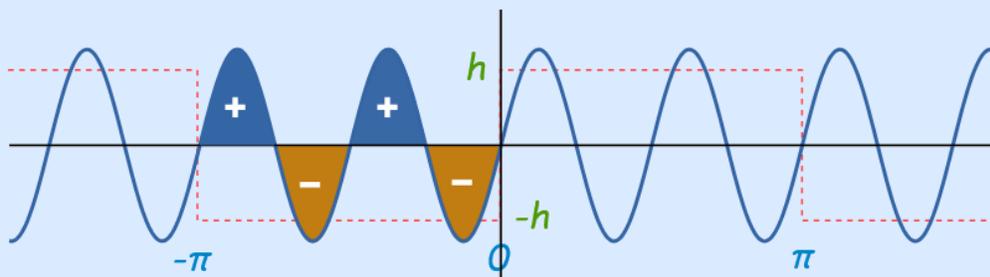


Again two areas cancel, but not the third

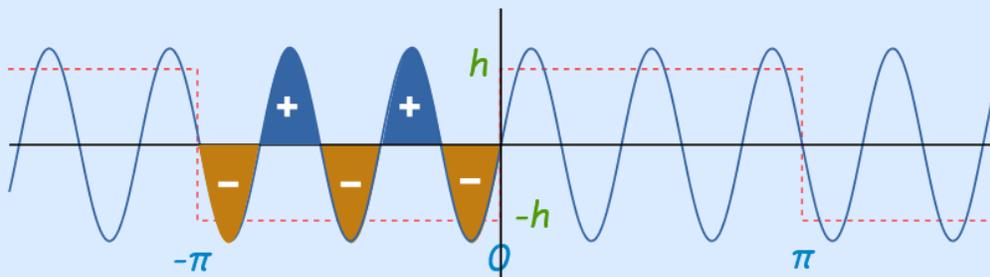
And we can conclude:

$$b_3 = \frac{b_1}{3} = \frac{4h}{3\pi}$$

The pattern continues:



When n is even the areas cancel for a result of zero.



When n is odd, all except one area cancel for a result of $1/n$.

So we can say

$$b_n = \frac{4h}{n\pi} \text{ when } n \text{ is odd, but } 0 \text{ otherwise}$$

And we arrive at our last step: putting the coefficients into the master formula:

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(nx \frac{\pi}{L}\right) + \sum_{n=1}^{\infty} b_n \sin\left(nx \frac{\pi}{L}\right)$$

And we know that:

- $a_0 = 0$
- $a_n = 0$ (all of them!),
- $b_n = 0$ when n is even
- $b_n = \frac{4h}{n\pi}$ when n is odd

So:

$$f(x) = \frac{4h}{\pi} \left[\sin(x) + \frac{\sin(3x)}{3} + \frac{\sin(5x)}{5} + \dots \right]$$

Direct current (DC) circuit protection | Eaton PSEC

Video - 9:59

Compared to alternating current (AC), direct current (DC) flows consistently in one direction therefore it requires additional mechanisms to safely interrupt circuits. Christy McElhinny, Field Application Engineer, explains the critical role DC circuit protection has within energy transition applications, which inherently use DC energy. This is especially true as energy transition applications such solar (also referred to as photovoltaic (PV) systems), battery storage, and wind generation are projected to increase in adoption. Though DC circuit protection has overlap in ratings familiar with AC systems, there also unique requirements of DC circuit protection within electrical vehicles (EV), EV chargers, energy storage, and PV systems. Christy also explains how a PV system is designed and operates by following a PV schematic. Also, the different design options that can be engineered and at which point DC circuit protection is required to be installed. Thank you for watching one of our many educational videos on the topic of power systems. Schedule a visit to one of Eaton's Power Systems Experience Centers in either Pittsburgh or Houston to learn more! To learn more about Eaton products and our Power Systems Experience Centers: <http://eaton.com/experience> To view more educational videos from Eaton's Power Systems Experience Centers: <http://videos.eaton.com/experience> Chapters 00:00 00:12 Intro 01:41 Why do energy transition applications use DC 03:06 Photovoltaic system design 04:50 Comparing DC and AC circuit protection 06:36 AC coupled PV system 07:21 DC coupled PV system 07:58 Reverse DC coupled PV system 08:33 Eaton DC protective devices and assemblies

Electrical arcs and sparks - why electricity jumps

When you hear a reference to Frankenstein one of the visuals you may think of are giant electrical sparks breathing “life” into Dr. Frankenstein’s creature. But what are electrical sparks? Electrical sparks, also known as arcing, is a natural phenomenon when electricity jumps from one point to another. Often occurring when you are connecting, or disconnecting, electrical devices. In this video Seth So dives into the topic sharing how arcs work with the “spark gap” demonstration installed at Eaton’s Power System Experience Center (PSEC). In the demo, one electrode is charged with a high-potential test kit and we see at what voltage the air gap is ionized and conducts to the ground electrode. When an arc is initiated, it’ll spark in open air. We also look at some of the technologies used to control arcing and make electrical systems safer. Some topics discussed include:

- Controlling clearances – how air gaps can be used to minimize the risk of arcing.
- Physical insulators – used when air gaps are not sufficient insulators.
- Creepage distance – The shortest distance between two conductive parts, measured along the surface of an insulator. Solid insulators and bushings have the “wavy” shape to increase their insulating properties.
- Fuses – we discuss expulsion and current limiting fuses.
- Vacuum bottles – switching device that uses a pair of separable contacts in an enclosed vacuum-tight chamber.
- Arc chutes – help collapse arc earlier by dividing arcs into smaller arcs.
- Eaton’s Arc Quenching System (AQS) – quenches arcs before it becomes an explosion. Learn about Arc Quenching Switchgear:

<https://www.youtube.com/watch?v=gdZBtrI5DxA&list=PL8XobqCtN9Z9zmxXF91EJpX2k8FjdRIEb&index=69> Thank you for watching one of our many educational videos on the topic of power systems. Schedule a visit to one of Eaton's Power Systems Experience Centers in either Pittsburgh or Houston to learn more! To learn more about Eaton products and our Power Systems Experience Centers: <http://eaton.com/experience> To view more educational videos from Eaton's Power Systems Experience Centers: <http://videos.eaton.com/experience> Chapters 00:00 Sketch 00:50 Introduction 01:30 What is electrical arcing? 02:30 How do electrical arcs work 04:00 How to control electrical arcing 05:19 How to safely break an arc 07:45 Switching quickly 08:18 Arc Quenching System (AQS)

Medium voltage motor controls | Eaton PSEC

Motors consume over 70% of the energy within industrial applications. This illustrates the reliance that industrial manufacturing has on motors, in turn making motor controls critical to that success. This is especially true for medium-voltage control (MVC) equipment that drive productivity, process performance and energy savings. In this video, Kyle Harrison, Product Manager Medium Voltage Control Assemblies, discusses medium voltage motor controls (MV MCs) to help us understand how they work and their role in industrial processes. Kyle explains that motor control centers (MCCs) are a centralized collection of equipment that control and protect motors. Also, compared to low voltage motors, medium voltage motors use smaller gauge wire, provide higher voltage, and require lower current to operate. Along with controlling motors, medium voltage motor controls can also be used to feed transformers and capacitor banks. Kyle also discusses the components that go into MV MCs and their role, including: • Isolation switch • Circuit breaker/fuse • Contactor • Protective relay • Control power transformer (CPT) • Voltage sensor • Meter We conclude by exploring real-world applications that use MV MCs such as mining, wastewater treatment facilities, and oil and gas operations. Looking for more on motor controls? Check out Kyle's other video on motor control fundamentals:

<https://www.youtube.com/watch?v=L46PWrQChA&list=PL8XobqCtN9Z9zmxXF91EJpX2k8FjdRIEb&index=1> Thank you for watching one of our many educational videos on the topic of power systems. Schedule a visit to one of Eaton's Power Systems Experience Centers in either Pittsburgh or Houston to learn more! To learn more about Eaton products and our Power Systems Experience Centers: <http://eaton.com/experience> To view more educational videos from Eaton's Power Systems Experience Centers: <http://videos.eaton.com/experience> Chapters 00:00 Promo 00:43 Low voltage vs medium voltage mcc 01:49 Core functions of MCCs 02:38 Medium voltage MCC design 03:30 Construction and components of MV control 06:59 MV motor control applications

Motor control fundamentals | Eaton PSEC

Electric motors are a critical part of our daily lives and as important is the ability to safely operate them. Motor starters provide that safety to both the operator and the motor itself. In this video Kyle Harrison, Product Manager Medium Voltage Control Assemblies, explains what motor starters are and how they work. Motor starters are electrical devices used to safely start and stop motors while also providing them with overcurrent and undervoltage protection.

Along with a breaker or fuse, the primary components that go into motor starters are contactors and relays. The purpose of the contactor is to establish and interrupt the power circuit to the motor. Relays protect motors from drawing too much current and overheating. Kyle also walks through the various motor starter designs and how they work including: • Across-the-line (ATL) starter (also referred to as full voltage starters) – and how inrush current affects motors and the electrical system • Reduced-voltage autotransformers • Reduced-voltage primary reactors • Solid-state soft starters (also referred to as reduced voltage soft starters (RVSS)) • Variable frequency drive (VFD) We also explain how motor starter design is based on system voltage and discuss motor starters applications. Learn about medium voltage motor controls:

https://www.youtube.com/watch?v=VhUjqYAoz_o&list=PL8XobqCtN9Z9zmxXF91EJpX2k8FjdRIEb&index=1

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Chapters
00:00 Intro
00:16 What are motor starters
01:29 How do motor starters work
02:20 Types of motor starters
08:31 Where are motor starters used

Jim Pytel – Big Bad Tech

Motor Family Tree (Full Lecture)

Jim Pytel 50:07

Mechanical Power: Torque and Rotational Speed (Full Lecture)

Jim Pytel 48:04

Electromagnetic Interaction (Full Lecture)

Jim Pytel 30:13

Rotating Magnetic Field (Full Lecture)

Jim Pytel 35:12

Motor Nameplates (Full Lecture)

Jim Pytel 26:35

Motor Nameplate Examples (Full Lecture)

Jim Pytel 13:54

Motor Connection Diagrams (Full Lecture)

Jim Pytel 29:50

Motor Connection Examples (Full Lecture)

Jim Pytel 11:29

Squirrel Cage Induction Motors: Mechanical Properties (Full Lecture)

Jim Pytel 39:01

Squirrel Cage Induction Motors: Electrical Characteristics (Full Lecture)

Jim Pytel 44:54

Inrush Current (Full Lecture)

Jim Pytel 19:34

Single Phasing (Full Lecture)

Jim Pytel 34:04

Asynchronous/Induction Generators (Full Lecture)

Jim Pytel 57:50

Synchronous Motors (Full Lecture)

Jim Pytel 29:08

Synchronous Motor Starters (Full Lecture)

Jim Pytel 28:28

Synchronous Condensers (Full Lecture)

Jim Pytel 15:05

Synchronous Generators (Full Lecture)

Jim Pytel 22:41

Loaded Synchronous Generators (Full Lecture)

Jim Pytel 29:21

Generator Demonstration (Full Lecture)

Jim Pytel 21:05

Generator Synchronization (Full Lecture)

Jim Pytel 31:02

Wound Rotor Induction Motors (Full Lecture)

Jim Pytel 23:21

Secondary Resistor Reduced Voltage Starters (Full Lecture)

Jim Pytel 21:11

Doubly Fed Induction Motors (Full Lecture)

Jim Pytel 37:48

Doubly Fed Induction Generators (Full Lecture)

Jim Pytel 37:43

Resumes, Cover Letters, and Interviews for Electro-Mechanical Technicians (Full Lecture)

Jim Pytel 44:12

Brushless DC and Permanent Magnet Synchronous Motors (Full Lecture)

Jim Pytel 57:31

What is a DC Motor?

A **DC motor (Direct Current motor)** converts **electrical energy** into **mechanical rotation** using the interaction between **magnetic fields and electric current**. Here's a simple explanation:

Basic Idea

When current flows through a wire in a magnetic field, it experiences a **force**.

In a DC motor, wires are arranged in loops (called the **armature**) inside a magnetic field.

These forces on the wires create a **rotating motion** — that's what drives the motor shaft.

Main Parts

1. **Stator** – the stationary part that provides the **magnetic field** (from permanent magnets or electromagnets).
 2. **Rotor (Armature)** – the rotating coil of wire carrying current.
 3. **Commutator** – a mechanical switch that reverses current direction in the coil every half turn, keeping rotation in one direction.
 4. **Brushes** – conductive contacts that deliver current from the DC source to the commutator.
 5. **Power Source** – provides DC voltage to drive current through the armature.
-

Step-by-Step Operation

1. DC voltage is applied to the motor terminals.
2. Current flows through the **brushes** into the **commutator** and **armature coils**.
3. The current-carrying armature sits in the **magnetic field** of the stator.
4. According to **Lorentz's Law ($F = BIL$)**, each wire in the magnetic field feels a force — one side is pushed up, the other down.
5. These opposite forces create **torque**, causing the rotor to turn.
6. When the rotor moves halfway around, the **commutator reverses current direction**, ensuring continuous rotation in the same direction.

⚡ Key Relationships

- **Torque (T)** is proportional to **armature current (I)**
 $\rightarrow T \propto I$
- **Speed (N)** is inversely proportional to **magnetic field strength (Φ)** and **armature current**
 $\rightarrow N \propto \frac{V-IR}{\Phi}$

🧠 In Simple Terms

- The **magnetic field** pulls and pushes on the **current-carrying wires**.
- The **commutator** keeps flipping the current so the torque stays in the same direction.
- This continuous push makes the motor **spin smoothly**.

Control of a DC Motor:

<https://www.youtube.com/watch?v=ctXdMkNCvLU>

Robert Jenkins

<https://www.youtube.com/watch?v=9kZS8qYPkrk>

EE for Everyone

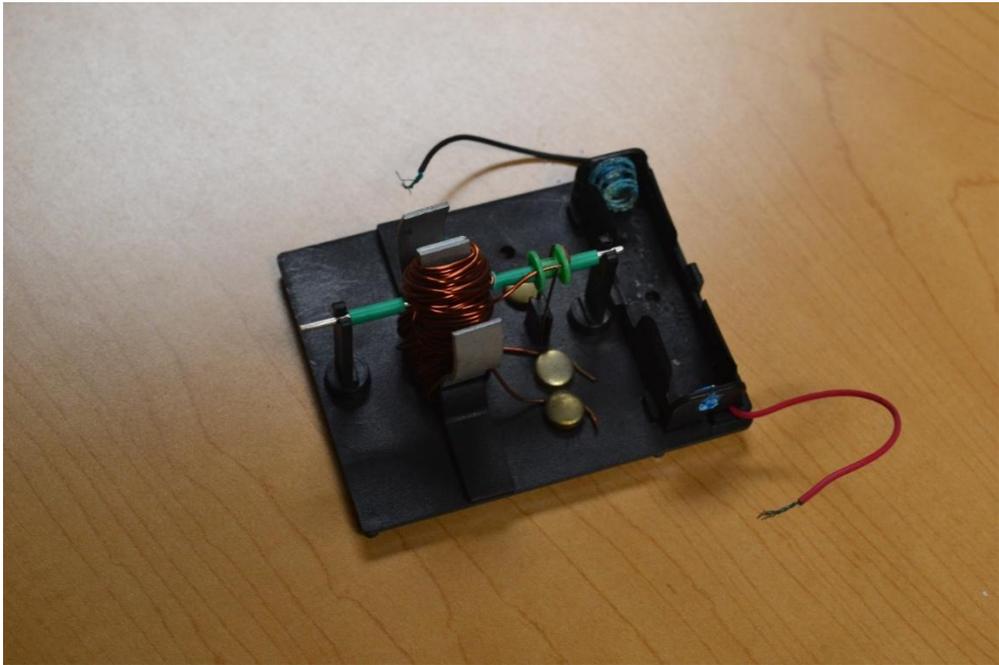
Old DC Motor made for 4-H project Demonstration:



ma



Slightly more modern kit DC motor:



Chapter 4

DC Machines

4.1 OPERATING PRINCIPLES

As indicated in Chapter 3, most electric machines operate on the basis of interaction between current-carrying conductors and electromagnetic fields. In particular, generator action is based on Faraday's law of electromagnetic induction, (2.1), which implies that a voltage (emf) is induced in a conductor moving in a region having flux lines at right angles to the conductor. That is, if a straight conductor of length l moves at velocity u (normal to its length) through a uniform magnetic field \mathbf{B} , the conductor itself always at right angles to \mathbf{B} , then only the velocity component u_{\perp} orthogonal to \mathbf{B} is effective in inducing the voltage e . In fact, the *Blu-rule* states:

$$e = Blu_{\perp} \quad (4.1)$$

It follows that the voltage e induced in an N -turn rectangular coil, of axial length l and radius r , rotating at a constant angular velocity ω in a uniform magnetic field \mathbf{B} (Fig. 4-1), is given by

$$e = 2BNlr\omega \sin \omega t = BNA\omega \sin \omega t \quad (4.2)$$

The second form of (4.2) holds for an arbitrary planar coil of area A . This voltage is available at the slip rings (or brushes), as shown in Fig. 4-1.

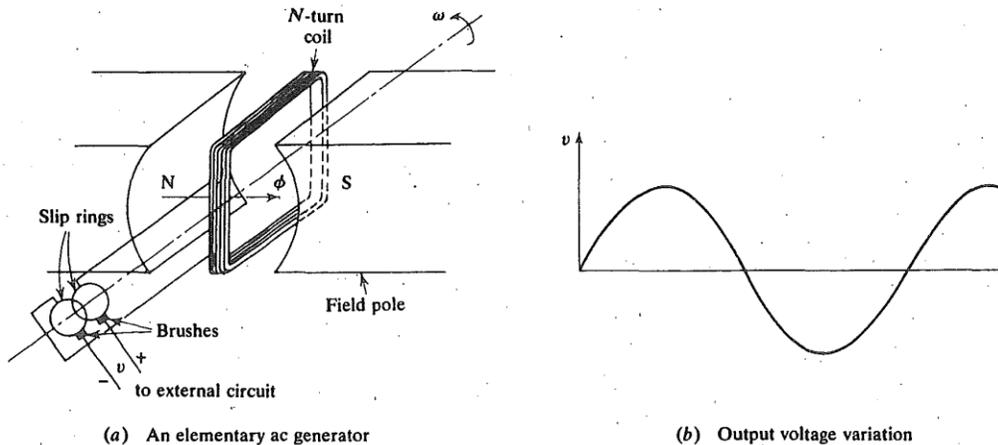


Fig. 4-1

The direction of the induced voltage is often determined by the *right-hand rule*, as depicted in Fig. 4-2(a). Clearly, this rule is equivalent to the vector version of (4.1):

$$\text{emf} = \int (\mathbf{lu} \times \mathbf{B}) \cdot d\mathbf{l}$$

Motor action is based on Ampere's law, (1.2), which we rewrite as the *Bli-rule*:

$$\mathbf{F} = B(li)_{\perp} \quad (4.3)$$

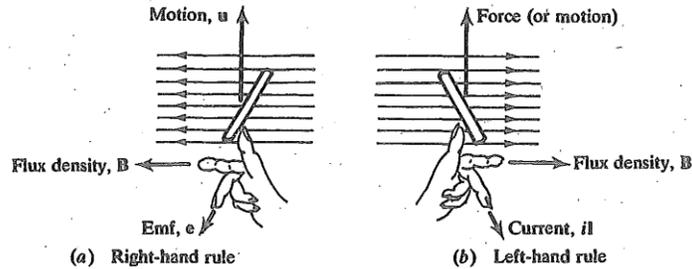


Fig. 4-2

Here, F is the magnitude of the force on a conductor carrying a directed current element il whose component normal to the uniform magnetic field B is $(il)_\perp$. The direction of the force may be obtained by the *left-hand rule*, shown in Fig. 4-2(b).

Just as an ac sinusoidal voltage is produced at the terminals of a generator, the torque produced by the coil fed at the brushes from an ac source would be alternating in nature, with a zero time-average value. If fed from a dc source, the resulting torque will align the coil (in a neutral position) as shown in Fig. 4-1(a). The time-average value of the torque will be zero.

4.2 COMMUTATOR ACTION

In order to get a unidirectional polarity at a brush, or to obtain a unidirectional torque from a coil in a magnetic field, the slip-ring-and-brush mechanism of Fig. 4-1(a) is modified to the one shown in Fig. 4-3(a). Notice that instead of two slip rings we now have one ring split into two halves that are insulated from each other. The brushes slide on these halves, known as *commutator segments*. It can be readily verified by applying the right-hand rule that such a commutator-brush system results in the brushes having definite polarities, corresponding to the output voltage waveform of Fig. 4-3(b). Thus the average output voltage is nonzero and we obtain a dc output at the brushes.

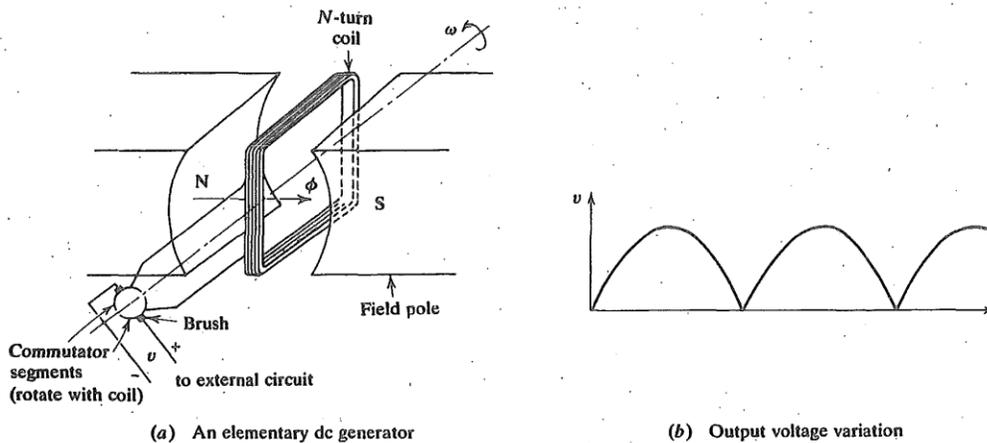


Fig. 4-3

It can also be verified, by applying the left-hand rule, that if the coil connected to the commutator-brush system is fed from a dc source, the resulting torque is unidirectional.

The commutator-brush mechanism is an integral part of usual dc machines, the only exception being the *Faraday disk*, or *homopolar machine*. (See Problem 7.24.)

4.3 ARMATURE WINDINGS AND PHYSICAL FEATURES

Figure 4-4 shows some of the important parts and physical features of a dc machine. (For the meaning of GNP and MNP, see Section 4.8.) The *field poles*, which produce the needed flux, are mounted on the *stator* and carry windings called *field windings* or *field coils*. Some machines carry several sets of field windings on the same *pole core*. To facilitate their assembly, the cores of the poles are built of sheet-steel laminations. (Because the field windings carry direct current, it is not electrically necessary to have the cores laminated.) It is, however, necessary for the *pole faces* to be laminated, because of their proximity to the *armature windings*. The *armature core*, which carries the armature windings, is generally on the *rotor* and is made of sheet-steel laminations. The *commutator* is made of hard-drawn copper segments insulated from one another by mica. As shown in Fig. 4-5, the armature windings are connected to the commutator segments over which the carbon *brushes* slide and serve as leads for electrical connection. The armature winding is the load-carrying winding.

The armature winding may be a *lap winding* [Fig. 4-5(a)] or a *wave winding* [Fig. 4-5(b)], and the various coils forming the armature winding may be connected in a series-parallel combination. It is found that in a simplex lap winding the number of paths in parallel, a , is equal to the number of poles, p ; whereas in a simplex wave winding the number of parallel paths is always 2.

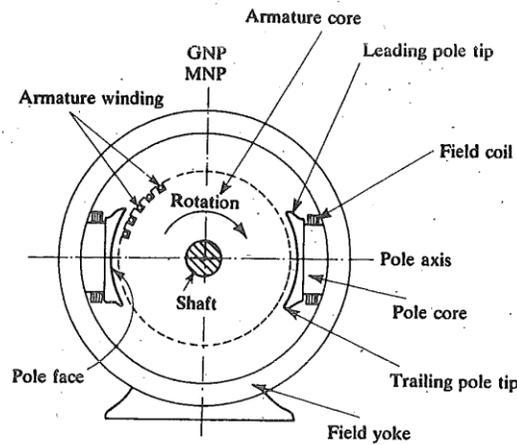


Fig. 4-4

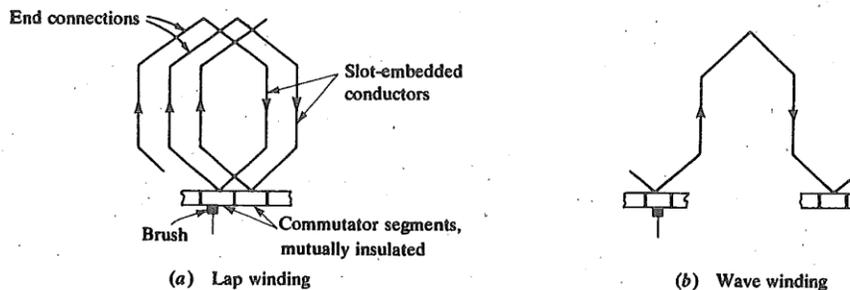


Fig. 4-5

4.4 EMF EQUATION

Consider a conductor rotating at n rpm in the field of p poles having a flux ϕ per pole. The total flux cut by the conductor in n revolutions is $p\phi n$; hence, the flux cut per second, giving the induced voltage e , is

$$e = \frac{p\phi n}{60} \quad (\text{V}) \quad (4.4)$$

If there is a total of z conductors on the armature, connected in a parallel paths, then the effective number of conductors in series is z/a , which produce the total voltage E in the armature winding. Hence, for the entire winding, (4.4) gives the *emf equation*:

$$E = \frac{p\phi n}{60} \frac{z}{a} = \frac{zp}{2\pi a} \phi \omega_m \quad (\text{V}) \quad (4.5)$$

where $\omega_m = 2\pi n/60$ (rad/s). This may also be written as

$$E = k_a \phi \omega_m = k_g \phi n \quad (4.6)$$

where $k_a \equiv zp/2\pi a$ (a dimensionless constant) and $k_g = zp/60a$. If the magnetic circuit is linear (i.e. if there is no saturation), then

$$\phi = k_f i_f \quad (4.7)$$

where i_f is the field current and k_f is a proportionality constant; and (4.6) becomes

$$E = k i_f \omega_m \quad (4.8)$$

where $k \equiv k_f k_a$, a constant. For a nonlinear magnetic circuit, E versus I_f is a nonlinear curve for a given speed, as shown in Fig. 4-6.

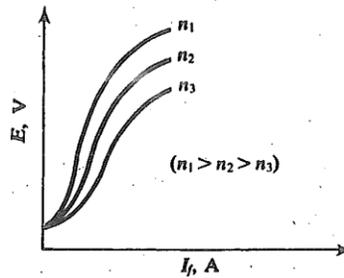


Fig. 4-6

4.5 TORQUE EQUATION

The mechanical power developed by the armature is $T_e \omega_m$, where T_e is the (electromagnetic) torque and ω_m is the armature's angular velocity. If this torque is developed while the armature current is i_a at an armature (induced) voltage E , then the armature power is Ei_a . Thus, ignoring any losses in the armature,

$$T_e \omega_m = Ei_a$$

which becomes, from (4.6),

$$T_e = k_a \phi i_a \quad (4.9)$$

This is known as the *torque equation*. For a linear magnetic circuit, (4.7) and (4.9) yield

$$T_e = k i_f i_a \quad (4.10)$$

where $k \equiv k_f k_a$, as in (4.8). Thus, k may be termed the *electromechanical energy-conversion constant*.

Notice that in (4.7) through (4.10) lowercase letters have been used to designate instantaneous values, but that these equations are equally valid under steady state.

4.6 SPEED EQUATION

The armature of a dc motor may be schematically represented as in Fig. 4-7. Under steady state we have

$$V - E = I_a R_a \tag{4.11}$$

Substituting (4.6) in (4.11) yields

$$\omega_m = \frac{V - I_a R_a}{k_a \phi} \tag{4.12}$$

which, for a linear magnetic circuit, becomes

$$\omega_m = \frac{V - I_a R_a}{k I_f} \tag{4.13}$$

An alternate form of (4.13) is

$$n = \frac{V - I_a R_a}{k_m I_f} = \frac{V - I_a R_a}{k_g \phi} \text{ (rpm)} \tag{4.14}$$

where $k_m \equiv 2\pi k/60$ ($\Omega \cdot \text{min}$). Equation (4.13) or (4.14) is known as the *speed equation*.

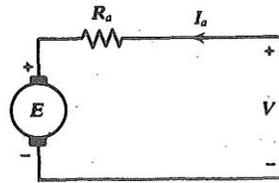
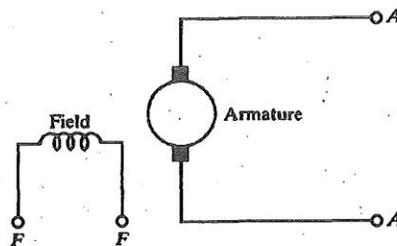


Fig. 4-7

4.7 MACHINE CLASSIFICATION

DC machines may be classified on the basis of the interconnections between the field and armature windings. See Fig. 4-8(a) to (g).



(a) Separately excited

Fig. 4-8(a)

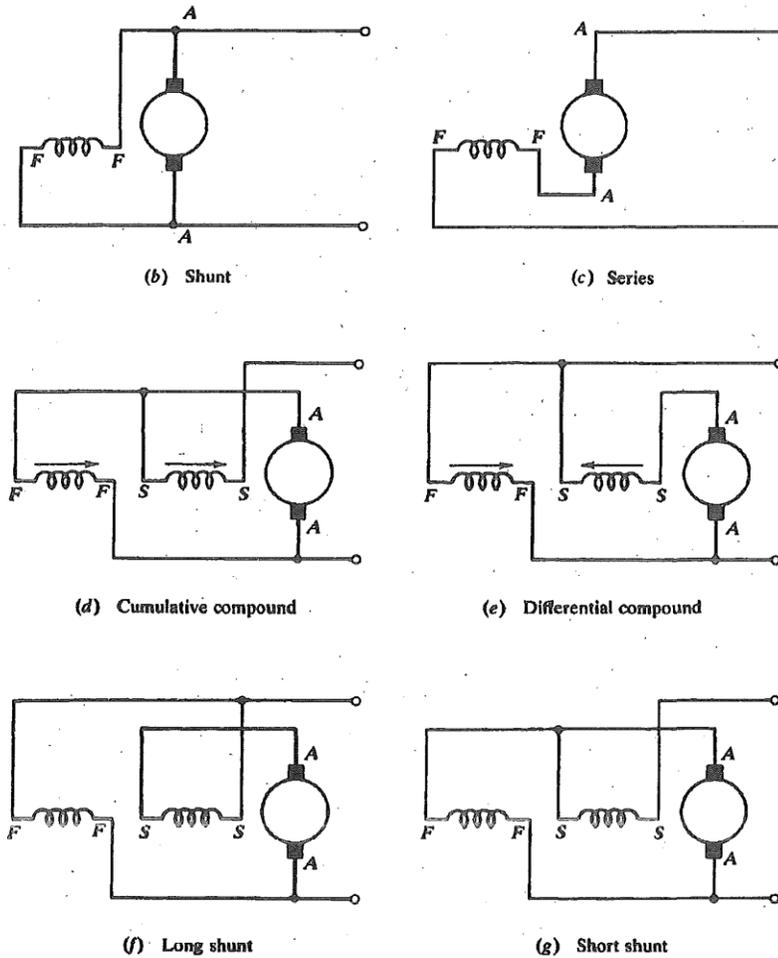


Fig. 4-8(b) to (g)

4.8 AIRGAP FIELDS AND ARMATURE REACTION

In the discussion so far, we have assumed no interaction between the fields produced by the field windings and by the current-carrying armature windings. In reality, however, the situation is quite different. Consider the two-pole machine shown in Fig. 4-9(a). If the armature does not carry any current (that is, if the machine is on no-load), the airgap field takes the form shown in Fig. 4-9(b). The *geometric neutral plane* and the *magnetic neutral plane* (GNP and MNP, respectively) are coincident. (Note: Magnetic lines of force intersect the MNP at right angles.) The brushes are located at the MNP for maximum voltage at the terminals. We now assume that the machine is on "load" and that the armature carries current. The direction of flow of current in the armature conductors depends on the location of the brushes. For the situation in Fig. 4-9(b), the direction of the current flow is the same as the direction of the induced voltages. In any event, the current-carrying armature conductors produce their own magnetic fields, as shown in Fig. 4-9(c), and the airgap field is now the resultant of the fields due to the field and armature windings. This resultant airgap field has the distorted form shown in Fig. 4-9(d). The interaction of the fields due to the armature and field windings is known as *armature reaction*. As a consequence of armature reaction, the airgap field is distorted

and the MNP is no longer coincident with the GNP. For maximum voltage at the terminals, the brushes have to be located at the MNP. Thus, one undesirable effect of armature reaction is that the brushes must be shifted constantly, since the deviation of the MNP from the GNP depends on the load (which presumably is always changing).

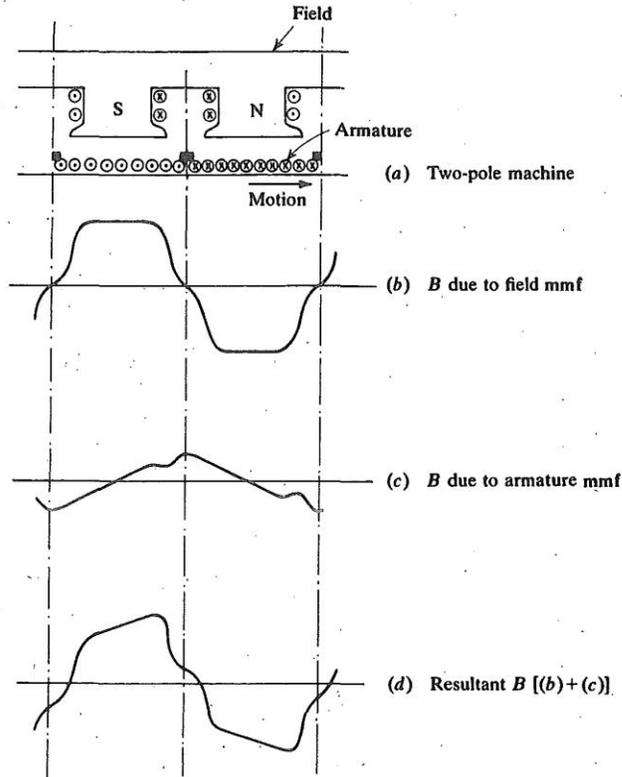


Fig. 4-9

The effect of armature reaction can be analyzed in terms of cross-magnetization and demagnetization, as shown in Fig. 4-10(a). The effect of cross-magnetization can be neutralized by means of *compensating windings*, as shown in Fig. 4-10(b). These are conductors embedded in pole faces, connected in series with the armature windings, and carrying currents in an opposite direction to the currents in the armature conductors that face them [Fig. 4-10(b)]. Once cross-magnetization has been neutralized, the MNP does not shift with load and remains coincident with the GNP at all loads. The effect of demagnetization can be com-

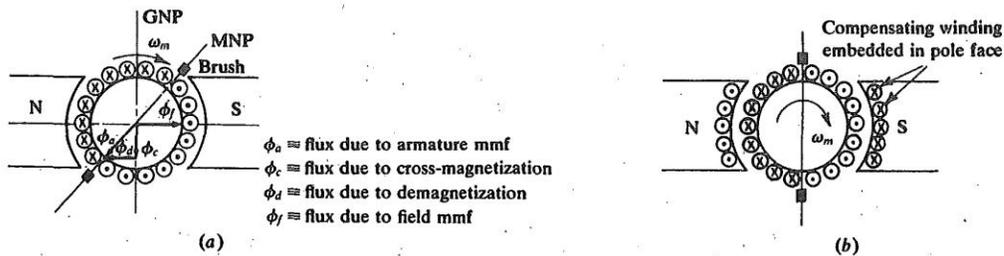


Fig. 4-10

compensated for by increasing the mmf on the main field poles. Because the net effect of armature reaction can be neutralized, we are justified in our preceding and succeeding discussions when we assume no "coupling" between the armature and field windings.

4.9 REACTANCE VOLTAGE AND COMMUTATION

In discussing the action of the commutator, we indicated that the direction of flow of current in a coil undergoing commutation reverses by the time the brush moves from one commutator segment to the other. This is schematically represented in Fig. 4-11. The flow of current in coil α for three different instants is shown. We have assumed that the current fed by a commutator segment is proportional to the area of contact between the brush and the segment. Thus, for satisfactory commutation, the direction of flow of current in coil α must completely reverse [Fig. 4-11(a) and (c)] by the time the brush moves from segment 2 to segment 3. The ideal situation is represented by the straight line in Fig. 4-12; it may be termed *straight-line commutation*. Because coil α has some inductance L , the change of current, ΔI , in a time Δt induces a voltage $L(\Delta I/\Delta t)$ in the coil. According to Lenz's law, the direction of this voltage, called *reactance voltage*, is opposite to the change (ΔI) which is causing it. As a result, the current in the coil does not completely reverse by the time the brush moves from one segment to the other. The balance of the "unreversed" current jumps over as a spark from the commutator to the brush, with the result that the commutator wears out from pitting. This departure from ideal commutation is also shown in Fig. 4-12.

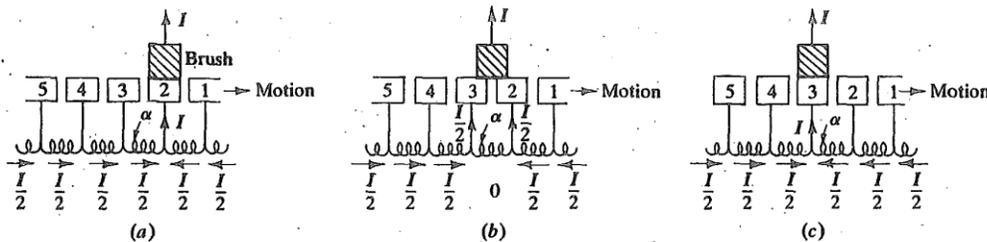


Fig. 4-11

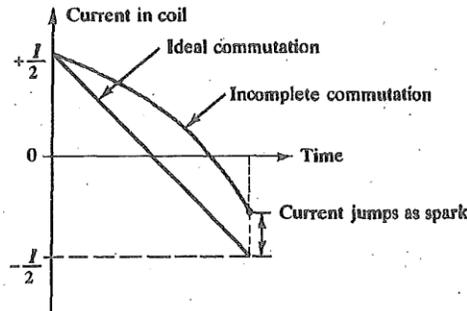


Fig. 4-12

The directions of the current flow and reactance voltage are shown in Fig. 4-13(a). Note that the direction of the induced voltage depends on the direction of rotation of the armature conductors and on the direction of the airgap field; it is given by $\mathbf{u} \times \mathbf{B}$ (or by the right-hand rule). Next, the direction of the current flow depends on the location of the brushes (or tapping points). Finally, the direction of the reactance voltage depends on the change in the direction of current flow and is determined from Lenz's law. For the brush position shown in Fig. 4-13(a), the reactance voltage retards the current reversal. If the brushes are advanced in the direction of rotation (for generator operation), we may notice, from Fig. 4-13(b), that the (rotation-) induced voltage opposes the reactance voltage, so that the current reversal is less impeded than when the reactance voltage acted alone, as in Fig. 4-13(a). We may further observe that the coil undergoing

commutation, being near the tip of the south pole, is under the influence of the field of a weak south pole. From this argument, we may conclude that commutation improves if we advance the brushes. But this is not a very practical solution. The same—perhaps better—results can be achieved if we keep the brushes at the GNP, or MNP, as in Fig. 4-13(a), but produce the "field of a weak south pole" by introducing appropriately wound auxiliary poles, called *interpoles* or *commutating poles*. See Fig. 4-13(c).

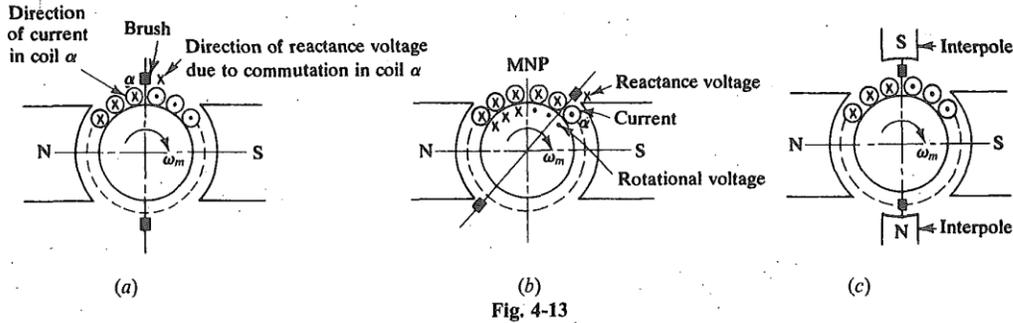


Fig. 4-13

4.10 EFFECT OF SATURATION ON VOLTAGE BUILDUP IN A SHUNT GENERATOR

Saturation plays a very important role in governing the behavior of dc machines. To observe one of its consequences, consider the self-excited shunt generator of Fig. 4-8(b). Under steady state,

$$V = I_f R_f \quad \text{and} \quad E = V + I_a R_a = I_f R_f + I_a R_a$$

These equations are represented by the upper straight lines in Fig. 4-14(a). Notice that the voltages V and E will keep building up and no equilibrium point can be reached. On the other hand, if we include the effect of saturation, as in Fig. 4-14(b), then point P , where the field-resistance line intersects the saturation curves defines the equilibrium.

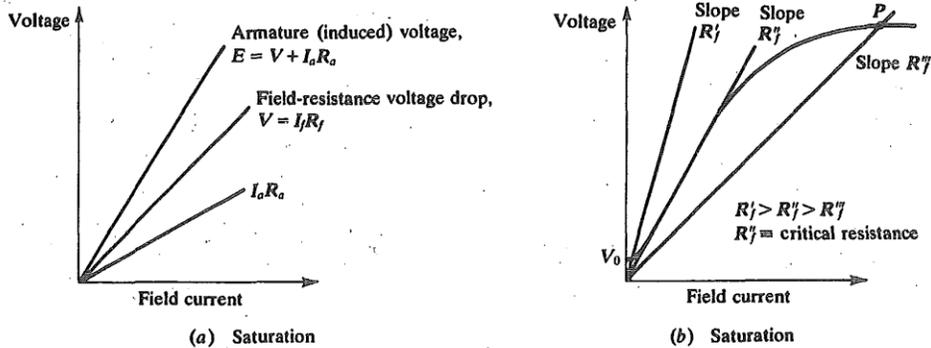


Fig. 4-14. No-load characteristic of a shunt generator.

Figure 4-14(b) indicates some residual magnetism, as measured by the small voltage V_0 . Also indicated in Fig. 4-14(b) is the *critical resistance*: a field resistance greater than the critical resistance (for a given speed) would not let the shunt generator build up an appreciable voltage. Finally, we should ascertain that the polarity of the field winding is such that a current through it produces a flux that aids the residual flux. If instead the two fluxes tend to neutralize, the machine voltage will not build up. To summarize, the conditions for the building up of a voltage in a shunt generator are the presence of residual flux (to provide starting voltage), field-circuit resistance less than the critical resistance, and appropriate polarity of the field winding.

4.11 LOSSES AND EFFICIENCY

Besides the volt-ampereage and speed-torque characteristics, the performance of a dc machine is measured by its efficiency:

$$\text{efficiency} \equiv \frac{\text{power output}}{\text{power input}} = \frac{\text{power output}}{\text{power output} + \text{losses}}$$

Efficiency may, therefore, be determined either from load tests or by determination of losses. The various losses are classified as follows:

1. **Electrical.** (a) Copper losses in various windings, such as the armature winding and different field windings. (b) Loss due to the contact resistance of the brush (with the commutator).
2. **Magnetic.** These are the iron losses and include the hysteresis and eddy-current losses in the various magnetic circuits, primarily the armature core and pole faces.
3. **Mechanical.** These include the bearing-friction, windage, and brush-friction losses.
4. **Stray-load.** These are other load losses not covered above. They are taken as 1 percent of the output (as a rule of thumb).

The power flow in a dc generator or motor is represented in Fig. 4-15, in which T_s denotes the shaft torque.

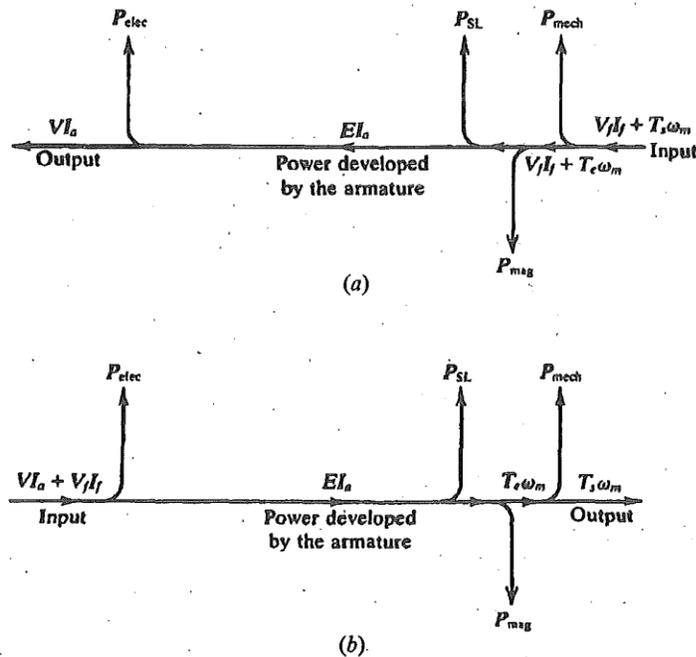


Fig. 4-15

4.12 MOTOR AND GENERATOR CHARACTERISTICS

Load characteristics of motors and generators are usually of greatest interest in determining potential applications of these machines. In some cases (as in Fig. 4-14), no-load characteristics are also of importance.

Typical load characteristics of dc generators are shown in Fig. 4-16, and Fig. 4-17 shows torque speed characteristics of dc motors.

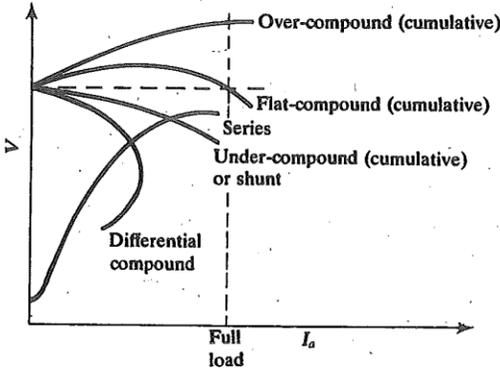


Fig. 4-16. Load characteristics of dc generators.

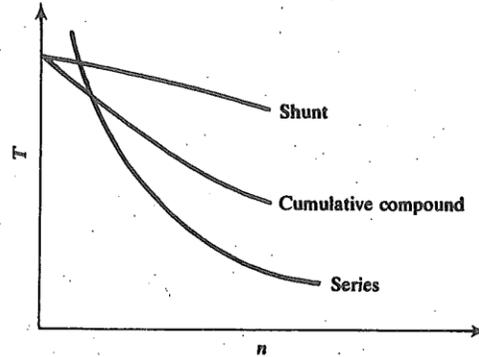


Fig. 4-17. Torque-speed characteristics of dc motors.

4.13 DC MOTOR DYNAMICS

A separately excited motor is represented in Fig. 4-18. For the armature circuit (of an idealized machine) we have

$$v = e + i_a R_a + L_a \frac{di_a}{dt} \tag{4.15}$$

$$e = k i_f \omega_m \tag{4.16}$$

and for the field circuit,

$$v_f = i_f R_f + L_f(i_f) \frac{di_f}{dt} \tag{4.17}$$

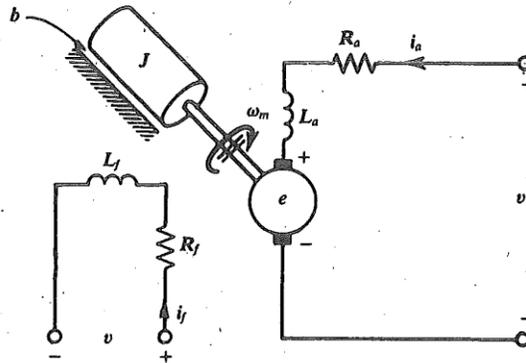


Fig. 4-18

The field-circuit inductance, $L_f(i_f)$, is shown as a nonlinear function of i_f to give generality to the set of equations. This nonlinear function is related to the magnetization curve of the machine or the flux-versus-ampere-turn characteristic of the magnetic circuit of the machine. Summation of torques acting on the motor shaft yields

$$T_e = b\omega_m + J \frac{d\omega_m}{dt} \quad (4.18)$$

$$T_e = k_f i_a \quad (4.19)$$

where b ($\text{N} \cdot \text{m} \cdot \text{s}/\text{rad}$) is a viscous damping coefficient representing mechanical loss and J ($\text{kg} \cdot \text{m}^2$) is the moment of inertia of the entire rotating system, including machine rotor, load, couplings, and shaft.

The set of equations (4.15) through (4.19) is nonlinear not only because of the nonlinear coefficients, such as L_f and, possibly, b , but also because of the product terms in (4.16) and (4.19). The set of state equations equivalent to the above set is useful in the analysis of a great number of machine problems. In order to apply these equations, the physical conditions of the specific problem must be introduced in an analytical manner. These conditions include numerical values for the R 's, the L 's, k , b , and J ; descriptions of the input terms, v and v_f ; and initial conditions for the state variables. Also, the equations themselves must be modified for different circuit configurations (e.g., for series-field excitation).

Solved Problems

- 4.1. Calculate the voltage induced in the armature winding of a 4-pole, lap-wound, dc machine having 728 active conductors and running at 1800 rpm. The flux per pole is 30 mWb.

Because the armature is lap wound, $p = a$, and

$$E = \frac{\phi n z}{60} \left(\frac{p}{a} \right) = \frac{(30 \times 10^{-3})(1800)(728)}{60} = 655.2 \text{ V}$$

- 4.2. What is the voltage induced in the armature of the machine of Problem 4.1, if the armature is wave wound?

For a wave-wound armature, $a = 2$. Thus,

$$E = \frac{(30 \times 10^{-3})(1800)(728)}{60} \left(\frac{4}{2} \right) = 1310.4 \text{ V}$$

- 4.3. If the armature in Problem 4.1 is designed to carry a maximum line current of 100 A, what is the maximum electromagnetic power developed by the armature?

Because there are 4 parallel paths ($a = p = 4$) in the lap-wound armature, each path can carry a maximum current of

$$\frac{I_a}{a} = \frac{100}{4} = 25 \text{ A}$$

Nevertheless, the power developed by the armature is

$$P_d = EI_a = (655.2)(100) = 65.5 \text{ kW}$$

- 4.4. By reconnecting the armature of Problem 4.1 in wave, will the developed power be changed?

No. In this case ($a = 2$), the line current is $I_a = 2 \times 25 = 50$ A (25 A being the limit that each path can carry). Hence,

$$P_d = (1310.4)(50) = 65.5 \text{ kW}$$

- 4.5. Calculate the electromagnetic torque developed by the armature described in Problem 4.1.

From the energy-conversion equation, $E I_a = T_e \omega_m$, and the result of Problem 4.3,

$$T_e = \frac{E I_a}{\omega_m} = \frac{65.5 \times 10^3}{2\pi(1800)/60} = 347.6 \text{ N} \cdot \text{m}$$

- 4.6. A dc machine has a 4-pole, wave-wound armature with 46 slots and 16 conductors per slot. If the induced voltage in the armature is 480 V at 1200 rpm, determine the flux per pole.

Here $z = 16 \times 46 = 736$, and so, from the emf equation

$$\phi = \frac{60E}{nz} \left(\frac{a}{p} \right) = \frac{(60)(480)}{(1200)(736)} \left(\frac{2}{4} \right) = 16.3 \text{ mWb}$$

- 4.7. Suppose that in Problem 4.6 the flux per pole remains 16.3 mWb, but the induced voltage measures only 410 V because of flux leakage. Evaluate the *leakage coefficient*, σ , where

$$\sigma \equiv \frac{\text{total flux}}{\text{useful flux}}$$

From the emf equation, the observed voltage is directly proportional to the effective flux. Hence,

$$\sigma = \frac{\text{voltage without leakage}}{\text{voltage with leakage}} = \frac{480}{410} = 1.17$$

- 4.8. A 4-pole, lap-wound armature has 144 slots with two coil sides per slot, each coil having two turns. If the flux per pole is 20 mWb and the armature rotates at 720 rpm, what is the induced voltage?

Substitute $p = a = 4$, $n = 720$, $\phi = 0.020$, and $z = 144 \times 2 \times 2 = 576$ in the emf equation to obtain

$$E = \frac{(0.020)(720)(576)}{60} \left(\frac{4}{4} \right) = 138.24 \text{ V}$$

- 4.9. A 10-turn square coil of side 200 mm is mounted on a cylinder 200 mm in diameter. The cylinder rotates at 1800 rpm in a uniform 1.1-T field. Determine the maximum value of the voltage induced in the coil.

From (4.2),

$$E_{\max} = BNA\omega = (1.1)(10)(0.200)^2(2\pi \times 1800)/60 = 82.94 \text{ V}$$

- 4.10. A 100-kW, 230-V, shunt generator has $R_a = 0.05 \Omega$ and $R_f = 57.5 \Omega$. If the generator operates at rated voltage, calculate the induced voltage at (a) full-load and (b) half full-load. Neglect brush-contact drop.

See Fig. 4-19; $I_f = 230/57.5 = 4 \text{ A}$.

$$(a) \quad I_L = \frac{100 \times 10^3}{230} = 434.8 \text{ A}$$

$$I_a = I_L + I_f = 434.8 + 4 = 438.8 \text{ A}$$

$$I_a R_a = (438.8)(0.05) = 22 \text{ V}$$

$$E = V + I_a R_a = 230 + 22 = 252 \text{ V}$$

$$(b) \quad I_L = 217.4 \text{ A}$$

$$I_a = 217.4 + 4 = 221.4 \text{ A}$$

$$I_a R_a = 11 \text{ V}$$

$$E = 230 + 11 = 241 \text{ V}$$

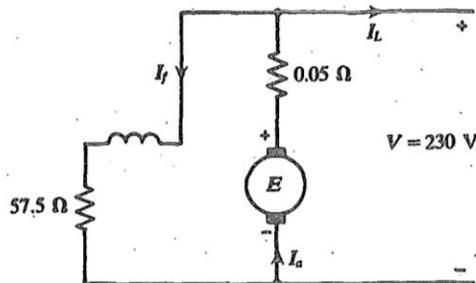


Fig. 4-19

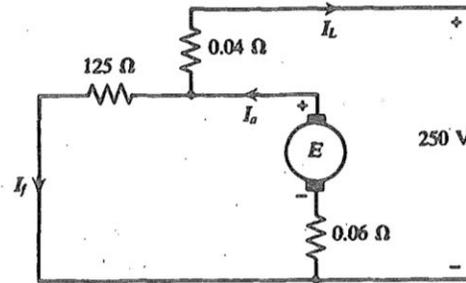


Fig. 4-20

- 4.11. A 50-kW, 250-V short-shunt compound generator has the following data: $R_a = 0.06 \Omega$, $R_{se} = 0.04 \Omega$, and $R_f = 125 \Omega$. Calculate the induced armature voltage at rated load and terminal voltage. Take 2 V as the total brush-contact drop.

See Fig. 4-20.

$$I_L = \frac{50 \times 10^3}{250} = 200 \text{ A}$$

$$I_L R_{se} = (200)(0.04) = 8 \text{ V}$$

$$V_f = 250 + 8 = 258 \text{ V}$$

$$I_a = 200 + 2.06 = 202.06 \text{ A}$$

$$I_a R_a = (202.06)(0.06) = 12.12 \text{ V}$$

$$E = 250 + 12.12 + 8 + 2 = 272.12 \text{ V}$$

4.12. Repeat Problem 4.11 for a long-shunt compound connection (Fig. 4-21).

$$I_L = 200 \text{ A}$$

$$I_f = \frac{250}{125} = 2 \text{ A}$$

$$I_a = 200 + 2 = 202 \text{ A}$$

$$I_a(R_a + R_{sc}) = 202(0.06 + 0.04) = 20.2 \text{ V}$$

$$E = 250 + 20.2 + 2 = 272.2 \text{ V}$$

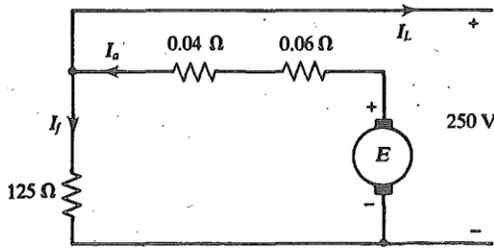


Fig. 4-21

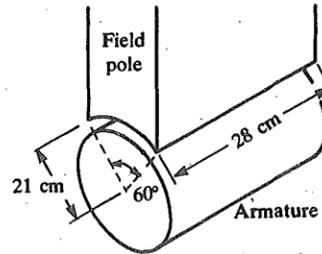


Fig. 4-22

4.13. The generator of Problem 4.10 has 4 poles, is lap wound with 326 armature conductors, and runs at 650 rpm on full-load. If the bore of the machine is 42 cm (in diameter), its axial length is 28 cm, and each pole subtends an angle of 60°, determine the airgap flux density.

A portion of the machine is illustrated in Fig. 4-22. From Problem 4.10,

$$E = 252 = \frac{\phi n z}{60} \left(\frac{p}{a} \right) \text{ whence } \phi = 71.35 \text{ mWb}$$

The pole surface area is

$$A = r\theta l = (0.21)(\pi/3)(0.28) = 0.0616 \text{ m}^2$$

Hence

$$B = \frac{\phi}{A} = \frac{71.35 \times 10^{-3}}{0.0616} = 1.16 \text{ T}$$

- 4.14. A separately excited dc generator has a constant loss of $P_c(W)$, and operates at a voltage V and armature current I_a . The armature resistance is R_a . At what value of I_a is the generator efficiency a maximum?

$$\text{output} = VI_a$$

$$\text{input} = VI_a + I_a^2 R_a + P_c$$

$$\text{efficiency } \eta = \frac{VI_a}{VI_a + I_a^2 R_a + P_c}$$

For η to be a maximum, $d\eta/dI_a = 0$, or

$$V(VI_a + I_a^2 R_a + P_c) - VI_a(V + 2I_a R_a) = 0 \quad \text{or} \quad I_a = \sqrt{\frac{P_c}{R_a}}$$

In other words, the efficiency is maximized when the armature loss, $I_a^2 R_a$, equals the constant loss, P_c .

- 4.15. The generator of Problem 4.10 has a total mechanical and core loss of 1.8 kW. Calculate (a) the generator efficiency at full-load and (b) the horsepower output from the prime mover to drive the generator at this load.

From Problem 4.10, $I_f = 4$ A and $I_a = 438.8$ A, so that

$$I_f^2 R_f = (16)(57.5) = 0.92 \text{ kW}$$

$$I_a^2 R_a = (438.8)^2(0.05) = 9.63 \text{ kW}$$

and $\text{total losses} = 0.92 + 9.63 + 1.8 = 12.35 \text{ kW}$

(a) $\text{output} = 100 \text{ kW}$

$$\text{input} = 100 + 12.35 = 112.35 \text{ kW}$$

$$\text{efficiency} = \frac{100}{112.35} = 89\%$$

(b) $\text{prime mover output} = \frac{112.35 \times 10^3 \text{ W}}{746 \text{ W/hp}} = 150.6 \text{ hp}$

- 4.16. (a) At what load does the generator of Problems 4.10 and 4.15 achieve maximum efficiency? (b) What is the value of this maximum efficiency?

(a) From Problem 4.15, the constant losses are

$$P_c = 920 + 1800 = 2720 \text{ W}$$

Hence, by Problem 4.14,

$$I_a = \sqrt{\frac{2720}{0.05}} = 233.24 \text{ A}$$

and $I_L = I_a - I_f = 233.24 - 4 = 229.24 \text{ A}$

(b) output power = $(229.24)(230) = 52.72 \text{ kW}$

$$I_a^2 R_a = P_c = 2.72 \text{ kW} \quad (\text{by Problem 4.14})$$

$$\text{input power} = 52.72 + 2(2.72) = 58.16 \text{ kW}$$

$$\text{maximum efficiency} = \frac{52.72}{58.16} = 90.6\%$$

- 4.17. The no-load (or saturation) characteristic of a shunt generator at 1200 rpm is shown in Fig. 4-23. The field has 500 turns per pole. (a) Determine the critical field resistance for self-excitation at 1200 rpm. (b) What is the total field-circuit resistance if the induced voltage is 230 V?

From Fig. 4-23:

(a) critical field resistance = $\frac{200}{1500/500} = 66.67 \Omega$

(b) field resistance at 230 V = $\frac{230}{2500/500} = 46 \Omega$

- 4.18. Let Fig. 4-23 represent the saturation curve at 1200 rpm of a series motor. The motor has $k_a = 40$ in (4.6), and is wound with 8 turns per pole. The total series-field and armature-circuit resistances are $25 \text{ m}\Omega$ and $50 \text{ m}\Omega$, respectively. Determine the flux per pole at (a) $E = 200 \text{ V}$, (b) an excitation of 2600 At per pole.

(a) From (4.6),

$$\phi = \frac{E}{k_a \omega_m} = \frac{200}{40(2\pi \times 1200)/60} = 39.8 \text{ mWb}$$

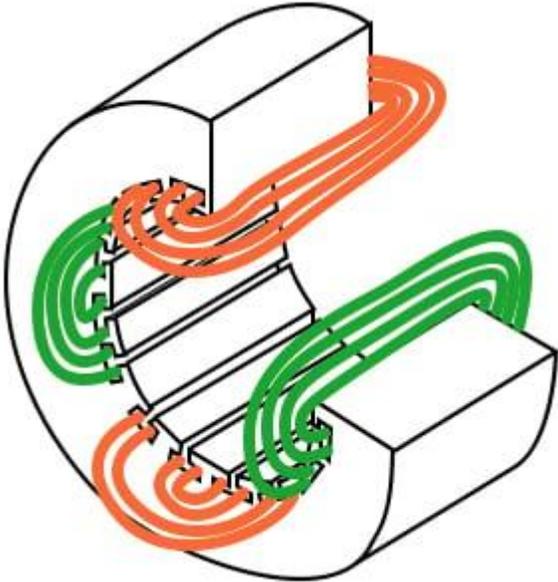
(b) At 2600 At per pole, $E \approx 232 \text{ V}$ (from Fig. 4-23). Thus,

$$\phi = \frac{232}{200} (39.8) = 46.15 \text{ mWb}$$

- 4.19. For a certain load, the motor of Problem 4.18 runs at 200 V while taking 325 A in current. If core loss is 220 W, and friction and windage loss is 40 W, determine (a) electromagnetic torque developed, (b) motor speed, (c) mechanical power output, and (d) motor efficiency.

At $325 \times 8 = 2600 \text{ At}$ per pole, the flux per pole is, from Problem 4.18(b),

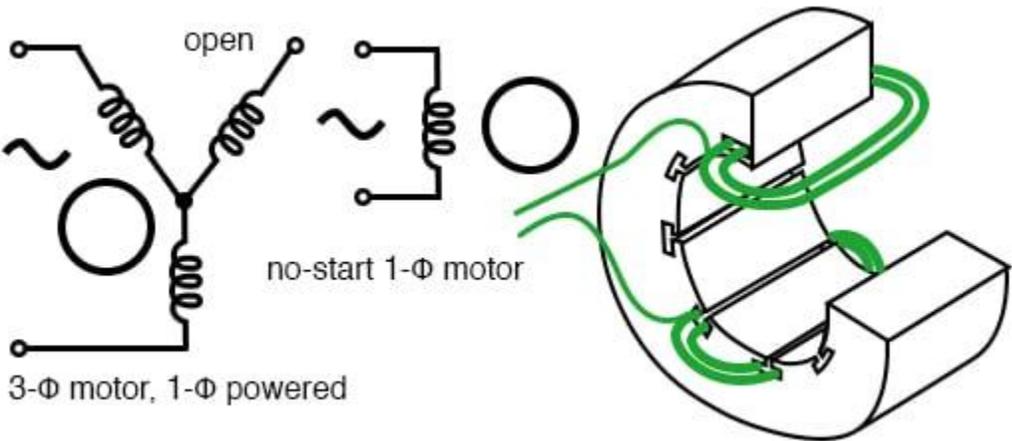
Single-phase Induction Motors



What's one way to solve the single phase problem?

Build a 2-phase motor, deriving 2-phase power from single phase.

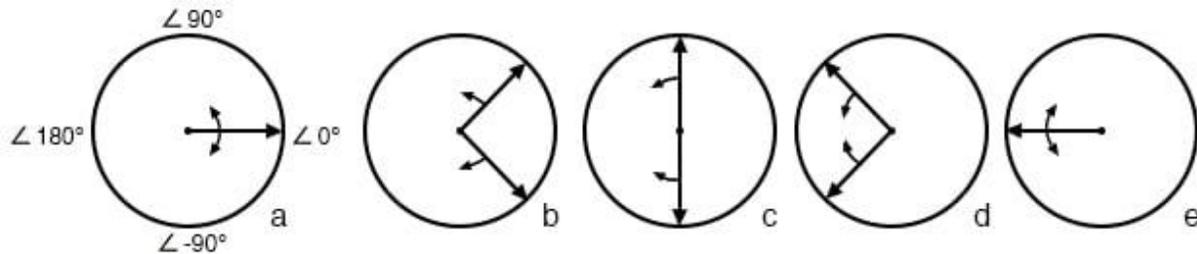
A three-phase motor may be run from a single-phase power source. However, it will not self-start. It may be hand started in either direction, coming up to speed in a few seconds. It will only develop 2/3 of the 3-φ power rating because one winding is not used.



3-φ motor runs from 1-φ power but does not start

Single Coil of a Single Phase Motor

The single coil of a single-phase induction motor does not produce a rotating magnetic field, but a pulsating field reaching maximum intensity at 0° and 180° electrical.



Single-phase stator produces a nonrotating, pulsating magnetic field

Another view is that the single-coil excited by a single-phase current produces two counter-rotating magnetic field phasors, coinciding twice per revolution at 0° (Figure above-a) and 180° (figure e). When the phasors rotate to 90° and -90° they cancel in figure c.

At 45° and -45° (figure b) they are partially additive along the $+x$ axis and cancel along the y -axis. An analogous situation exists in figure d. The sum of these two phasors is a phasor stationary in space, but alternating polarity in time. Thus, no starting torque is developed.

However, if the rotor is rotated forward at a bit less than the synchronous speed, It will develop maximum torque at 10% slip with respect to the forward rotating phasor. Less torque will be developed above or below 10% slip.

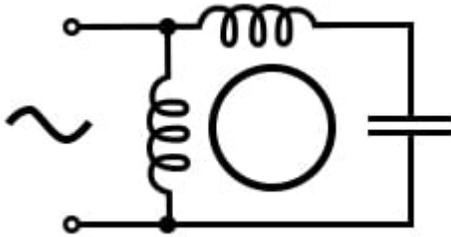
The rotor will see 200% - 10% slip with respect to the counter-rotating magnetic field phasor. Little torque (see torque vs slip curve) other than a double frequency ripple is developed from the counter-rotating phasor. Thus, the single-phase coil will develop torque, once the rotor is started.

If the rotor is started in the reverse direction, it will develop a similar large torque as it nears the speed of the backward rotating phasor.

Single-phase induction motors have a copper or aluminum squirrel cage embedded in a cylinder of steel laminations, typical of polyphase induction motors.

Permanent-Split Capacitor Motor

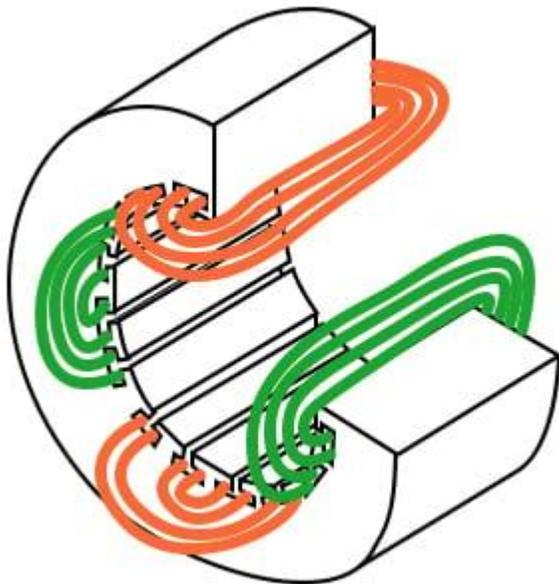
One way to solve the single phase problem is to build a 2-phase motor, deriving 2-phase power from single phase. This requires a motor with two windings spaced apart 90° electrical, fed with two phases of current displaced 90° in time. This is called a permanent-split [capacitor](#) motor.



Permanent-split capacitor induction motor

This type of motor suffers increased current magnitude and backward time shift as the motor comes up to speed, with torque pulsations at full speed. The solution is to keep the capacitor (impedance) small to minimize losses.

The losses are less than for a shaded pole motor. This motor configuration works well up to 1/4 horsepower (200 watts), though, usually applied to smaller motors. The direction of the motor is easily reversed by switching the capacitor in series with the other winding. This type of motor can be adapted for use as a servo motor, described elsewhere in this chapter.



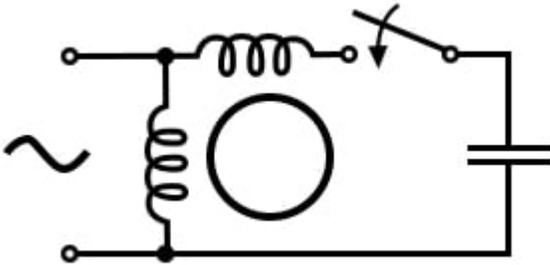
Single-phase induction motor with embedded stator coils

Single-phase induction motors may have coils embedded into the stator for larger size motors. Though, the smaller sizes use less complex to build concentrated windings with salient poles.

Capacitor-Start Induction Motor

In the figure below a larger capacitor may be used to start a single-phase induction motor via the auxiliary winding if it is switched out by a centrifugal switch once the motor is up to speed. Moreover, the auxiliary winding may be many more turns of heavier wire than used in a resistance split-phase motor to mitigate excessive temperature rise.

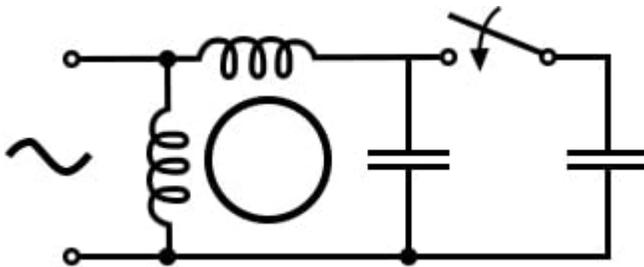
The result is that more starting torque is available for heavy loads like air conditioning compressors. This motor configuration works so well that it is available in multi-horsepower (multi-kilowatt) sizes.



Capacitor-start induction motor

Capacitor-Run Motor Induction Motor

A variation of the capacitor-start motor (figure below) is to start the motor with a relatively large capacitor for high starting torque, but leave a smaller value capacitor in place after starting to improve running characteristics while not drawing excessive current. The additional complexity of the capacitor-run motor is justified for larger size motors.



Capacitor-run motor induction motor

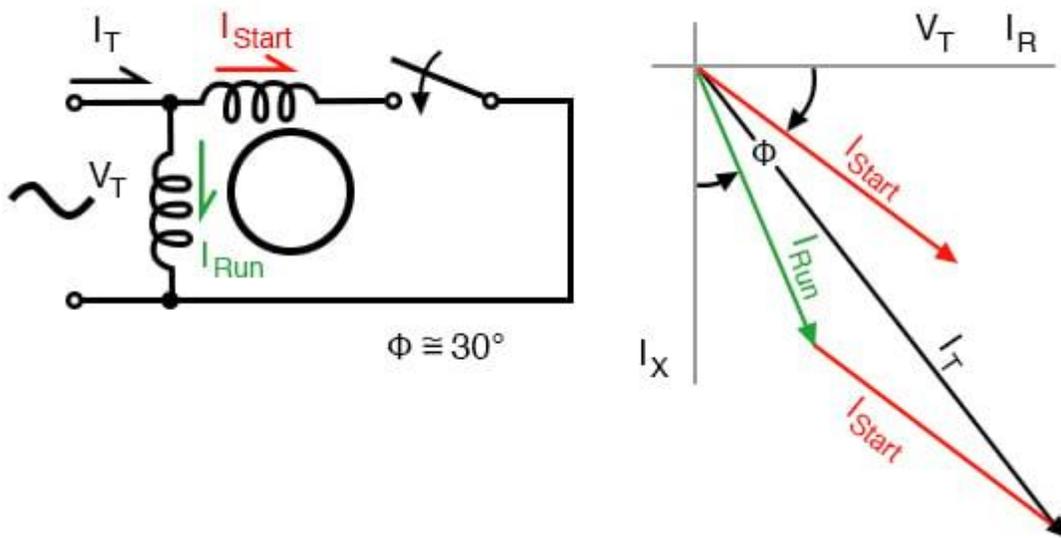
A motor starting capacitor may be a double-anode non-polar electrolytic capacitor which could be two + to + (or - to -) series-connected polarized electrolytic capacitors. Such AC rated electrolytic capacitors have such high losses that they can only be used for intermittent duty (1 second on, 60 seconds off) like motor starting.

A capacitor for motor running must not be of electrolytic construction, but a lower loss polymer type.

Resistance Split-Phase Motor Induction Motor

If an auxiliary winding of much fewer turns, a smaller wire is placed at 90° electrical to the main winding, it can start a single-phase induction motor. With lower inductance and higher resistance, the current will experience less phase shift than the main winding.

About 30° of phase difference may be obtained. This coil produces a moderate starting torque, which is disconnected by a centrifugal switch at 3/4 of synchronous speed. This simple (no capacitor) arrangement serves well for motors up to 1/3 horsepower (250 watts) driving easily started loads.



Resistance split-phase motor induction motor

This motor has more starting torque than a shaded pole motor (next section), but not as much as a two-phase motor built from the same parts. The current density in the auxiliary winding is

so high during starting that the consequent rapid temperature rise precludes frequent restarting or slow starting loads.

Nola Power Factor Corrector

Frank Nola of NASA proposed a power factor corrector for improving the efficiency of AC induction motors in the mid-1970s. It is based on the premise that induction motors are inefficient at less than full load. This inefficiency correlates with a low power factor.

The less than unity power factor is due to magnetizing current required by the stator. This fixed current is a larger proportion of total motor current as the motor load is decreased. At light load, the full magnetizing current is not required. It could be reduced by decreasing the applied voltage, improving the power factor and efficiency.

The power factor corrector senses power factor, and decreases motor voltage, thus restoring a higher power factor and decreasing losses.

Since single-phase motors are about 2 to 4 times as inefficient as three-phase motors, there are potential energy savings for 1- ϕ motors. There are no savings for a fully-loaded motor since all the stator magnetizing current is required.

The voltage cannot be reduced. But there are potential savings from a less than fully loaded motor. A nominal 117 VAC motor is designed to work at as high as 127 VAC, as low as 104 VAC. That means that it is not fully loaded when operated at greater than 104 VAC, for example, a 117 VAC refrigerator.

It is safe for the power factor controller to lower the line voltage to 104-110 VAC. The higher the initial line voltage, the greater the potential savings. Of course, if the power company delivers closer to 110 VAC, the motor will operate more efficiently without any add-on device.

Any substantially idle, 25% FLC or less, a single-phase induction motor is a candidate for a PFC. Though, it needs to operate a large number of hours per year. And the more time it idles, as in lumber saw, punch press, or conveyor, the greater the possibility of paying for the controller in a few years operation.

It should be easier to pay for it by a factor of three as compared to the more efficient 3- ϕ -motor. The cost of a PFC cannot be recovered for a motor operating only a few hours per day.

Summary: Single-phase induction motors

- *Single-phase induction motors* are not self-starting without an auxiliary stator winding driven by an out of phase current of near 90° . Once started the auxiliary winding is optional.
- The auxiliary winding of a permanent split *capacitor motor* has a capacitor in series with it during starting and running.
- A *capacitor-start induction motor* only has a capacitor in series with the auxiliary winding during starting.
- A *capacitor-run motor* typically has a large non-polarized electrolytic capacitor in series with the auxiliary winding for starting, then a smaller non-electrolytic capacitor during running.
- The auxiliary winding of a *resistance split-phase motor* develops a phase difference versus the main winding during starting by virtue of the difference in resistance.

Tesla Polyphase Induction Motors

Most AC motors are induction motors. Induction motors are favored due to their ruggedness and simplicity. In fact, 90% of industrial motors are induction motors.

Nikola Tesla conceived the basic principles of the polyphase induction motor in 1883 and had a half horsepower (400 watts) model by 1888. Tesla sold the manufacturing rights to George Westinghouse for \$65,000.

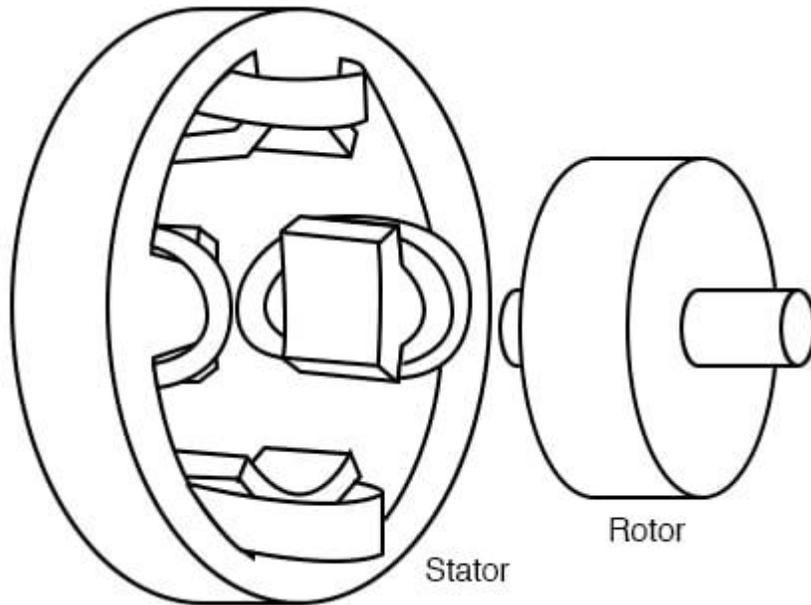
Most large (> 1 hp or 1 kW) industrial motors are *polyphase induction motors*. By polyphase, we mean that the stator contains multiple distinct windings per motor pole, driven by corresponding time-shifted sine waves.

In practice, this is two or three phases. Large industrial motors are 3-phase. While we include numerous illustrations of two-phase motors for simplicity, we must emphasize that nearly all polyphase motors are three-phase.

By *induction motor*, we mean that the stator windings induce a current flow in the rotor conductors, like a transformer, unlike a brushed DC commutator motor.

AC Induction Motor Construction

An induction motor is composed of a rotor, known as an armature, and a stator containing windings connected to a polyphase energy source as shown in the figure below. The simple 2-phase induction motor below is similar to the 1/2 horsepower motor which Nikola Tesla introduced in 1888.



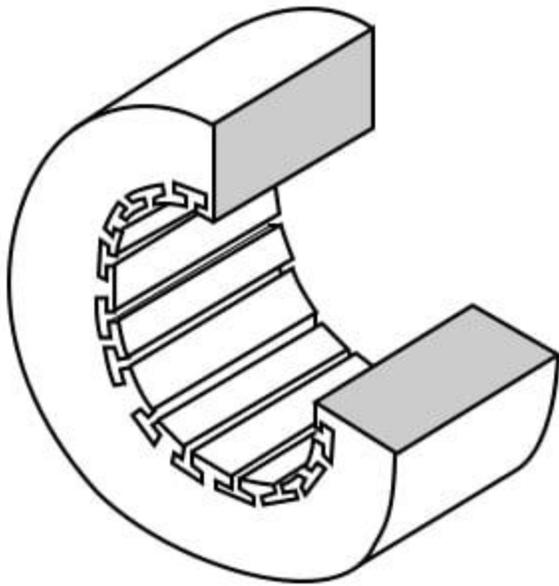
Tesla polyphase induction motor

The stator in the figure above is wound with pairs of coils corresponding to the phases of electrical energy available. The 2-phase induction motor stator above has 2-pairs of coils, one pair for each of the two phases of AC.

The individual coils of a pair are connected in series and correspond to the opposite poles of an electromagnet. That is, one coil corresponds to an N-pole, the other to an S-pole until the phase of AC changes polarity. The other pair of coils is oriented 90° in space to the first pair.

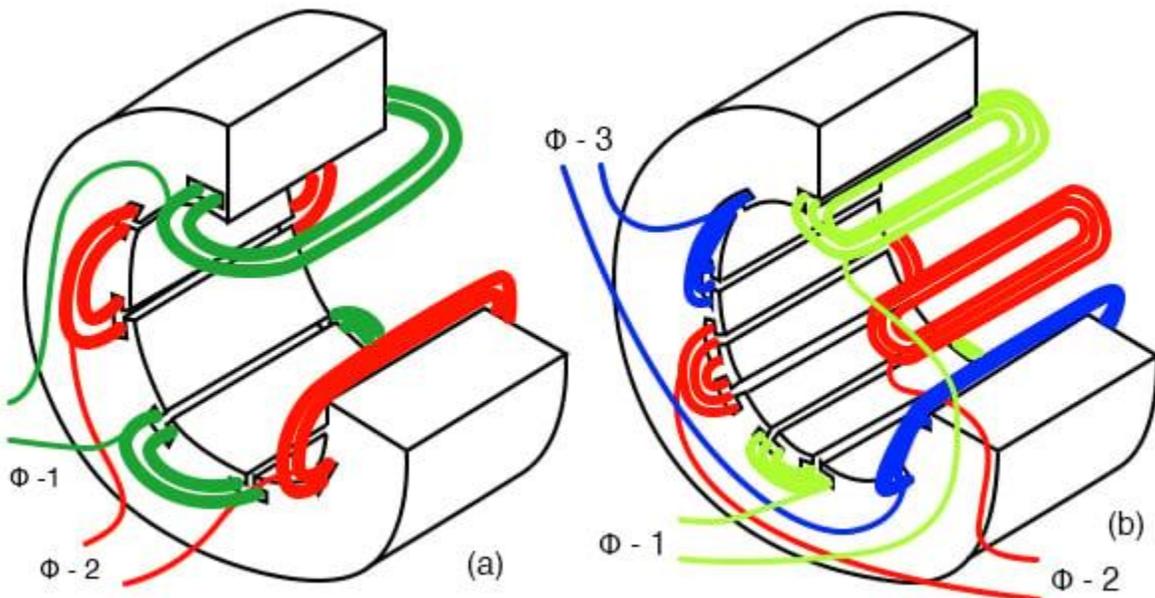
This pair of coils is connected to AC shifted in time by 90° in the case of a 2-phase motor. In Tesla's time, the source of the two phases of AC was a 2-phase alternator.

The stator in the figure above has *salient*, obvious protruding poles, as used on Tesla's early induction motor. This design is used to this day for sub-fractional horsepower motors (<50 watts). However, for larger motors, less torque pulsation and higher efficiency results if the coils are embedded into slots cut into the stator laminations (figure below).



Stator frame showing slots for windings

The stator laminations are thin insulated rings with slots punched from sheets of electrical grade steel. A stack of these is secured by end screws, which may also hold the end housings.



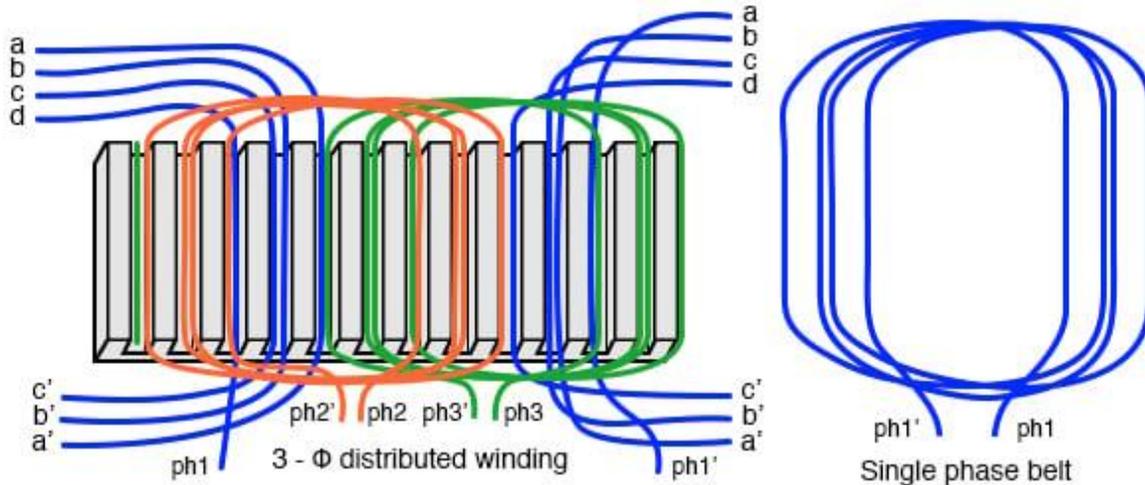
Stator with (a) 2- ϕ and (b) 3- ϕ windings

In the figure above, the windings for both a two-phase motor and a three-phase motor have been installed in the stator slots. The coils are wound on an external fixture, then worked into the slots. Insulation wedged between the coil periphery and the slot protects against abrasion.

Actual stator windings are more complex than the single windings per pole in the figure above. Comparing the 2- ϕ motor to Tesla's 2- ϕ motor with salient poles, the number of coils is the same. In actual large motors, a pole winding is divided into identical coils inserted into many smaller slots than above.

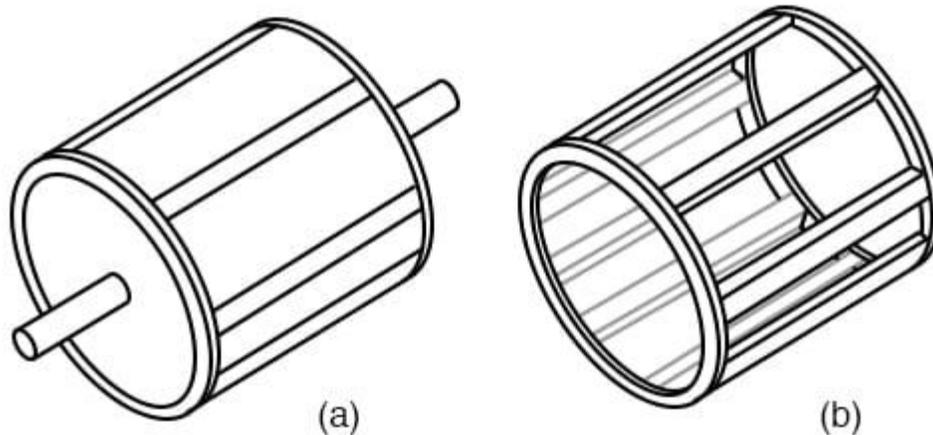
This group is called a *phase belt* (see the figure below). The distributed coils of the phase belt cancel some of the odd harmonics, producing a more sinusoidal magnetic field distribution across the pole. This is shown in the [synchronous motor section](#).

The slots at the edge of the pole may have fewer turns than the other slots. Edge slots may contain windings from two phases. That is, the phase belts overlap.



The key to the popularity of the AC induction motor is its simplicity as evidenced by the simple rotor (figure below). The rotor consists of a shaft, a steel laminated rotor, and an embedded copper or aluminum *squirrel cage*, shown at (b) removed from the rotor.

As compared to a DC motor armature, there is no commutator. This eliminates the brushes, arcing, sparking, graphite dust, brush adjustment and replacement, and re-machining of the commutator.



Laminated rotor with (a) embedded squirrel cage, (b) conductive cage removed from the rotor

The squirrel cage conductors may be skewed, twisted, with respect to the shaft. The misalignment with the stator slots reduces torque pulsations.

Both rotor and stator cores are composed of a stack of insulated laminations. The laminations are coated with insulating oxide or varnish to minimize eddy current losses. The alloy used in the laminations is selected for low hysteresis losses.

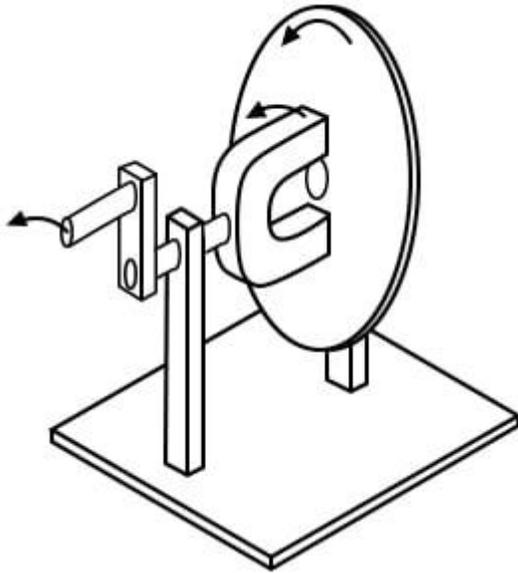
Theory of Operation of Induction Motors

A short explanation of operation is that the stator creates a rotating magnetic field which drags the rotor around.

The theory of operation of induction motors is based on a rotating magnetic field. One way of creating a rotating magnetic field is to rotate a permanent magnet. If the moving magnetic lines of flux cut a conductive disk, it will follow the motion of the magnet.

The lines of flux cutting the conductor will induce a voltage, and consequent current flow, in the conductive disk. This current flow creates an electromagnet whose polarity opposes the motion of the permanent magnet– *Lenz's Law*.

The polarity of the electromagnet is such that it pulls against the permanent magnet. The disk follows with a little less speed than the permanent magnet.



Rotating magnetic field produces torque in conductive disk

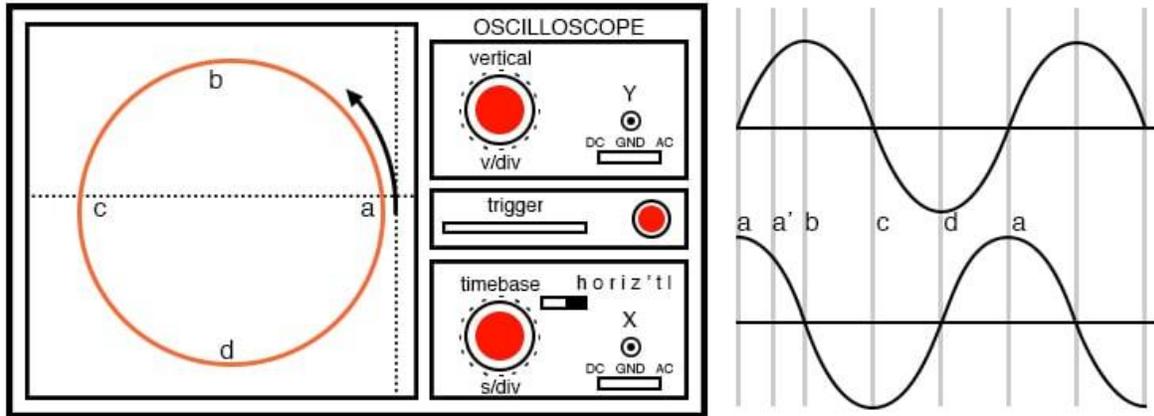
The torque developed by the disk is proportional to the number of flux lines cutting the disk and the rate at which it cuts the disk. If the disk were to spin at the same rate as the permanent magnet, there would be no flux cutting the disk, no induced current flow, no electromagnetic field, no torque.

Thus, the disk speed will always fall behind that of the rotating permanent magnet, so that lines of flux cut the disk induce a current, create an electromagnetic field in the disk, which follows the permanent magnet.

If a load is applied to the disk, slowing it, more torque will be developed as more lines of flux cut the disk. Torque is proportional to *slip*, the degree to which the disk falls behind the rotating magnet. More slip corresponds to more flux cutting the conductive disk, developing more torque.

An analog automotive eddy-current speedometer is based on the principle illustrated above. With the disk restrained by a spring, disk and needle deflection is proportional to the magnet rotation rate.

A rotating magnetic field is created by two coils placed at right angles to each other, driven by currents which are 90° out of phase. This should not be surprising if you are familiar with oscilloscope Lissajous patterns.

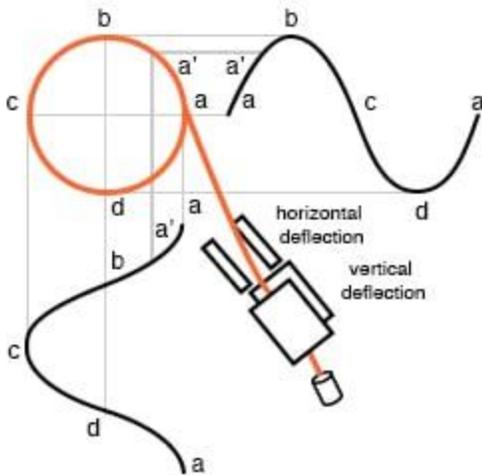


Out of phase (90°), sine waves produce circular Lissajous pattern

In the figure above, a circular Lissajous is produced by driving the horizontal and vertical oscilloscope inputs with 90° out of phase sine waves. Starting at (a) with maximum “X” and minimum “Y” deflection, the trace moves up and left toward (b).

Between (a) and (b) the two waveforms are equal to 0.707 Vpk at 45°. This point (0.707, 0.707) falls on the radius of the circle between (a) and (b). The trace moves to (b) with minimum “X” and maximum “Y” deflection. With maximum negative “X” and minimum “Y” deflection, the trace moves to (c).

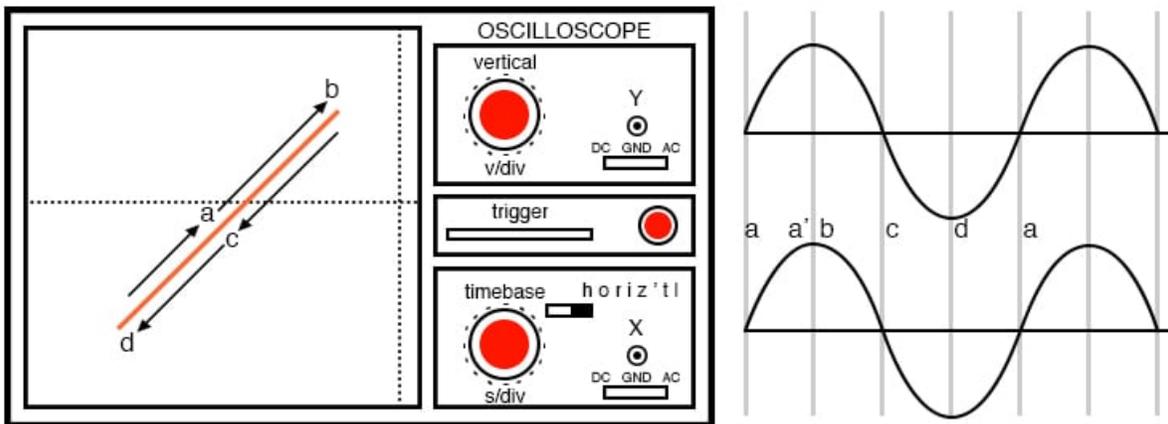
Then with minimum “X” and maximum negative “Y”, it moves to (d), and on back to (a), completing one cycle.



X-axis sine and Y-axis cosine trace circle

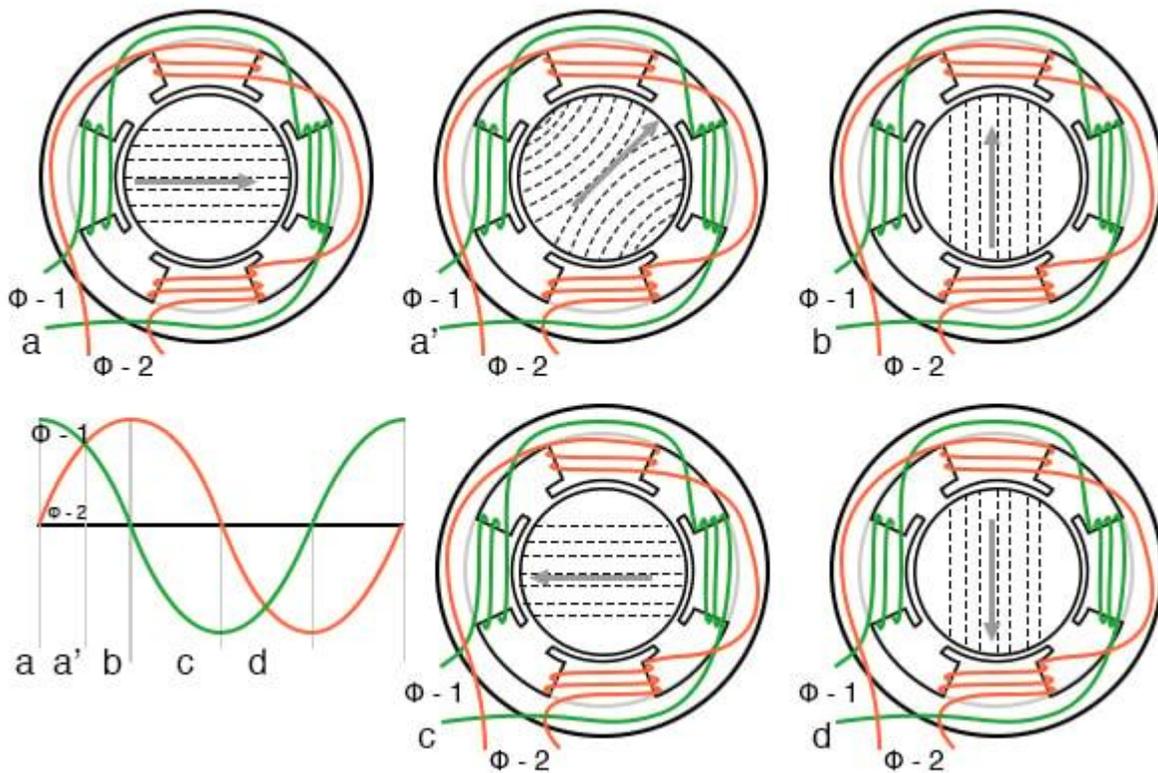
The figure shows the two 90° phase-shifted sine waves applied to oscilloscope deflection plates which are at right angles in space. The combination of 90° phased sine waves and right angle deflection, results in a two-dimensional pattern– a circle. This circle is traced out by a counterclockwise-rotating electron beam.

For reference, the figure below shows why in-phase sine waves will not produce a circular pattern. Equal “X” and “Y” deflection moves the illuminated spot from the origin at (a) up to the right (1,1) at (b), back down left to origin at (c), down left to (-1,-1) at (d), and back upright to the origin. The line is produced by equal deflections along both axes; $y=x$ is a straight line.



No circular motion from in-phase waveforms

If a pair of 90° out of phase sine waves produces a circular Lissajous, a similar pair of currents should be able to produce a circular rotating magnetic field. Such is the case for a 2-phase motor. By analogy three windings placed 120° apart in space, and fed with corresponding 120° phased currents will also produce a rotating magnetic field.



Rotating magnetic field from 90° phased sine waves

As the 90° phased sine waves, the figure above, progress from points (a) through (d), the magnetic field rotates counterclockwise (figures a-d) as follows:

- (a) $\phi-1$ maximum, $\phi-2$ zero
- (a') $\phi-1$ 70%, $\phi-2$ 70%
- (b) $\phi-1$ zero, $\phi-2$ maximum

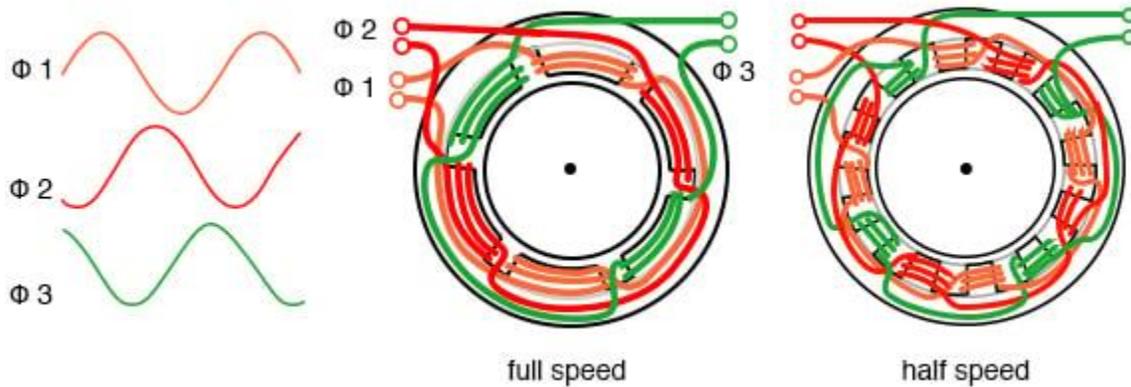
- (c) ϕ -1 maximum negative, ϕ -2 zero
- (d) ϕ -1 zero, ϕ -2 maximum negative

Full Motor Speed and Synchronous Motor Speed

The rotation rate of a stator rotating magnetic field is related to the number of pole pairs per stator phase. The “full speed” figure below has a total of six poles or three pole-pairs and three phases. However, there is but one pole pair per phase.

The magnetic field will rotate once per sine wave cycle. In the case of 60 Hz power, the field rotates at 60 times per second or 3600 revolutions per minute (rpm). For 50 Hz power, it rotates at 50 rotations per second or 3000 rpm. The 3600 and 3000 rpm, are the *synchronous speed* of the motor.

Though the rotor of an induction motor never achieves this speed, it certainly is an upper limit. If we double the number of motor poles, the synchronous speed is cut in half because the magnetic field rotates 180° in space for 360° of the electrical sine wave.



Doubling the stator poles halves the synchronous speed

The synchronous speed is given by:

$N_s = 120 \cdot f / P$ N_s = synchronous speed in rpm f = frequency of applied power, Hz P = total number of poles per phase, a multiple of 2

Example: The “half speed” figure above has four poles per phase (3-phase). The synchronous speed for 50 Hz power is: $S = 120 \cdot 50 / 4 = 1500$ rpm

The short explanation of the induction motor is that the rotating magnetic field produced by the stator drags the rotor around with it.

The longer more correct explanation is that the stator's magnetic field induces an alternating current into the rotor squirrel cage conductors which constitutes a transformer secondary. This induced rotor current, in turn, creates a magnetic field.

The rotating stator magnetic field interacts with this rotor field. The rotor field attempts to align with the rotating stator field. The result is the rotation of the squirrel cage rotor. If there were no mechanical motor torque load, no bearing, windage, or other losses, the rotor would rotate at the synchronous speed.

However, the *slip* between the rotor and the synchronous speed stator field develops torque. It is the magnetic flux cutting the rotor conductors as it slips which develops torque. Thus, a loaded motor will slip in proportion to the mechanical load.

If the rotor were to run at synchronous speed, there would be no stator flux cutting the rotor, no current induced in the rotor, no torque.

Torque in Induction Motors

When power is first applied to the motor, the rotor is at rest, while the stator magnetic field rotates at the synchronous speed N_s . The stator field is cutting the rotor at the synchronous speed N_s . The current induced in the rotor shorted turns is maximum, as is the frequency of the current, the line frequency.

As the rotor speeds up, the rate at which stator flux cuts the rotor is the difference between synchronous speed N_s and actual rotor speed N , or $(N_s - N)$. The ratio of actual flux cutting the rotor to synchronous speed is defined as *slip*:

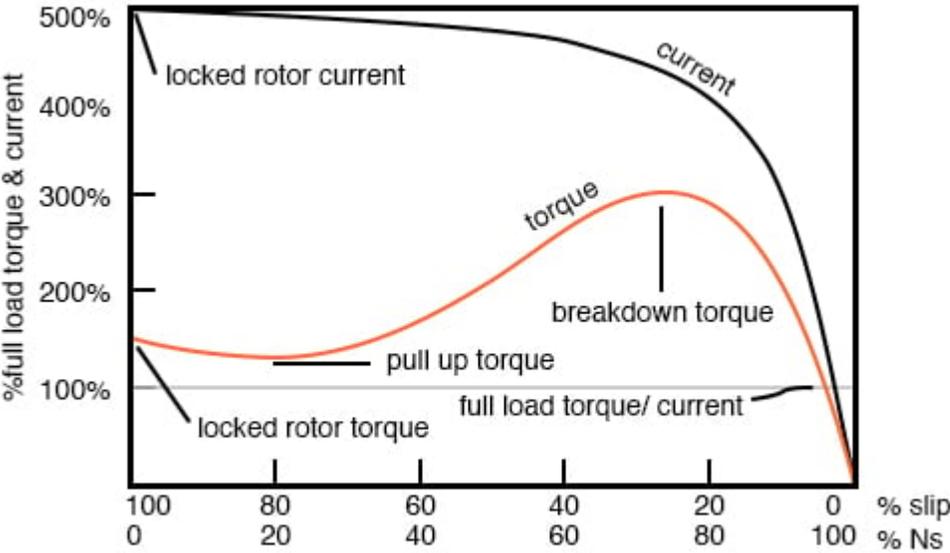
$$s = (N_s - N)/N_s \text{ where: } N_s = \text{synchronous speed, } N = \text{rotor speed}$$

The frequency of the current induced into the rotor conductors is only as high as the line frequency at the motor start, decreasing as the rotor approaches synchronous speed. *Rotor frequency* is given by:

$$f_r = s \cdot f \text{ where: } s = \text{slip, } f = \text{stator power line frequency}$$

Slip at 100% torque is typically 5% or less in induction motors. Thus for $f = 50 \text{ Hz}$ line frequency, the frequency of the induced current in the rotor $f_r = 0.05 \cdot 50 = 2.5 \text{ Hz}$. Why is it so low? The stator magnetic field rotates at 50 Hz. The rotor speed is 5% less.

The rotating magnetic field is only cutting the rotor at 2.5 Hz. The 2.5 Hz is the difference between the synchronous speed and the actual rotor speed. If the rotor spins a little faster, at the synchronous speed, no flux will cut the rotor at all, $f_r = 0$.



Torque and speed vs %Slip. $\%N_s = \%Synchronous \text{ Speed}$

The figure above graph shows that starting torque known as *locked rotor torque* (LRT) is higher than 100% of the *full load torque* (FLT), the safe continuous torque rating. The locked rotor torque is about 175% of FLT for the example motor graphed above.

Starting current known as *locked rotor current* (LRC) is 500% of *full load current* (FLC), the safe running current. The current is high because this is analogous to a shorted secondary on a transformer. As the rotor starts to rotate the torque may decrease a bit for certain classes of motors to a value known as the *pull-up torque*.

This is the lowest value of torque ever encountered by the starting motor. As the rotor gains 80% of synchronous speed, torque increases from 175% up to 300% of the full load torque. This *breakdown torque* is due to the larger than normal 20% slip.

The current has decreased only slightly at this point but will decrease rapidly beyond this point. As the rotor accelerates to within a few percents of synchronous speed, both torque and current will decrease substantially. Slip will be only a few percents during normal operation.

For a running motor, any portion of the torque curve below 100% rated torque is normal. The motor load determines the operating point on the torque curve. While the motor torque and current may exceed 100% for a few seconds during starting, continuous operation above 100% can damage the motor.

Any motor torque load above the breakdown torque will stall the motor. The torque, slip, and current will approach zero for a “no mechanical torque” load condition. This condition is analogous to an open secondary transformer.

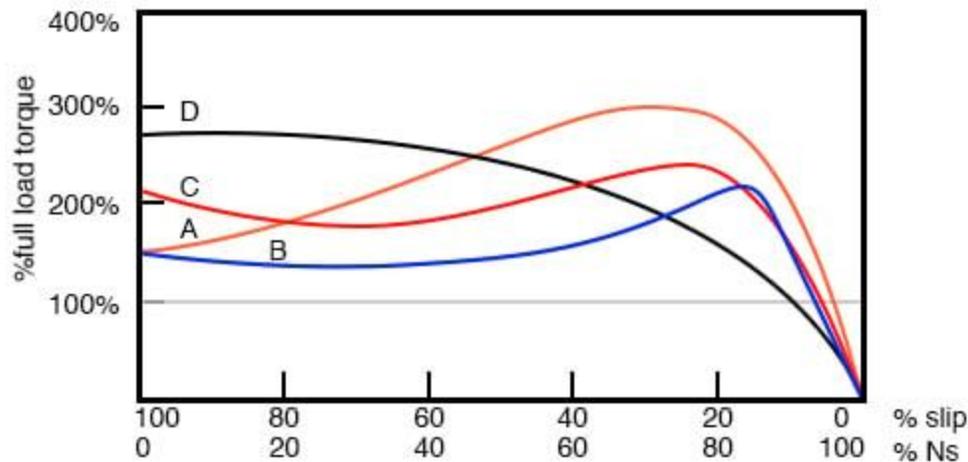
There are several basic induction motor designs showing considerable variation from the torque curve above. The different designs are optimized for starting and running different types of loads. The locked rotor torque (LRT) for various motor designs and sizes ranges from 60% to 350% of full load torque (FLT).

Starting current or locked rotor current (LRC) can range from 500% to 1400% of full load current (FLC). This current draw can present a starting problem for large induction motors.

NEMA and IEC Motor Classes

Various standard classes (or designs) for motors, corresponding to the torque curves (figure below) have been developed to better drive various type loads. The National Electrical Manufacturers Association (NEMA) has specified motor classes A, B, C, and D to meet these drive requirements.

Similar International Electrotechnical Commission (IEC) classes N and H correspond to NEMA B and C designs respectively.



Characteristics for NEMA designs

All motors, except class D, operate at 5% slip or less at full load.

- **Class B (IEC Class N)** motors are the default motor to use in most applications. With a starting torque of LRT = 150% to 170% of FLT, it can start most loads, without excessive starting current (LRT). Efficiency and power factor are high. It typically drives pumps, fans, and machine tools.
- **Class A** starting torque is the same as class B. Drop out torque and starting current (LRT) is higher. This motor handles transient overloads as encountered in injection molding machines.
- **Class C (IEC Class H)** has higher starting torque than class A and B at LRT = 200% of FLT. This motor is applied to hard-starting loads which need to be driven at constant speed like conveyors, crushers, and reciprocating pumps and compressors.
- **Class D** motors have the highest starting torque (LRT) coupled with low starting current due to high slip (5% to 13% at FLT). The high slip results in lower speed. The speed regulation is poor. However, the motor excels at driving highly variable speed loads like those requiring an energy storage flywheel. Applications include punch presses, shears, and elevators.
- **Class E** motors are a higher efficiency version of class B.
- **Class F** motors have much lower LRC, LRT, and break down torque than class B. They drive constant, easily started loads.

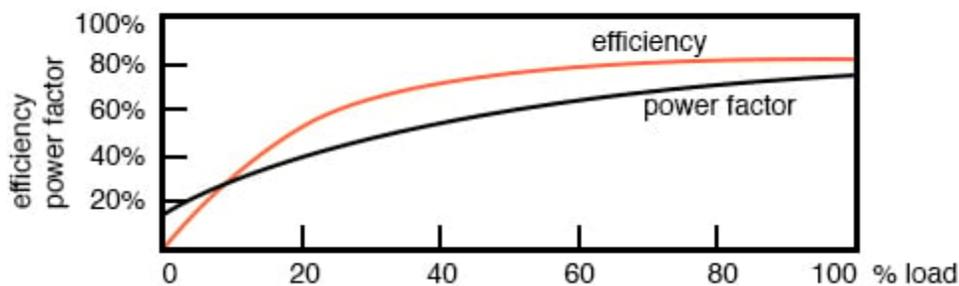
Power Factor in Induction Motors

Induction motors present a lagging (inductive) power factor to the power line. The power factor in large fully loaded high-speed motors can be as favorable as 90% for large high-speed motors. At 3/4 full load, the largest high-speed motor power factor can be 92%.

The power factor for small low-speed motors can be as low as 50%. At starting, the power factor can be in the range of 10% to 25%, rising as the rotor achieves speed.

Power factor (PF) varies considerably with the motor mechanical load (figure below). An unloaded motor is analogous to a transformer with no resistive load on the secondary. Little resistance is reflected from the secondary (rotor) to the primary (stator).

Thus the power line sees a reactive load, as low as 10% PF. As the rotor is loaded an increasing resistive component is reflected from the rotor to stator, increasing the power factor.



Induction motor power factor and efficiency

Efficiency in Induction Motors

Large three-phase motors are more efficient than smaller 3-phase motors, and most all single-phase motors. Large induction motor efficiency can be as high as 95% at full load, though 90% is more common.

Efficiency for a lightly loaded or no-loaded induction motor is poor because most of the current is involved with maintaining the magnetizing flux. As the torque load is increased, more current is consumed in generating torque, while current associated with magnetizing remains fixed. Efficiency at 75% FLT can be slightly higher than that at 100% FLT.

Efficiency is decreased a few percents at 50% FLT and decreased a few more percents at 25% FLT. Efficiency only becomes poor below 25% FLT. The variation of efficiency with loading is shown in the figure above.

Induction motors are typically oversized to guarantee that their mechanical load can be started and driven under all operating conditions. If a polyphase motor is loaded at less than 75% of rated torque where efficiency peaks, efficiency suffers only slightly down to 25% FLT.

Nola Power Factor Corrector

Frank Nola of NASA proposed a power factor corrector (PFC) as an energy-saving device for single-phase induction motors in the late 1970s. It is based on the premise that a less than fully loaded induction motor is less efficient and has a lower power factor than a fully-loaded motor. Thus, there is energy to be saved in partially loaded motors, 1- ϕ motors in particular.

The energy consumed in maintaining the stator magnetic field is relatively fixed with respect to load changes. While there is nothing to be saved in a fully-loaded motor, the voltage to a partially loaded motor may be reduced to decrease the energy required to maintain the magnetic field.

This will increase the power factor and efficiency. This was a good concept for the notoriously inefficient single phase motors for which it was intended.

This concept is not very applicable to large 3-phase motors. Because of their high efficiency (90%+), there is not much energy to be saved. Moreover, a 95% efficient motor is still 94% efficient at 50% full load torque (FLT) and 90% efficient at 25% FLT.

The potential energy savings in going from 100% FLT to 25% FLT is the difference in efficiency $95\% - 90\% = 5\%$. This is not 5% of the full load wattage but 5% of the wattage at the reduced load. The Nola power factor corrector might be applicable to a 3-phase motor which idles most of the time (below 25% FLT), like a punch press.

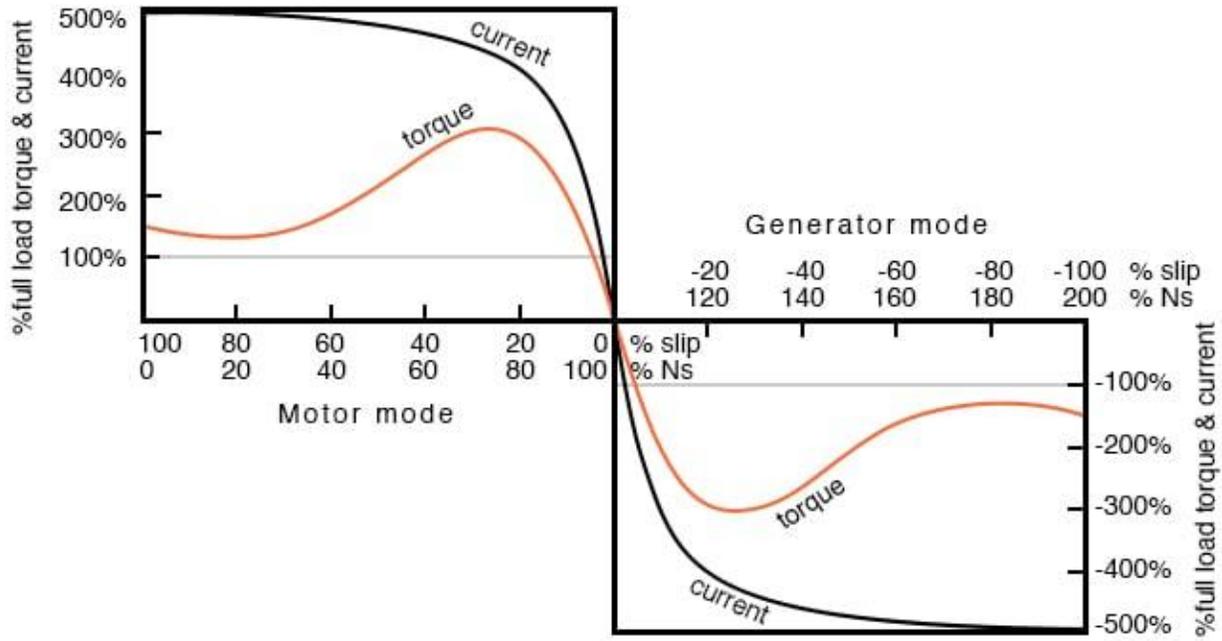
The payback period for the expensive electronic controller has been estimated to be unattractive for most applications. Though, it might be economical as part of an electronic motor starter or speed Control.

Induction Motors as Alternators

An induction motor may function as an alternator if it is driven by a torque at greater than 100% of the synchronous speed (figure below). This corresponds to a few % of "negative" slip, say -1% slip.

This means that as we are rotating the motor faster than the synchronous speed, the rotor is advancing 1% faster than the stator rotating magnetic field. It normally lags by 1% in a motor.

Since the rotor is cutting the stator magnetic field in the opposite direction (leading), the rotor induces a voltage into the stator feeding electrical energy back into the power line.



Negative torque makes induction motor into a generator

Such an *induction generator* must be excited by a “live” source of 50 or 60 Hz power. No power can be generated in the event of a power company power failure. This type of alternator appears to be unsuited as a standby power source.

As an auxiliary power wind turbine generator, it has the advantage of not requiring an automatic power failure disconnect switch to protect repair crews.

It is fail-safe.

Small remote (from the power grid) installations may be made self-exciting by placing [capacitors](#) in parallel with the stator phases. If the load is removed residual magnetism may generate a small amount of current flow.

This current is allowed to flow by the capacitors without dissipating power. As the generator is brought up to full speed, the current flow increases to supply a magnetizing current to the

stator. The load may be applied at this point. Voltage regulation is poor. An induction motor may be converted to a self-excited generator by the addition of capacitors.

Startup procedure is to bring the wind turbine up to speed in motor mode by application of normal power line voltage to the stator. Any wind-induced turbine speed in excess of synchronous speed will develop negative torque, feeding power back into the power line, reversing the normal direction of the electric kilowatt-hour meter.

Whereas an induction motor presents a lagging power factor to the power line, an induction alternator presents a leading power factor. Induction generators are not widely used in conventional power plants.

The speed of the steam turbine drive is steady and controllable as required by synchronous alternators. Synchronous alternators are also more efficient.

The speed of a wind turbine is difficult to control and subject to wind speed variation by gusts. An induction alternator is better able to cope with these variations due to the inherent slip. This stresses the gear train and mechanical components less than a synchronous generator.

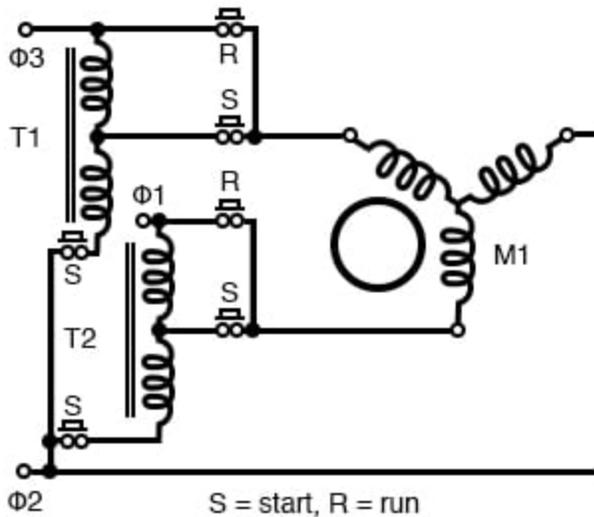
However, this allowable speed variation only amounts to about 1%. Thus, a direct line connected induction generator is considered to be fixed-speed in a wind turbine (See [Doubly-fed induction generator](#) for a true variable speed alternator).

Multiple generators or multiple windings on a common shaft may be switched to provide a high and low speed to accommodate variable wind conditions.

Motor Starting and Speed Control

Some induction motors can draw over 1000% of full load current during starting; though, a few hundred percents is more common. Small motors of a few kilowatts or smaller can be started by direct connection to the power line.

Starting large motors can cause line voltage sag, affecting other loads. Motor-start rated circuit breakers (analogous to slow blow fuses) should replace standard circuit breakers for starting motors of a few kilowatts. This breaker accepts high overcurrent for the duration of starting.



Autotransformer induction motor starter

Motors over 50 kW use motor starters to reduce line current from several hundred to a few hundred percents of full load current. An intermittent duty autotransformer may reduce the stator voltage for a fraction of a minute during the start interval, followed by application of full line voltage as in the figure above.

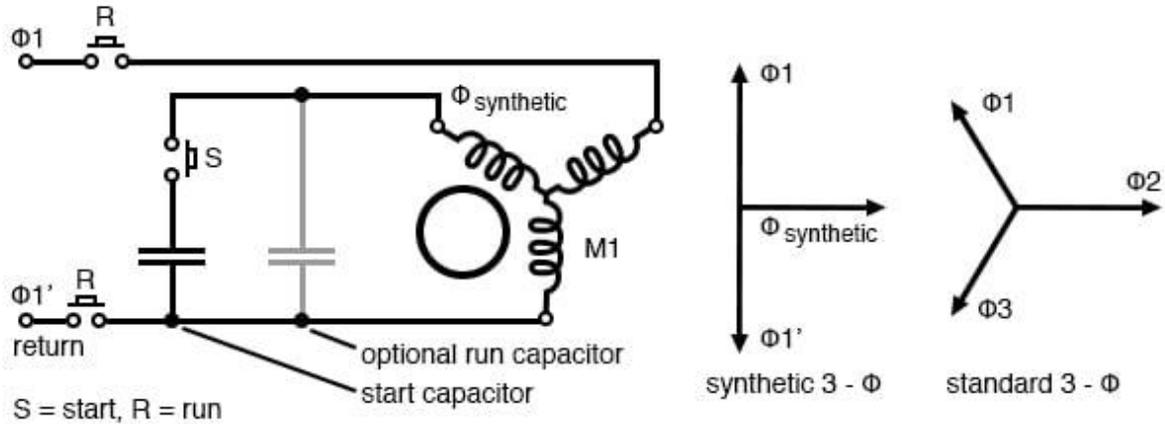
Closure of the S contacts applies reduced voltage during the start interval. The S contacts open and the R contacts close after starting. This reduces starting current to, say, 200% of full load current. Since the autotransformer is only used for the short start interval, it may be sized considerably smaller than a continuous-duty unit.

Running Three-Phase Motors on Single-Phase Provisions

Three-phase motors will run on single phase as readily as single-phase motors. The only problem for either motor is starting. Sometimes 3-phase motors are purchased for use on single-phase if three-phase provisioning is anticipated.

The power rating needs to be 50% larger than for a comparable single phase motor to make up for one unused winding. Single-phase is applied to a pair of windings simultaneous with a start capacitor in series with the third winding.

The start switch is opened in the figure below upon motor start. Sometimes a smaller capacitor than the start capacitor is retained while running.

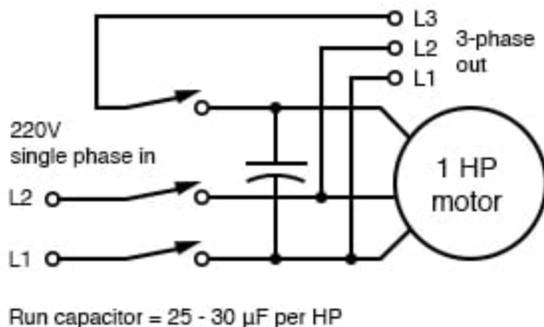


Starting a three-phase motor on single phase

The circuit in the figure above for running a three-phase motor on single phase is known as a *static phase converter* if the motor shaft is not loaded. Moreover, the motor acts as a 3-phase generator.

Three-phase power may be tapped off from the three stator windings for powering other 3-phase equipment. The capacitor supplies a *synthetic* phase approximately $\angle 90^\circ$ between the $\angle 180^\circ$ single-phase power source terminals for starting.

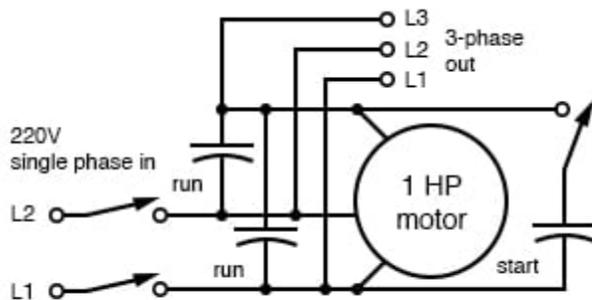
While running, the motor generates approximately standard 3- ϕ , as shown above. Matt Isserstedt shows a complete design for powering a home machine shop.



Self-starting static phase converter. Run capacitor = 25-30 μF per HP. Adapted from Figure 7, Hanrahan

Since a static phase converter has no torque load, it may be started with a capacitor considerably smaller than a normal start capacitor. If it is small enough, it may be left in the circuit as a run-capacitor (See the figure above).

However, smaller run-capacitors result in better 3-phase power output. Moreover, adjustment of these capacitors to equalize the currents as measured in the three phases results in the most efficient machine. However, a large start capacitor is required for about a second to quickly start the converter. Hanrahan provides construction details.

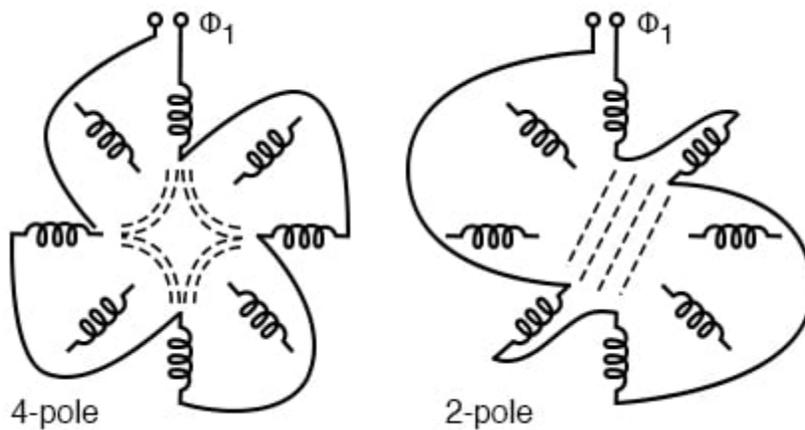


Start capacitor = 50 - 100 $\mu\text{F}/\text{HP}$. Run capacitors = 12 - 16 $\mu\text{F}/\text{HP}$.

More efficient static phase converter. Start capacitor = 50-100 $\mu\text{F}/\text{HP}$. Run capacitors = 12-16 $\mu\text{F}/\text{HP}$. Adapted from Figure 1, Hanrahan

Induction Motors with Multiple Fields

Induction motors may contain multiple field windings, for example, a 4-pole and an 8-pole winding corresponding to 1800 and 900 rpm synchronous speeds. Energizing one field or the other is less complex than rewiring the stator coils.



Multiple fields allow speed change

If the field is segmented with leads brought out, it may be rewired (or switched) from 4-pole to 2-pole as shown above for a 2-phase motor. The 22.5° segments are switchable to 45° segments. Only the wiring for one phase is shown above for clarity.

Thus, our induction motor may run at multiple speeds. When switching the above 60 Hz motor from 4 poles to 2 poles the synchronous speed increases from 1800 rpm to 3600 rpm.

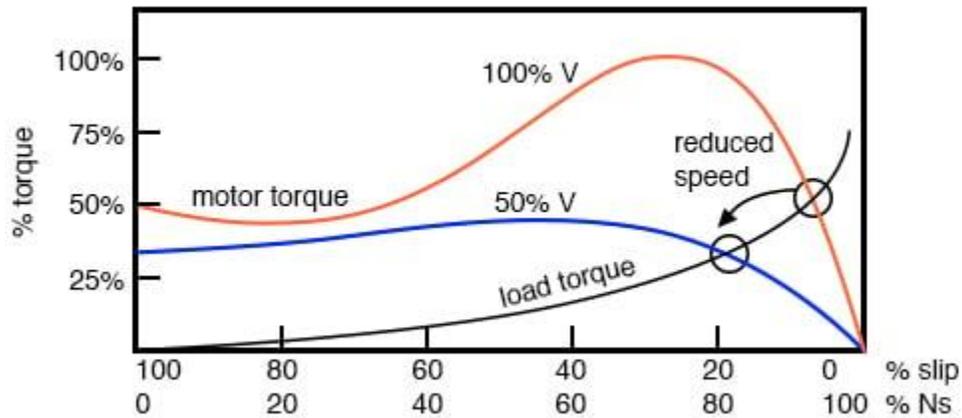
Q: If the motor is driven by 50 Hz, what would be the corresponding 4-pole and 2-pole synchronous speeds?

A:

$$N_s = 120f/P = 120 \cdot 50/4 = 1500 \text{ rpm (4-pole)} \quad N_s = 3000 \text{ rpm (2-pole)}$$

Induction Motors with Variable Voltage

The speed of small squirrel cage induction motors for applications such as driving fans may be changed by reducing the line voltage. This reduces the torque available to the load which reduces the speed (see figure below).

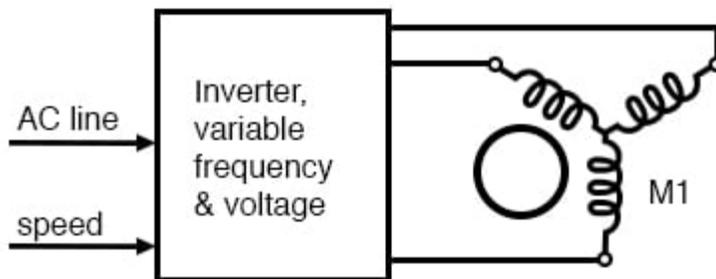


Variable voltage controls induction motor speed

Electronic Speed Control in Induction Motors

Modern solid-state electronics increase the options for speed control. By changing the 50 or 60 Hz line frequency to higher or lower values, the synchronous speed of the motor may be changed. However, decreasing the frequency of the current fed to the motor also decreases reactance X_L which increases the stator current.

This may cause the stator magnetic circuit to saturate with disastrous results. In practice, the voltage to the motor needs to be decreased when the frequency is decreased.



Electronic variable speed drive

Conversely, the drive frequency may be increased to increase the synchronous speed of the motor. However, the voltage needs to be increased to overcome increasing reactance to keep current up to a normal value and maintain torque.

The inverter approximates sine waves to the motor with pulse width modulation outputs. This is a chopped waveform which is either on or off, high or low, the percentage of “on” time corresponds to the instantaneous sine wave voltage.

Once electronics is applied to induction motor control, many control methods are available, varying from the simple to complex:

- *Scalar Control*: Low-cost method described above to control only voltage and frequency, without feedback.
- *Vector Control*: Also known as a vector phase control. The flux and torque producing components of stator current are measured or estimated on a real-time basis to enhance the motor torque-speed curve. This is computation intensive.
- *Direct Torque Control*: An elaborate adaptive motor model allows more direct control of flux and torque without feedback. This method quickly responds to load changes.

Tesla Polyphase Induction Motors Summary

- A *polyphase induction motor* consists of a polyphase winding embedded in a laminated stator and a conductive squirrel-cage embedded in a laminated rotor.
- Three-phase currents flowing within the stator create a rotating magnetic field which induces a current and consequent magnetic field in the rotor. Rotor torque is developed as the rotor slips a little behind the rotating stator field.
- Unlike single-phase motors, polyphase induction motors are *self-starting*.
- *Motor starters* minimize loading of the power line while providing a larger starting torque than required during running. Line current reducing *starters* are only required for large motors.
- Three-phase motors will run on single phase if started.
- A *static phase converter* is a three-phase motor running on single phase having no shaft load, generating a 3-phase output.
- *Multiple field windings* can be rewired for multiple discrete motor speeds by changing the number of poles.

Linear Induction Motors

The wound stator and the squirrel cage rotor of an induction motor may be cut at the circumference and unrolled into a linear induction motor. The direction of linear travel is controlled by the sequence of the drive to the stator phases.

The linear induction motor has been proposed as a drive for high-speed passenger trains. Up to this point, the linear induction motor with the accompanying magnetic repulsion levitation system required for a smooth ride has been too costly for all but experimental installations.

However, the linear induction motor is scheduled to replace steam-driven catapult aircraft launch systems on the next generation of the naval aircraft carrier, CVNX-1, in 2013. This will increase efficiency and reduce maintenance.

Motor Calculations Part 1: Motors and Branch-Circuit Conductors – Mike Holt

Overcurrent and short-circuit protection aren't the same for motors

The best method for providing overcurrent protection for most circuits is to use a circuit breaker that combines overcurrent protection with short-circuit and ground-fault protection. However, this isn't usually the best choice for motors. With rare exceptions, the best method for providing overcurrent protection in these cases is to separate the overload protection devices from the short-circuit

The best method for providing overcurrent protection for most circuits is to use a circuit breaker that combines overcurrent protection with short-circuit and ground-fault protection. However, this isn't usually the best choice for motors. With rare exceptions, the best method for providing overcurrent protection in these cases is to separate the overload protection devices from the short-circuit and ground-fault protection devices (**Fig. 1**).

Motor overload protection devices like heaters protect the motor, the motor control equipment, and the branch-circuit conductors from motor overload and the resultant excessive heating (430.31). They don't provide protection against short-circuits or ground-fault currents. That's the job of the branch and feeder breakers, which don't provide motor overload protection. This arrangement makes motor calculations different from those used for other types of loads. Let's look at how to apply Art. 430, starting at the motor.

Overload protection. Motor overload devices are often integrated into the motor starter. But you can use a separate overload device like a dual-element fuse, which is usually located near the motor starter, not the supply breaker.

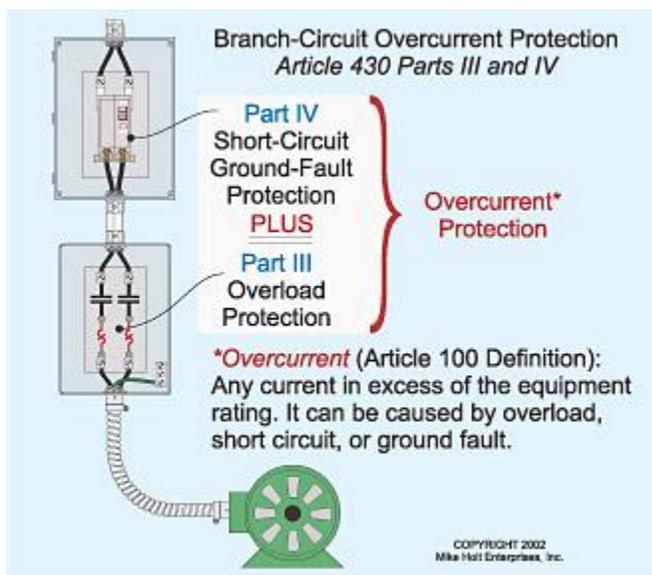


Fig. 1. Overcurrent protection is generally accomplished by separating the overload protection from the short-circuit and ground-fault protection device.

If you use fuses, you must provide one for each ungrounded conductor (430.36 and 430.55). Thus, a 3-phase motor requires three fuses. Keep in mind that these devices are at the load end of the branch circuit and that they don't provide short-circuit or ground-fault protection.

Motors rated more than 1 hp without integral thermal protection and motors rated 1 hp or less that are automatically started [430.32(C)] must have an overload device sized per the motor nameplate current rating [430.6(A)]. You must size the overload devices no larger than the requirements of 430.32. Motors with a nameplate service factor (SF) rating of 1.15 or more must have an overload protection device sized no more than 125% of the motor nameplate current rating.

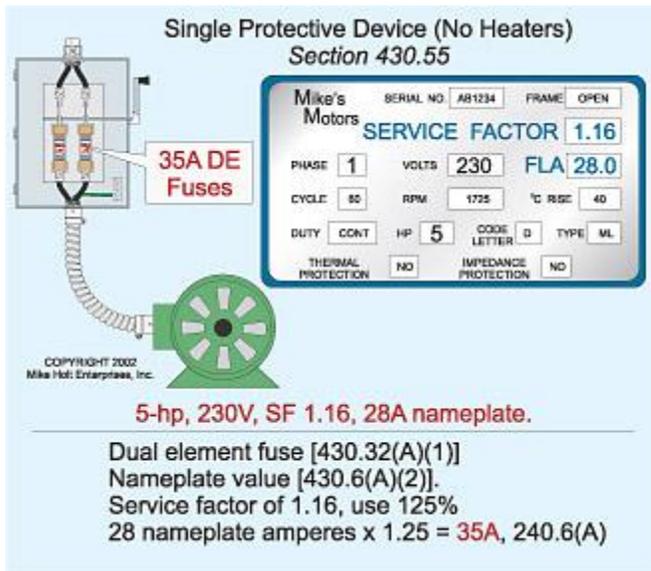


Fig. 2. When working with motors that have a service factor rating of 1.15 or higher, size overload protection devices no more than 125% of the motor nameplate rating.

Let's look at **Fig. 2** and work through a sample calculation.

Example No. 1: Suppose you use a dual-element fuse for overload protection. What size fuse do you need for a 5-hp, 230V, single-phase motor with a service factor of 1.16 if the motor nameplate current rating is 28A?

- (a) 25A
- (c) 35A
- (b) 30A
- (d) 40A

The overload protection shall be sized according to the motor nameplate current rating [430.6(A), 430.32(A)(1), and 430.55].

You also have to consider another factor: nameplate temperature rise. For motors with a nameplate temperature rise rating not over 40°C, size the overload protection device no more than 125% of the motor nameplate current rating. Thus, $28A \times 1.25 = 35A$ [240.6(A)]

Sizing Fuses for Overcurrent Protection (No Heaters)
Section 430.55

70A DE Fuses

**FLA = 60A
FLC = 65A**

50-hp, 460V, 3-phase Rise 39°C, 60A FLA.

Dual element fuse [430.32(A)(1)]
Nameplate (FLA) value to be used.
Temperature Rise 39°C, use 125%
60A nameplate x 1.25 = 75A
Next size *down = **70A DE Fuses** [240.6(A)]
**The next size up rule does not apply when fuses are used for overload protection.*

Fig. 3. Size the overload protection device of a motor with a nameplate temperature rise rating of 40°C or less at no more than 125% of the motor nameplate current rating.

Let's look at **Fig. 3** and work through another example problem.

Example No. 2: Again, suppose you're using a dual-element fuse for the overload protection. What size fuse do you need for a 50-hp, 460V, 3-phase motor that has a temperature rise of 39°C and motor nameplate current rating of 60A (FLA)?

- (a) 40A
- (c) 60A
- (b) 50A
- (d) 70A

The overload protection is sized per the motor nameplate current rating, not the motor full load current (FLC) rating. Thus, $60A \times 1.25 = 75A$. Overload protection shall not exceed 75A, so you need to use a 70A dual-element fuse [240.6(A) and 430.32(A)(1)].

Motors that don't have a service factor rating of 1.15 or higher or a temperature rise rating of 40°C and less must have an overload protection device sized at not more than 115% of the motor nameplate ampere rating (430.37).

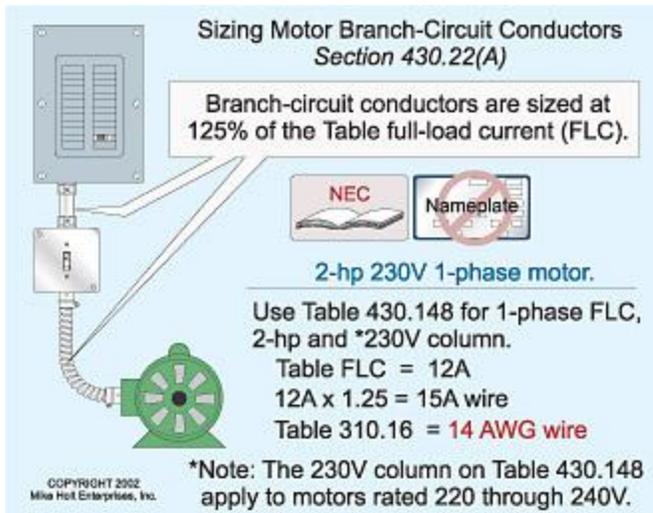


Fig. 4. Refer to Table 310.16 when selecting the proper size conductor to serve a single motor.

Sizing branch-circuit conductors. Branch-circuit conductors that serve a single motor must have an ampacity of not less than 125% of the motor's FLC as listed in Tables 430.147 through 430.150 [430.6(A)]. You must select the conductor size from Table 310.16 according to the terminal temperature rating (60°C or 75°C) of the equipment [110.14(C)]. Let's reinforce this concept by working through a sample calculation. Refer to **Fig. 4**.

Example No. 3: What size THHN conductor do you need for a 2-hp, 230V, single-phase motor?

- (a) 14 AWG
- (c) 10 AWG
- (b) 12 AWG
- (d) 8 AWG

Let's walk through the solution:

Step 1: Conductor sized no less than 125% of motor FLC

Step 2: Table 430.148 shows the FLC of 2-hp, 230V, single-phase as 12A

Step 3: $12A \times 1.25 = 15A$

Step 4: Per Table 310.16, you need to use 14 AWG THHN rated 20A at 60°C

The minimum size conductor the NEC permits for building wiring is 14 AWG [310.5]. However, local codes and many industrial facilities have requirements that 12 AWG be used as the smallest branch-circuit wire. So in this example you might need to use 12 AWG instead of 14 AWG.

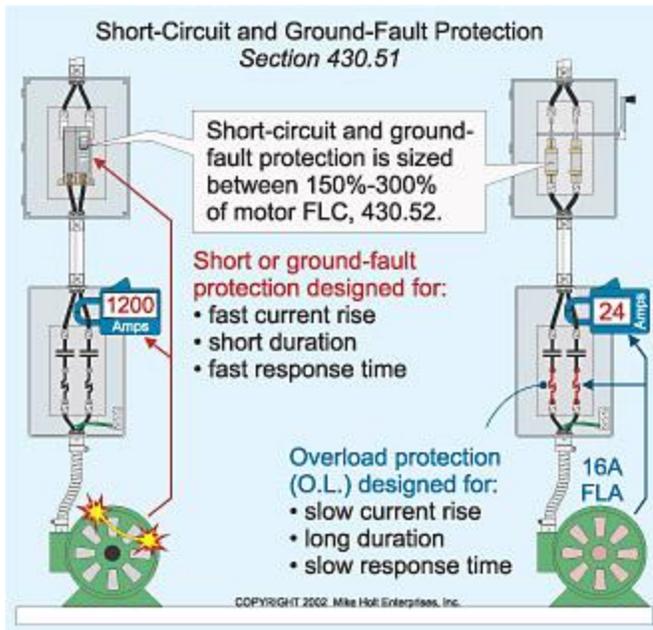


Fig. 5. Short-circuit and ground-fault protection devices are designed for fast current rise, short-duration events. On the other hand, overload protection devices are designed for slow current rate, long-duration situations.

Branch-circuit protection for short-circuits and ground-faults. Branch-circuit short-circuit and ground-fault protection devices protect the motor, motor control apparatus, and conductors against short circuits or ground faults. They don't protect against an overload (430.51) (**Fig. 5**).

The short-circuit and ground-fault protection device required for motor circuits isn't the type required for personnel (210.8), feeders (215.9 and 240.13), services (230.95), or temporary wiring for receptacles (527.6).

Per 430.52(C), you must size the short-circuit and ground-fault protection for the motor branch circuit — except those that serve torque motors — so they're no greater than the percentages listed in Table 430.52.

When the short-circuit and ground-fault protection device value that you find in Table 430.52 doesn't correspond to the standard rating or setting of overcurrent protection devices as listed in 240.6(A), use the next higher protection device size [430.52(C)(1) Ex. 1].

Did that statement stop you? Does it strike you as incorrect? That's a common response, but remember, motors are different than other system components. Motor overload protection devices, such as heaters and fuses, protect the motor and other items from overload. The short-circuit and ground-fault protection doesn't need to perform this function. Therefore, oversizing won't compromise protection. Undersizing will prevent the motor from starting.

Use the following two-step process to determine what percentage from Table 430.52 you should use to size the motor branch-circuit short-circuit ground-fault protection device.

Step 1: Locate the motor type on Table 430.52.

Step 2: Select the percentage from Table 430.52 according to the type of protection device, such as non-time delay (one-time), dual-element fuse, or inverse-time circuit breaker. Don't forget to use the next higher protection device size when necessary.

Let's see if you have this concept down with a short quiz. Of the following statements, which one is true? Use Table 430.52 to look up the numbers.

1. The branch-circuit short-circuit protection (non-time delay fuse) for a 3-hp, 115V, single-phase motor shall not exceed 110A.
2. The branch-circuit short-circuit protection (dual-element fuse) for a 5-hp, 230V, single-phase motor shall not exceed 50A.
3. The branch-circuit short-circuit protection (inverse-time breaker) for a 25-hp, 460V, 3-phase synchronous motor shall not exceed 70A.

Let's address each question individually. We'll be referring to 430.53(C)(1) Ex. 1 and Table 430.52.

1. Per Table 430.148, $34A \times 3.00 = 102A$. The next size up is 110A. So this is true.
2. Per Table 430.148, $28A \times 1.75 = 49A$. The next size up is 50A. So, this is also true.
3. Per Table 430.150, $26A \times 2.50 = 65A$. The next size up is 70A. This is also true.

Remember the following important principles:

- You must size the conductors at 125% of the motor FLC [430.22(A)].
- You must size the overloads no more than 115% to 125% of the motor nameplate current rating, depending on the conditions [430.32(A)(1)].
- You must size the short-circuit ground-fault protection device from 150% to 300% of the motor FLC [Table 430.52].

If you put all three of these together, you can see the branch-circuit conductor ampacity (125%) and the short-circuit ground-fault protection device (150% to 300%) aren't related.

This final example should help you see if you've been paying attention.

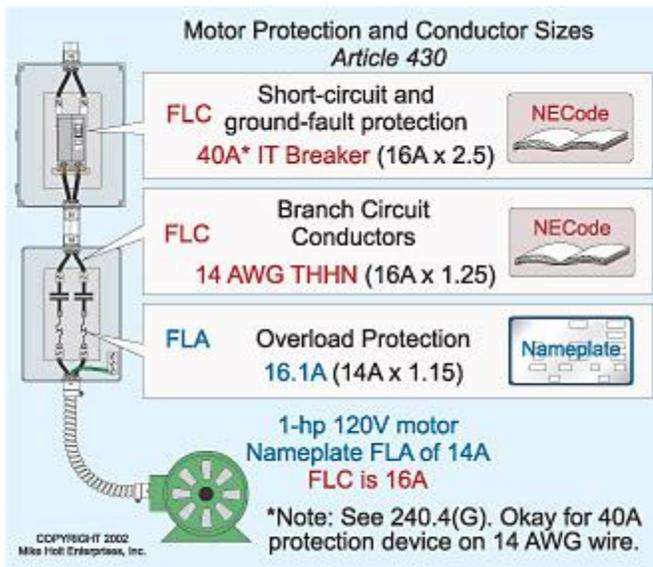


Fig. 6. Although this example may bother some people, the 14 AWG THHN conductors and motor are protected against overcurrent by the 16A overload protection device and the 40A short-circuit protection device.

Example No. 4: Are any of the following statements true for a 1-hp, 120V motor, nameplate current rating of 14A? Refer to **Fig. 6**.

- (a) The branch-circuit conductors can be 14 AWG THHN.
- (b) Overload protection is from 16.1A.
- (c) Short-circuit and ground-fault protection is permitted to be a 40A circuit breaker.
- (d) All of these are true.

Walking through each of these, you can see:

- (a) The conductors are sized per 430.22(A): $16A \times 1.25 = 20A$; Table 310.16 requires 14 AWG at 60°C.
- (b) Per 430.32(A)(1), overload protection is sized as follows: $14A$ (nameplate) $\times 1.15 = 16.1A$.
- (c) Short-circuit and ground-fault protection is determined based on 430.52(C)(1): $16A \times 2.50 = 40A$ circuit breaker.

Therefore all three statements are true.

The 16A overload protection device protects the 14 AWG conductors from overcurrent, while the 40A short-circuit protection device protects them from short circuits. This example

illustrates the sometimes confusing fact that when you're doing motor calculations, you're actually calculating overcurrent and short-circuit protection separately.

Motor calculations have long been a source of confusion and errors for many people. Understanding what makes these calculations different should help you do your motor calculations correctly every time. Next month we'll look at sizing motor feeders in Part 2

Motor Calculations Part 2: Feeders – Mike Holt

Part 1 of this two-part series explained how to size overload protection devices and short-circuit and ground-fault protection for motor branch circuits. Understanding the key point of that article, which was that motor overload protection requires separate calculations from short-circuit and ground-fault protection, clears up a common source of confusion and a point of error. But another source of confusion arises when it comes to sizing short-circuit and ground-fault protection for a feeder that supplies more than one motor. Let's look again at branch-circuit calculations and then resolve the feeder issues so your calculations will always be correct.

Branch-circuit conductors and protection devices

Per 430.6(A), branch-circuit conductors to a single motor must have an ampacity of not less than 125% of the motor full load current (FLC) as listed in Tables 430.147 through 430.150. To illustrate this, let's size the branch-circuit conductors (THHN) and short-circuit ground-fault protection device for a 3-hp, 115V, single-phase motor. The motor FLA is 31A, and dual-element fuses for short-circuit and ground-fault protection are in use (**Fig. 1**).

- Per Table 430.148, the FLC current is 34A.
- $34A \times 125\% = 43A$.
- Per Table 310.16 (60°C terminals [110.14(C)(1)(a)]), the conductor must be a 6 AWG THHN rated 55A.

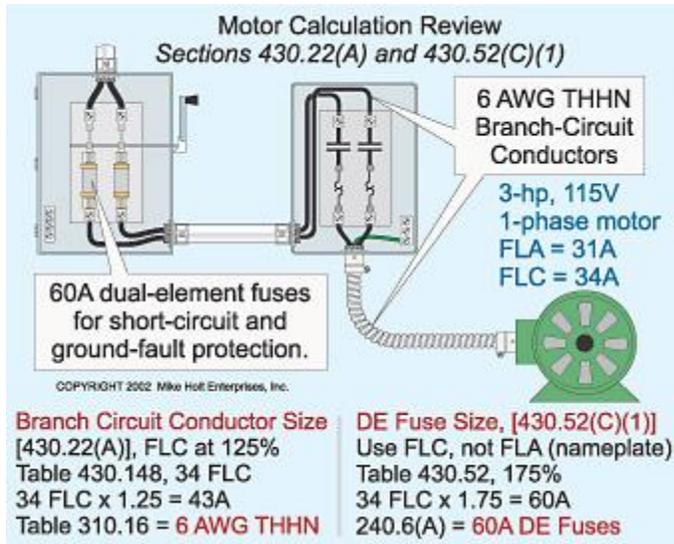


Fig. 1. Don't make the mistake of using a motor's FLA nameplate rating when using the short-circuit and ground-fault protection devices. You must use the FLC rating given in Table 430.148.

Per the motor FLC listed in Table 430.52, size the branch-circuit short-circuit and ground-fault protection devices by using multiplication factors based on the type of motor and protection device. When the protection device values determined from Table 430.52 don't correspond with the standard rating of overcurrent protection devices listed in 240.6(A), you must use the next higher overcurrent protection device. To illustrate this, let's use the same motor as in the previous example.

- Per 240.6(A), multiply 34A×175%
- You need a 60A dual-element fuse.

To explore this example further, see Example No. D8 in Annex D of the 2002 NEC. Once you've sized the motor overloads, branch-circuit conductors, and branch-circuit protective devices, you're ready to move on to the next step.

Motor feeder conductor calculations

From 430.24, you can see that conductors that supply several motors must have an ampacity not less than:

- 125% of the highest-rated motor FLC [430.17], plus
- The sum of the FLCs of the other motors (on the same phase), as determined by 430.6(A), plus
- The ampacity required to supply the other loads on that feeder.

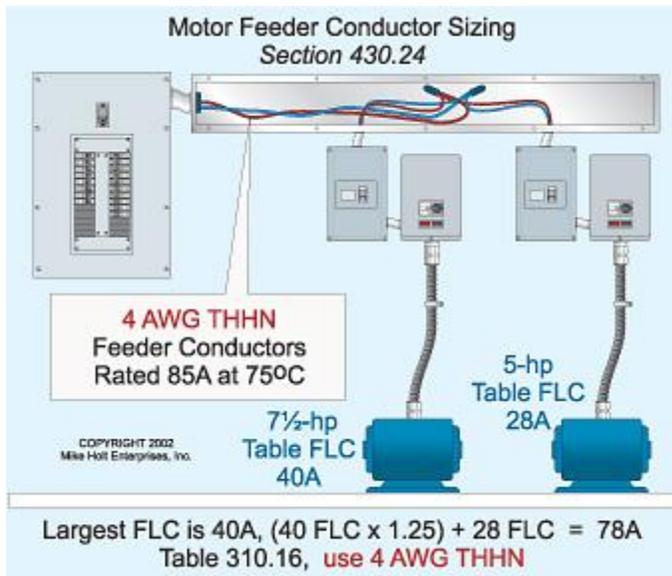


Fig. 2. Motor feeder conductors shall be sized not less than 125% of the largest motor FLC plus the sum of the FLCs of the other motors on the same phase.

Use Fig. 2 and solve the following problem.

Example No. 1. For what ampacity must you size the feeder conductor if it supplies the following two motors? The terminals are rated for 75°C.

- One 7.5-hp, 230V (40A), single-phase motor
- One 5-hp, 230V (28A), single-phase motor

- (a) 50A
- (b) 60A
- (c) 70A
- (d) 80A

Let's walk through the solution.

- The largest motor is 40A.
- $40A \times 1.25 + 28A = 78A$.
- 80A is the closest selection that's at least 78A.

What size conductor would give us this ampacity?

- (a) 2 AWG
- (b) 4 AWG

- (c) 6 AWG
- (d) 8 AWG

Per Table 310.16, a 6 AWG conductor rated at 75°C provides 65A of ampacity, so it's too small. However, a 4 AWG conductor provides 85A of ampacity, which will accommodate the necessary 78A. Therefore, you need to size this feeder conductor at 4 AWG.

Next, we have to determine what size overcurrent protection device (OCPD) we must provide for a given feeder.

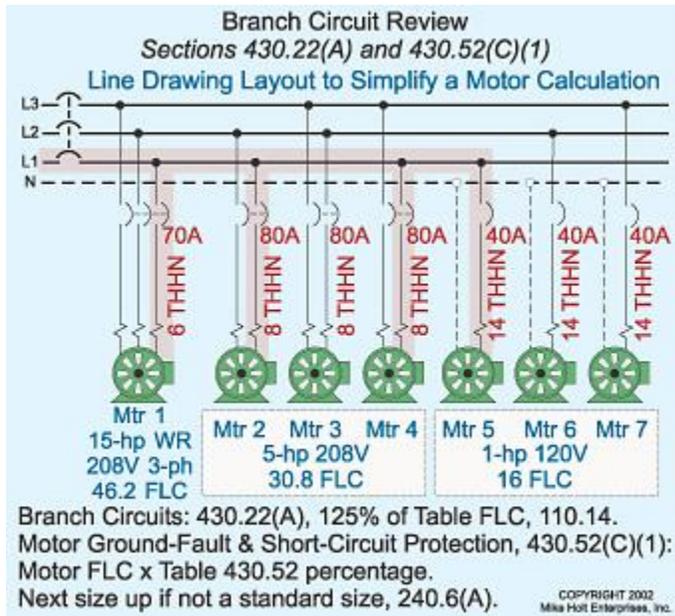


Fig. 3. To size overcurrent protection devices for each feeder, start by determining the ampacities required for each motor and move on from there.

Example No. 2. Using a slightly more complex example, try sizing the feeder conductor (THHN) and protection device (inverse-time breakers, 75°C terminal rating) for the following motors (**Fig. 3**):

- Three 1-hp, 120V, single-phase motors
- Three 5-hp, 208V, single-phase motors
- One wound-rotor, 15-hp, 208V, 3-phase motor

Refer to 240.6(A), 430.52(C)(1), Table 430.148, and Table 430.52. Start by determining the ampacities required for each size of motor, then walk through each step until you arrive at the correct OCPD size.

- 1-hp motor: FLC is 16A.
16A×250%=40A
- 5-hp motor: FLC is 30.8A.
30.8A×250%=77A (Next size up is 80A.)
- 15-hp motor: FLC is 46.2A.

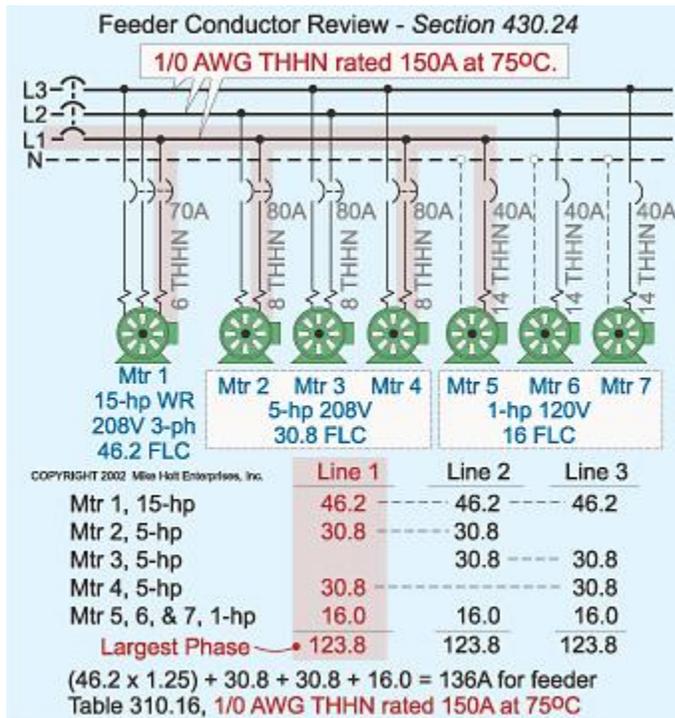


Fig. 4. Each motor's FLC will come into play when sizing the conductor.

46.2A×150% (wound-rotor) 569A (Next size up is 70A.)

Now, let's look at the feeder conductor. Conductors that supply several motors must have an ampacity of not less than 125% of the highest-rated motor FLC (430.17), plus the sum of the other motor FLCs [430.6(A)] on the same phase (**Fig. 4**).

Continuing with this example, add up all the ampacities, multiplying the highest rated motor by 125%. Thus:

- (46.2A×1.25)+30.8A+30.8A+16A=136A.

Table 310.16 shows you need 1/0 AWG THHN because at 150A it's the smallest conductor that accommodates the 136A of ampacity we're working with. When sizing the feeder conductor, be sure to include only the motors that are on the same phase. For that reason, these calculations only involve four motors.

You must provide the feeder with a protective device with a rating or setting not greater than the largest rating or setting of the branch-circuit short-circuit and ground-fault protective device (plus the sum of the full-load currents of the other motors of the group) [430.62(A)].

Remember, motor feeder conductors must be protected against the overcurrent that results from short circuits and ground faults but not those that result from motor overload.

When sizing the feeder protection, be sure to include only the motors that are on the same phase.

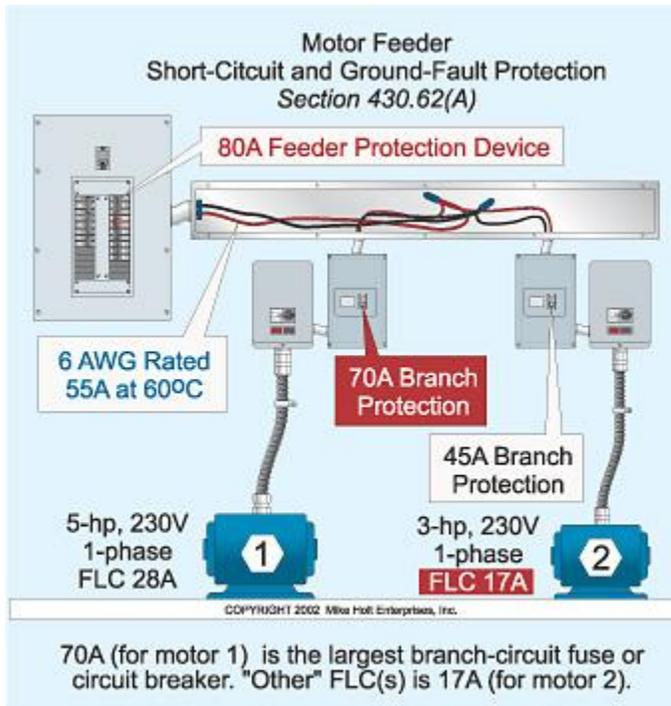


Fig. 5. In this example, the largest branch-circuit fuse or circuit breaker allowed for Motor 1 is 70A.

Refer to **Fig. 5** for this sample motor feeder protection calculation.

Example No. 3. What size feeder protection (inverse-time breaker) do you need for the following two motors?

- 5-hp, 230V, single-phase motor
- 3-hp, 230V, single-phase motor

- (a) 30A breaker
- (b) 40A breaker
- (c) 50A breaker
- (d) 80A breaker

Let's walk through the solution.

Step 1: Get the motor FLC from Table 430.148.

- A 5-hp motor FLC is 28A.
- A 3-hp motor FLC is 17A.

Step 2: Size the branch-circuit protection per the requirements of 430.52(C)(1), Table 430.52, and 240.6(A)

- 5-hp: $28A \times 2.5 = 70A$
- 3-hp: $17A \times 2.5 = 42.5A$ (Next size up is 45A.)

Step 3: Size the feeder conductor per 430.24(A).

- The largest motor is 28A.
- $(28A \times 1.25) + 17A = 52A$
- Table 310.16 shows 6 AWG rated 55A at 60°C as the smallest conductor with sufficient ampacity.

Step 4: Size the feeder protection per 430.62.

- It must not be greater than the 70A protection of the branch circuit plus the 17A of the other motor, which is the total of all loads on that feeder.
- $70A + 17A = 87A$

Choose the next size down, which is 80A.

How can you be safe if you're selecting the next size down instead of the next size up? Remember, you've already accounted for all the loads, and the NEC requires that you not exceed the protection of the branch circuit. Again, keep in mind that you aren't calculating for motor overload protection. Motor calculations are different from other calculations. With motor feeders, you're calculating for protection from short circuits and ground faults, only — not overload. Motor calculations get confusing if you forget there's a division of responsibility in the protective devices. To get your calculations right, you must separately calculate the motor overload protection (typically near the motor), branch-circuit protection (from short circuits and ground faults), and feeder-circuit protection (from short circuits and ground faults). Remember that overload protection is only at the motor.

Any time you find yourself confused, just refer to NEC Figure 430.1. It shows the division of responsibility between different forms of protection in motor circuits. Example D8 in Annex D of the 2002 NEC illustrates this with actual numbers. Keeping this division of responsibility in mind will allow you to make correct motor calculations every time.

LESSON 9a 3- ϕ Motors

TABLE of SECTIONS

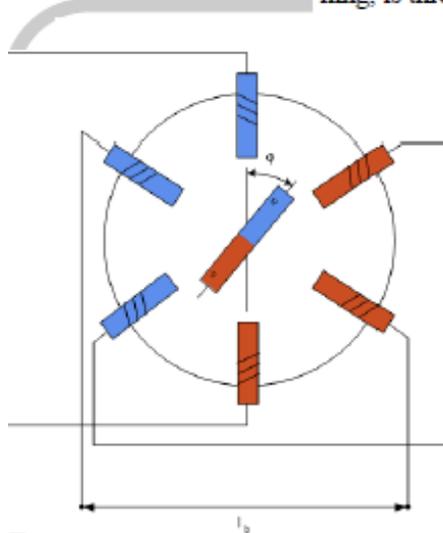
Preview

The most common class of 3 phase load is the 3 phase electric motor. A 3 phase induction motor has a simple design, inherently high starting torque, and high efficiency. Such motors are applied in industry for pumps, fans, blowers, compressors, conveyor drives, and many other types of motor-driven equipment. A 3 phase motor is more compact and less costly than a 1-phase motor of the same voltage class and rating; also 1-phase AC motors above 10 HP (7.5 kW) are not as efficient and thus not usually manufactured.



Section 9a-1 Introduction to 3-Phase Motors

Fig 9a-1 Cutaway of a 3- ϕ AC Motor



More efficient than single phase power which splits the phase to get the motor running, is three phase current. In this manner, the phase difference between windings in a motor is provided automatically by the electrical service.

Because the current draw in a three phase motor is actually through three conductors, it is a little more difficult to measure. Then too, because they are out-of-phase with each other, the total current draw is not the same as the sum of the three current draws from each phase.

In fact, the total current draw in a three phase motor is found from the POWER (VA) by the equation

$$\text{Apparent Power (VA)} = \sqrt{3} \text{ Volts (Amps)} = 1.732 \text{ VA}$$

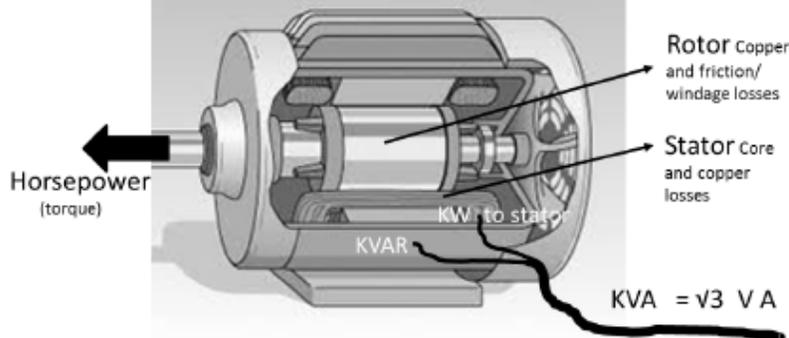
Example: An ammeter placed on a 208V, 3 ϕ motor indicated 28.6 amps draw. What is the Apparent Power of the motor?

$$\begin{aligned} \text{Solution: Apparent Power} &= 1.732 (208) (28.6) = 10006 \text{ VA} \\ &= 10 \text{ KVA} \end{aligned}$$

Section 9a-2 Power Factor

As the diagram below shows, the KVA power to the motor consists of a real or true part (kW) which actually runs the motor and a reactive part which supports the induction characteristics of the motor. When power is connected to a motor, the electromagnetic flux in the many windings of the motor interfere with each other, and create many induced voltages either increase current or reduce it. The result is that the current no longer tracks voltage, but rather there is a phase difference Θ between them. In a motor, often the current lags the voltage by this phase amount. It is the reactive power or KVAR that supports this phase difference in voltage and current. The ratio of the apparent power that is kW (The true power) to the total KVA is called power factor.

Fig 9a-2 Power Flow in a 3- ϕ AC Motor



ages either increase current or reduce it. The result is that the current no longer tracks voltage, but rather there is a phase difference Θ between them. In a motor, often the current lags the voltage by this phase amount. It is the reactive power or KVAR that supports this phase difference in voltage and current. The ratio of the apparent power that is kW (The true power) to the total KVA is called power factor.

Since the current lags the voltage in an induction motor, is said that the motor has a lagging power factor and that the power factor is equal to the cosine of the phase angle ($\text{pf} = \cos \Theta$). Using this relationship, the true power of the motor is then calculated as

$$\text{True Power (kW)} = \text{pf (Apparent Power)} = \sqrt{3} \text{ Volts (Amps) pf} = 1.732 \text{ VA cos } \Theta$$

Sample Problem 9a-1

An 208V 3- ϕ induction motor is measured to draw 8.76 kW true power with a .85 lagging power factor. What is the full load amp draw?

Solution: Amps = True Power / ($\sqrt{3}$ Volts pf) = $8760 / (1.732 \times 208 \times .85) = 28.6$ amps

Section 9a-3 Efficiency

As Figure 9a-2 shows, the kW that supplies the stator and rotor is not all converted into rotational work of the shaft, but rather some is lost in a variety of ways; I^2R loss in the windings of both stator and rotor (if the rotor is wire wound), core losses which amount to eddy current losses, losses due to mechanical friction, primarily in the bearings of the motor, and air friction (windage—the rotor acts as a fan to move air out of the way). Therefore the output power of the motor does not match the input power. Typically the output power is rated in Horsepower and although there is a conversion factor of

$$1 \text{ Hp} = 746 \text{ watts}$$

A motor does not generate Hp at this rate, due to the inefficiencies of the motor. Therefore a motor $\text{Hp} = \text{kW} (746) (\text{eff})$

Sample Problem 9a-2

A 208V, 3 ϕ motor is rated at 12 Hp. Its efficiency is .9. What is the input rating of the motor in kW?

Solution: Hp is the output power: Output power = .746 (12) = 9 kW
The electrical power (input) = output power/eff = 9 / .9 = 10kW

In SI units, Hp is not used and therefore the power input to the motor and the power output to the motor are both rated in watts or kW. Therefore it is necessary to be very clear whether kW is kW input or kW output.

Sample Problem 9a-3

A 208V, 3 ϕ motor is rated at 9 kW output. Its efficiency is .9. What is the input rating of the motor in kW?

Solution: The electrical power (input) = output power/eff = 9 / .9 = 10kW

The true rating of a motor is in Hp or output kW.

The efficiency of the motor in the example was .9, but that is very difficult to determine for a given motor. In fact, it can be done in the lab in two ways: by dynamometer testing and by electrical testing;

Electrical determination of motor efficiency - In this method the kW output = kW input - I^2R losses (copper losses in stator + rotor) - friction/windage - core losses. Each one of these losses are difficult to measure.

Sample Problem 9a-4

A 208V, 3 ϕ motor has an input rating of the motor of 10 kW and draws 8.76 amps. If the winding resistance is 1 Ω , friction/windage losses are 250W, and core losses are 630W, what is the efficiency of the motor?

Solution: The total losses are = $(8.76)^2 (1) + 295 + 630 = 1002$ W
Eff = $(10kW - 1.002kW)/10kW = .9$

Mechanical determination of motor efficiency - In this method, the motor is connected to a prony brake or an electric dynamometer. A wattmeter measures the input power while the prony brake measures torque. Then

$$\text{Hp} = \text{torque (ft-lbs) (rpm)} / 5252$$

Fig 9a-3 Prony brake measures torque by friction



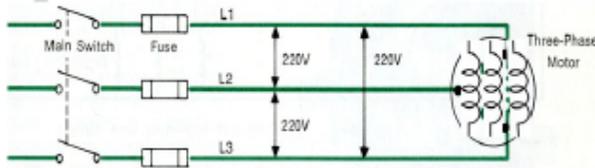
Sample Problem 9a-5

A 208V, 3 ϕ motor draws 10 kW on a wattmeter at full load, which develops 63 ft-lbs of torque on a prony brake at 1000 rpm. What is the efficiency of this motor?

Solution: Hp = $63 (1000) / 5252 = 12$
Eff = output power/ input power = $12 (746) / 10,000 = .9$

Section 9a-4 3- ϕ Motors—Wye and Delta

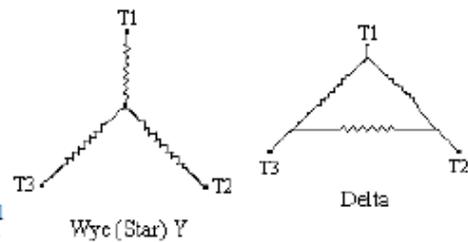
Fig 9a-4 Three phase motor consists of three windings connected to three phase power



Three-phase electric motors are more efficient than single-phase motors and consume less current for a given horsepower rating. Because a three-phase motor requires less amperage than a single-phase motor of the same horsepower rating, the motor can be wired using smaller size conductors.

A three phase motor consists of three windings that are connected to the three phases of electrical power. However, this can be done in two different configurations. The windings can be connected in a star (or Y- wye) connection or in a delta connection.

Fig 9a-5 Three phase motor connections



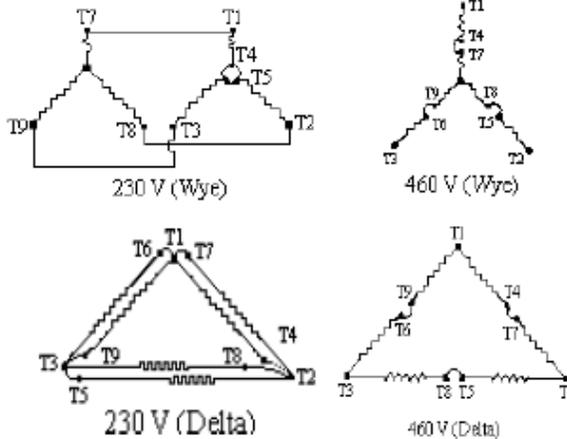
When the windings of a 3-phase motor are connected in STAR:

the voltage applied to each winding is reduced to only $(1/\sqrt{3})$ [1 divided by root three] of the voltage applied to the winding when it is connected directly across two incoming power service lines in DELTA.

the current per winding is reduced to only $(1/\sqrt{3})$ [1 divided by root three] of the normal running current taken when it is connected in DELTA.

so, because of the Power Law V [in volts] $\times I$ [in amps] = P [in watts], the total output power when the motor is connected in STAR is: $P_S = [V_L \times (1/\sqrt{3})] \times [I_D \times (1/\sqrt{3})] = P_D \times (1/3)$ [one third of the power in DELTA] where: V_L is the line-to-line voltage of the incoming 3-phase power service I_D is the line current drawn in DELTA P_S is the total power the motor can produce when running in STAR P_D is the total power it can produce when running in DELTA.

Fig 9a-6 Six winding motors allow for different supply voltages



A further disadvantage when the motor is connected in STAR is that its total output torque is only $1/3$ of the total torque it can produce when running in DELTA.

A motor designed to operate at a single three ϕ voltage (208,230,460) requires only three leads and is suitable for full voltage starting. The internal connections of the motor coils may be wye (Y) (also known as star (A) or delta ()). This type of motor requires no connection diagram because the electrician simply connects the three motor leads (which may be labeled T1, T2 and T3) to the respective starter terminals, which connect to power supply lines, L1, L2 and L3. See the figure below for the connection schematics.

Many OEM's and most distributors like to stock motors that may be used with various power supplies. For this reason we find many motors rated for dual voltage. The most common domestic motor in NEMA frames is the 9 lead, dual voltage motor rated 230/460 volts. Note, the voltage ration is 1:2. For 230 volt operation, the coils are

For example, say you have a motor where all six ends of the three windings are brought out, and that each winding is designed to operate at 230V. If you connect the windings in delta, then the three power leads must supply 230V measured from line-to-line. If instead the three windings are connected in wye, then the three power leads would have to supply $230V \times \sqrt{3} \approx 399V$ measured from line-to-line in order for 230V to be placed across each winding.

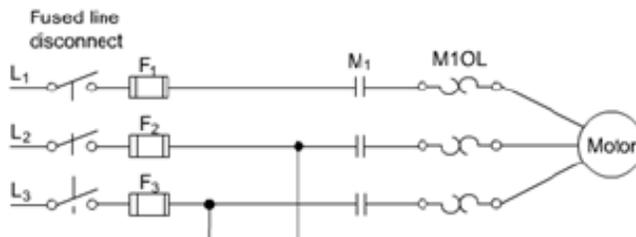
In the US, it is not uncommon to see motors up to around 25HP wye connected, and delta connected above that. It is common to find nine leads which locks your connection into either a wye or a delta right out of the box. The thing to remember is that, for either voltage, you always connect the incoming lines to motor leads one, two, and three. The problem is what to do with the other leads

Wye-Delta motors have six leads which allow the motor winding to be connected in either a wye or a delta configuration. To achieve a reduced voltage start, the windings are connected in the wye, reducing the voltage across each winding to 58% of line voltage. This reduces the starting current and torque approximately 1/3 of full voltage starting values. After starting, the motor windings are reconnected in the delta configuration.

Closed transition starters are provided with an additional contactor and resistor bank used to maintain voltage at the motor terminals during the transition from start to run. A wye-delta starter does not truly reduce the voltage applied to the motor terminals but provides the effect of reduced voltage starting and is classified as such.

Section 9a-4 3- ϕ Motor Starters and Circuits

Fig 9a-7 Schematic of a motor starter

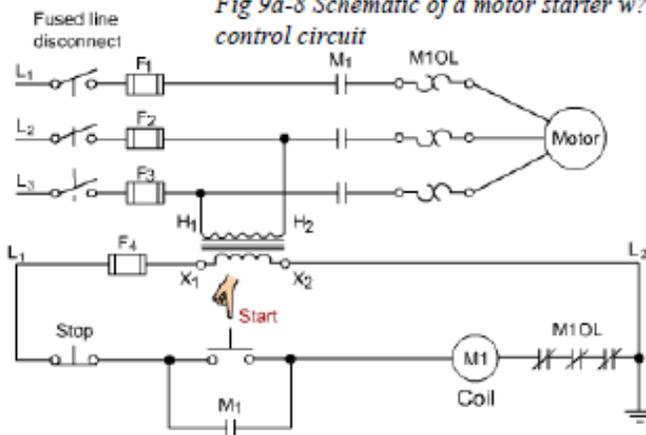


closed by the control section. When closed, current flows between the power and the motor. Overload heaters (resettable fuses) protect a motor by sensing the current going to the motor. Many of these use small heaters, often bi-metallic elements that bend when warmed by current to the motor. When current is too high for too long, heaters open the relay contacts carrying current to the coil of the contactor.

To connect a 3- ϕ motor to the building power supply usually requires a separate enclosure called a motor starter. Typically it consists of a manual switch, called a line disconnect consisting of a single throw, triple pole switch with fuses or circuit breakers and overload heaters between the power line connections and the motor. <http://www.wisc-online.com/objects/ViewObject.aspx?ID=IAU9007>

Normally open contacts labeled M1 are part of each power line and are in open condition until they are

Fig 9a-8 Schematic of a motor starter w/ control circuit



The control circuit is typically created at a lower voltage by a transformer that is built into the motor starter. A very simple control circuits shown in Fig 9a-8 which starts the motor by pushing a simple start button and stops the motor with a push button stop button. The M1OL contacts in the control circuit imply that the heaters not only have contacts in the high voltage section, but also in the control circuit. When an overload trips, the coil M1 drops out and all the power to the motor is cut off. The motor cannot then be turned back on until the overload cools, and even when it cools, the motor is not started automatically but only when the start button is once again pushed.

Section 9a-5 3- ϕ Single Phasing

When a three phase motor is "single phased", it is a power system problem, not a motor problem. A three phase motor needs three EQUAL phases in order to operate properly. When the symmetry of the motor is interrupted by the loss of a phase, the motor will die quickly unless the controls have single phase protection. Many heater type overload relays do not have this.

Single phasing occurs as a result of several possibilities. A loose wire, a bad connection, bad starter contacts, overload relay problems, a bad breaker, a blown fuse, and other things can cause this destructive condition. Obvious signs are a louder than normal humming from the motor and/or a shaft that vibrates rather than rotating.

Testing for this possibility needs to be done quickly since motors are not happy with this condition at all. The obvious test is to look at the current in each phase. This is where multiple meters will help so you can see all three phases at once. You can also look at the voltage, again with multiple meters if possible. I look at the phase to ground readings first. The phase to ground voltage will equal the phase to phase voltage divided by 1.73; thus 480 volts phase to phase will be 277 volts phase to ground. The advantage of taking phase to ground measurements is that each reading is independent of whatever is happening in the other phases. However, you can read phase to phase if you want. You would see an unbalance there too. The phase to ground reading will show you the bad phase, though; this will make troubleshooting easier.

Table 430-7(b). Locked-Rotor Indicating Code Letters

Code Letter	Kilovolt-Amperes per Horsepower with Locked Rotor	Code Letter	Kilovolt-Amperes per Horsepower with Locked Rotor
A	0-3.14	L	9.0-9.99
B	3.15-3.54	M	10.0-11.19
C	3.55-3.99	N	11.2-12.49
D	4.0-4.49	P	12.5-13.99
E	4.5-4.99	R	14.0-15.99
F	5.0-5.59	S	16.0-17.00
G	5.6-6.29	T	18.0-19.99
H	6.3-7.09	U	29.0-22.39
J	7.1-7.99	V	22.4 and up
K	8.0-8.99		

Section 9a-6 AC Locked Motor Amps

For three phase motors, sizing the overload components may require a determination of the locked rotor amperage. NEC code has a table which calculated LRA based on the motor type and the locked rotor KVA.

Sample Problem 9a-6

What is the LRA of a 480V, 400 Hp Type G motor?

Solution:

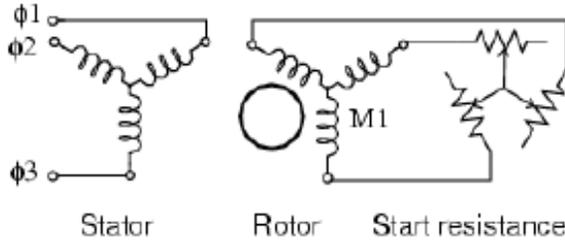
$$\text{LR KVA} = 6 \text{ (ave) KVA/HP (400)} = 2400 \text{ KVA LR}$$

$$\text{or } I_{LR} = \text{KVA}_{LR} / \sqrt{3} \text{ V} = 2400000 / \sqrt{3} (480) = 2890 \text{ LRC}$$

Section 9a-7 Start Resistance and Wire Wound Rotors

Squirrel cage 3- ϕ induction motors draw 500% to over 1000% of full load current (FLC) during starting. While this is not a severe problem for small motors, it is for large (10's of kW) motors. Placing resistance in series with the rotor windings not only decreases start current, locked rotor current (LRC), but also increases the starting torque, locked rotor torque (LRT). Figure 9a-10 shows that by increasing the rotor resistance from R_0 to R_1 to R_2 , the breakdown torque peak is shifted left to zero speed. Note that this torque peak is much higher than the starting torque available with no rotor resistance (R_0). Slip is proportional to rotor resistance, and pullout torque is proportional to slip. Thus, high torque is produced while starting.

Fig 9a-10 Resistance in the windings

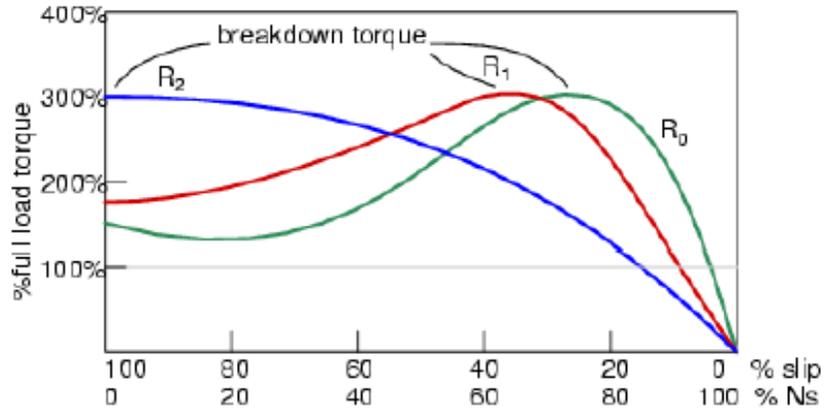


A wound rotor induction motor has a stator like the squirrel cage induction motor, but a rotor with insulated windings brought out via slip rings and brushes. However, no power is applied to the slip rings. Their sole purpose is to allow resistance to be placed in series with the rotor windings while starting. (Figure below) This resistance is shorted out once the motor is started to make the rotor look electrically like the squirrel cage counterpart.

The resistance decreases the torque available at full running speed. But that resistance is shorted out by the time the rotor is started. A shorted rotor operates like a squirrel cage rotor. Heat generated during starting is mostly dissipated external to the motor in the starting resistance. The complication and maintenance associated with brushes and slip rings is a disadvantage of the wound rotor as compared to the simple squirrel cage rotor.

This motor is suited for starting high inertial loads. A high starting resistance makes the high pull out torque available at zero speed. For comparison, a squirrel cage rotor only exhibits pull out (peak) torque at 80% of its' synchronous speed. The equations that were presented for motors all work the same for generators.

Fig 9a-11 Breakdown torque peak is shifted to zero speed by increasing rotor resistance.



Section 9a-3 AC Alternators/Generators

Generators are machines that convert rotational (mechanical) motion into electrical energy (VA). They look very much like the page 1 diagram of a 3- ϕ AC motor, but the rotor is spun by an outside source (gasoline engine, etc). In this case the rotor is called an armature and it consists of wire windings which have voltage and current induced in them (excited) due to moving through a magnetic flux created by the stator, which is called the field windings.

In a generator, an armature is spun inside a magnetic field. The armature consists of several coils which generate electrical energy as they spin through the magnetic fields. Brushes conduct the electrical energy to a wire. The strength of the electrical output can be controlled by varying the strength of the stationary magnetic *field* that is outside the spinning armature.

The equations that were presented for motors all work the same for generators.

Sample Problem 9a-7

What is the maximum current generated from a 3 ϕ generator rated at 100 kVA and 120V.

Solution: For maximum current, assume efficiency and pf = 1. Then

$$\begin{aligned} VA &= \sqrt{3} \text{ Volts (Amps)} \\ \text{and} \quad \text{Amps} &= 100,000 / (1.732)(120) = 481 \end{aligned}$$

An AC alternator is a type of generator where the armature is stationary and the field winding rotate inside. The mathematics is basically the same for an alternator, as for a generator.

The definition of efficiency must be modified to reflect that the output of the generator is electricity, while the input is the horsepower from the driving engine.

$$\text{Eff} = \text{Watts from wattmeter} / \text{Watts from Hp}$$

Also the rating of a generator is either KVA or WATTS but KVA refers to the Apparent POWER and kW refers to the true power. $\text{kW} = \text{KVA} \times \text{pf}$

The following equations apply: The induced or excitation voltage equals the sum of the terminal voltage plus the impedance drop in the generator (alternator).

$$V_E = V_i = V + IZ = V + I(R + jX)$$

$$\begin{aligned} p &= 120 f/p & \text{where: } p &= \text{the number of poles} \\ & & f &= \text{the frequency (Hertz)} \\ & & n &= \text{rpm} = 2 \pi \text{ radians/min.} \end{aligned}$$

Sample Problem 9a-8

An alternator has a 12 V terminal voltage and internal resistance of 3 ohms. If the field current at rated speed is 3 amps, what is the required excitation voltage?

$$\text{Solution: } V_E = V + IZ = 12 + 3 \times 3 = 21 \text{ V}$$

$$\text{Generator output} = \text{kW} = \text{kVA} \cdot \text{power factor (pf)}$$

$$\text{Generator input} = \text{output} + \text{losses}$$

$$= \text{output} + 3I^2fR_e + I_{f(d.c.)}^2 R_{f(d.c.)} + P_{\text{stray power}} + \text{Friction} + \text{Windage}$$

$$\text{Efficiency} = \frac{\text{output}}{\text{input}} = \frac{\text{output}}{\text{output} + \text{losses}}$$

$$\text{Drive shaft torque (ft-lb)} = \text{Hp} \times 5252 / \text{rpm}$$

Lesson 9a Problems

A four-pole 3ϕ induction motor is rated at 500 Hp running at 1750 rpm on 60 hz three-phase power. It draws 125 amps on 2,300 volts.

- 1) What is the synchronous speed of the motor (rpm) ?
(A) 3600 (B) 3550 (C) 1800 (D) 1750
2. What is the slip of the motor (%) ?
(A) 50 (B) 20 (C) 5 (D) 2.77
3. Slip is defined as $\text{slip} = (\text{synchronous rpm} - \text{full load rpm}) / \text{synchronous rpm}$
Another parameter is sometimes used called speed regulation. It is defined almost the same :
$$\text{SR} = (\text{No load rpm} - \text{full load rpm}) / \text{full load rpm}$$

What is the speed regulation of the motor (%)?
(A) 52 (B) 22 (C) 5.4 (D) 2.85
4. What is the torque generated at full speed (ft-lb)?
(A) 125 (B) 250 (C) 375 (D) 1500
5. According to NEC Table 430.250, the typical full load current draw for this motor is 118 amps. If this motor is classified as Type B, what is the locked rotor amps (LRA), according to NEC code?
(A) 815 (B) 615 (C) 415 (D) 215
6. How many times greater than the FLA is the LRA?
(A) 2.0 (B) 3.3 (C) 5.0 (D) 6.5
7. If the power factor of this motor is .90 lagging, what is the implied efficiency of the motor (%)?
(A) 73 (B) 80 (C) 83 (D) 90
8. The footnote on NEC Table 430.250 indicates that if the pf = .9, the amperage from Question 4 must be increase by a factor of 1.1 . Using this value of amperage, what is the assumed efficiency of the 2300V, 500 Hp motor on this Table?
(A) 73 (B) 80 (C) 83 (D) 90
- 9) A four pole, 3- ϕ induction motor is rated at 500 Hp at 1750 rpm (on 60 Hz). It draws 125 amps at 2300 volts at a power factor of .92. What is the full load KVA of the motor, the KVAR, and the efficiency?
- 10) A 50 hp, 440 V, 60 Hz, 3-phase, wye-connected synchronous motor operates at 1725 rpm at full load and 1780 rpm at no load. What is the slip of the motor, in %, at full load? What is the speed regulation at full load?
- 11) In NEC code Table 430.250 (similar to Figure 7a-5), the full-load current from a 100 hp, 460 V three-phase motor is 124 amps. What efficiency does this assume? A) .95 B) .90 C) .85 D) .75

Lesson 9a Problems

- 11) A generator provides 400 VAC at 500 Hz turning at 14,000 rpm under full load.
 (A) How many poles does it have?
 (B) What is the slip (%)?
 (C) What is the speed regulation?
- 12) If the motor in the above problem runs at a .8 power factor (lagging) at full load, what is the kVA demand load for this motor? A) 123.5 B) 247 C) 276 D) 299
- 13) In the above problem, how much capacitance kVAR is needed to bring the total power factor up to .95? A) 247 B) 123.5 C) 41.6 D) 25.4

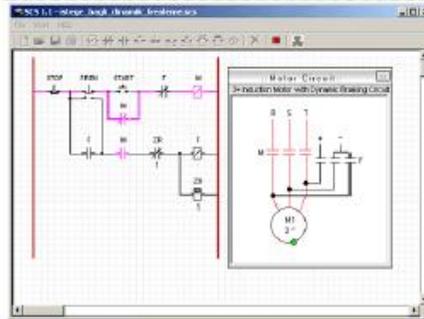
Applications of ELECTRICAL ENGINEERING

LESSON 10a Motor Control Circuits

Preview

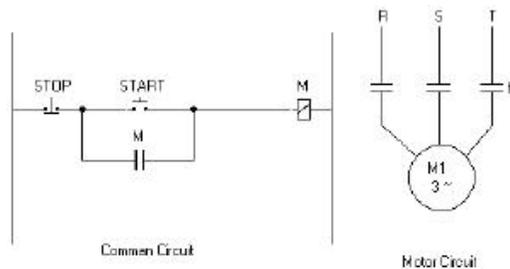
TABLE of SECTIONS

Motor Starting circuits for three phase motors allow the motor to start, stop, and operate based on controls and switches



Section 10a-1 Basic Latch Circuit

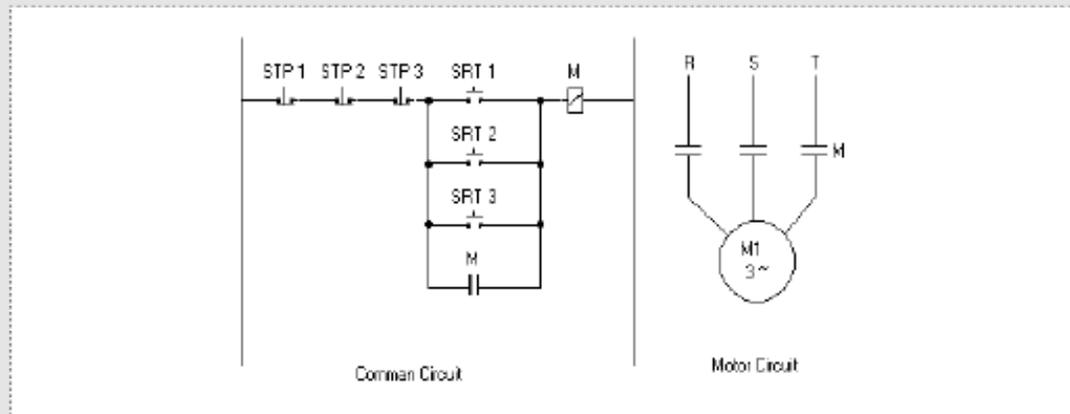
Button lock circuit



This circuit shows how to use relay contact to lock button position. When "START" button pressed "M" relay activates and motor starts to rotate. At the same time one contact of relay "M" locks start button so that current continue to pass through relay "M". Stop button is used to cut the energy from relay "M", which will cut energy from motor.

Section 10a-2 Multiple Latch Circuit

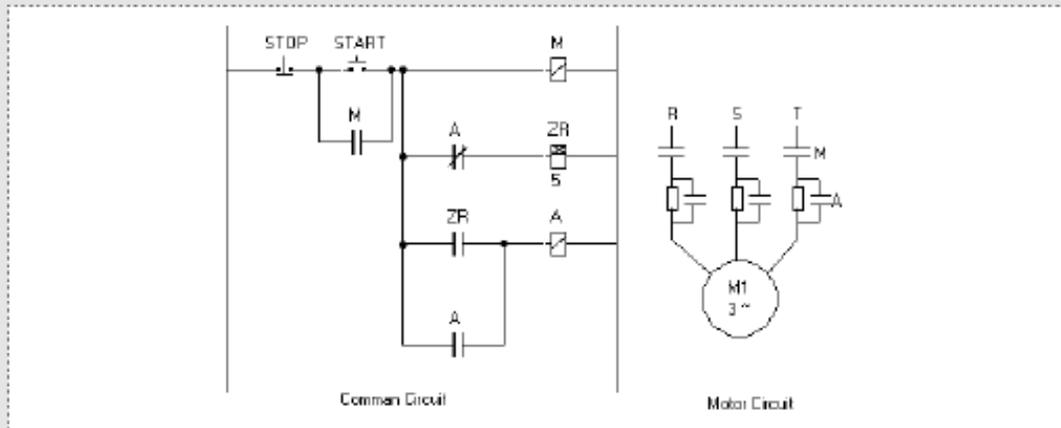
Remote control circuit



This circuit is used to control motor from different locations. All stop buttons connected serially and start buttons connected parallel to each other. We can start and stop motor by using any start and stop buttons from any location.

Section 10a-3 Soft Starting Circuit

Motor Starter circuit with single resistance

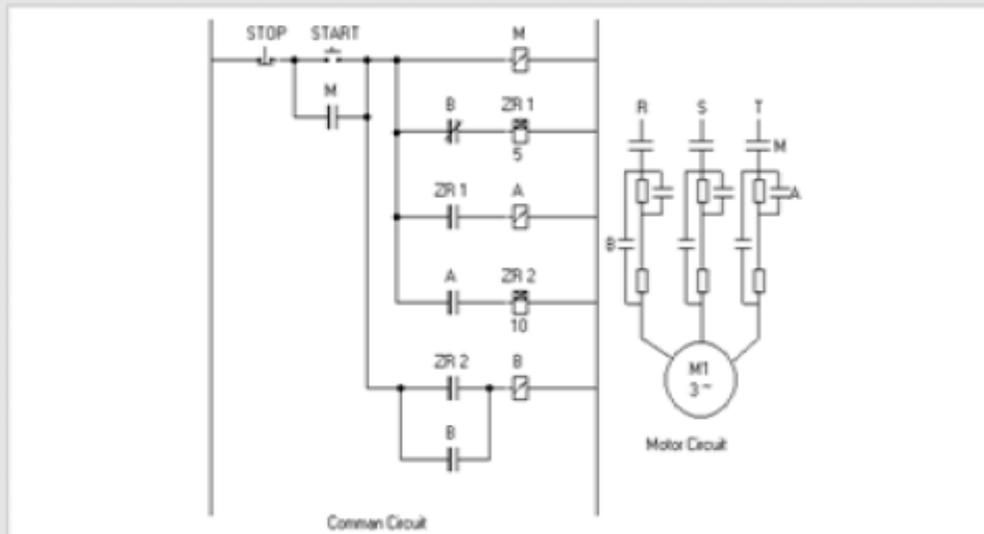


When the stator windings of an induction motor are connected directly to its 3-phase supply, a very large current (5-8 times full load current) flows initially. This surge current reduces as the motor accelerates up to its running speed. When very large motors are started direct-on-line they cause a disturbance of voltage (voltage dip) on the supply lines due to the large starting current surge. This voltage disturbance may result in the malfunction of other electrical equipment connected to the supply. To limit the starting current some large induction motors are started at reduced voltage and then have the full supply voltage reconnected when they have run up to near rated speed.

This circuit is used to start high power induction motors. When we click on start button "M" relay and "ZR" time relay activated. Contactors of relay "M" connect motor to 3 phase power supply and auxiliary contact used to keep relay in power. Motor voltage is reduced by resistances serially connected to motor. Time relay "ZR" starts to count down. When time relay finishes counting, relay "A" is activated by contactor of time relay. Contactors of relay "A" bypass resistances used in motor circuit and motor continues rotating with full supply voltage. Timer of time relay "ZR" must be arranged to count until motor gets to rated speed. Button Stop is used to cut power off from motor and all relays used in command circuit.

Section 10a-4 Three stage Starting

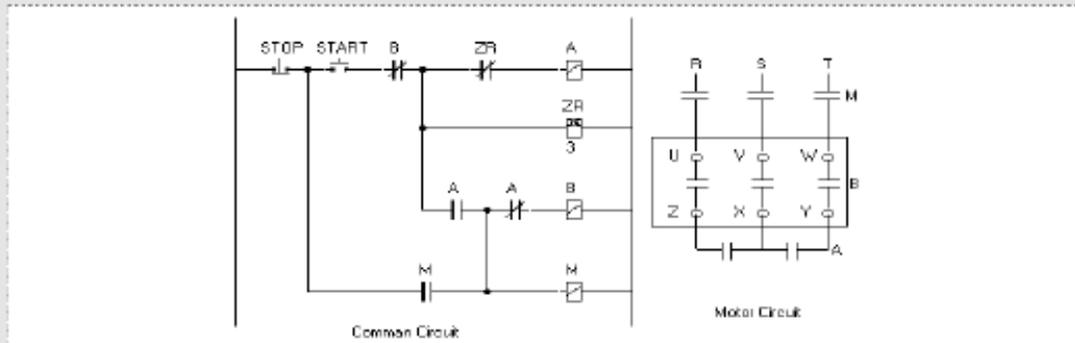
Motor starter circuit with double resistance



High power motors always need starter circuit for starting to work. When we click on start button motor starts to work with low voltage because of usage of two resistances. First timer starts to count to specified time. When timer finishes counting relay "A" activates which used to bypass first resistance in motor circuit and starts second timer. When second timer finishes counting relay "B" activates and both resistances bypassed in motor circuit.

Section 10a-5 Star-Delta Starting Circuit

Star- Delta starter circuit



This circuit is used to start motor working using star connection for some period and then switch to delta connection. This method is used to start high power motors.

When we click on "START" button, "A" relay is activated and its contact activates relay "M", which connects motor to power supply.

Other contacts of relay "A" connect motor windings in star connection. Motor starts rotate using low voltage.

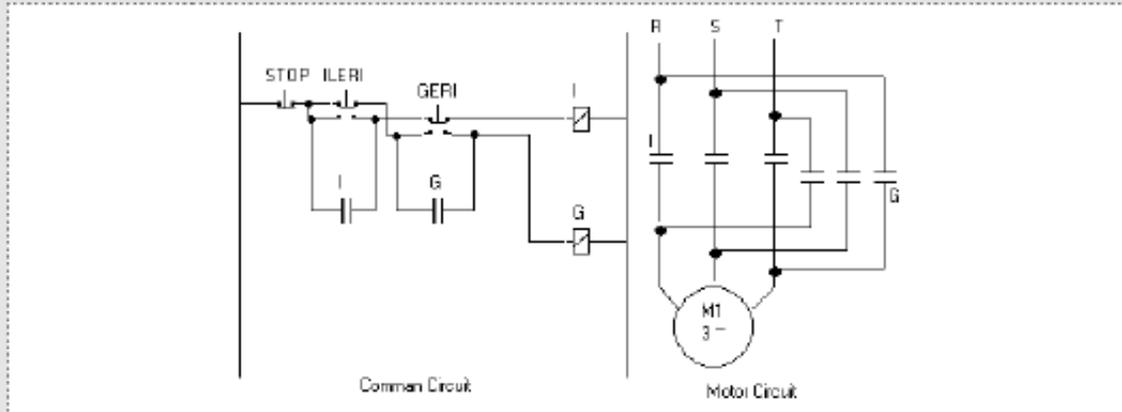
Time relay "ZR" is also activated by "START" button. When time relay finishes counting its normally closed contact activates and relay "A" is deactivated, which activates relay "B".

Relay "B" converts motor connection type to delta. Auxiliary contact of relay "B" deactivates time relay "ZR".

When we click on "STOP" button all relays deactivated and motor circuit cut off power supply.

Section 10a-6 Reversing Motor Circuit

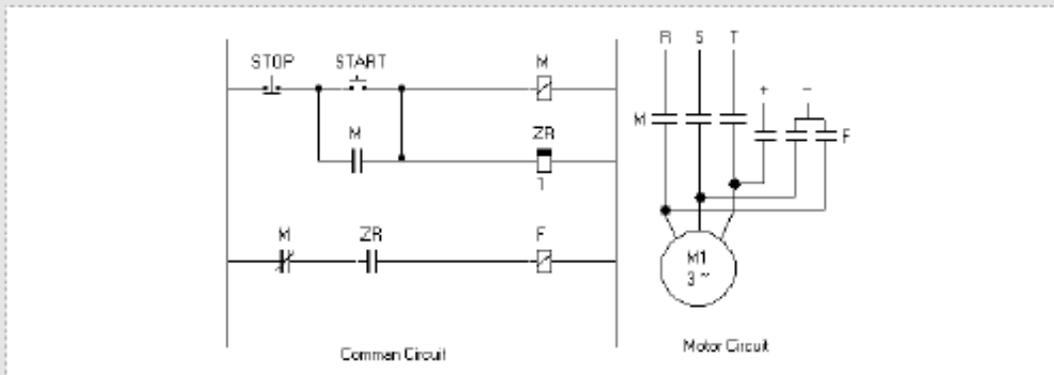
Changing rotation direction with protection lock



We have two buttons to rotate motor in separate directions. "ILERI" button activates Relay "I" which rotates motor forward direction and "GERI" button activates relay "G" which rotates motor in backward direction. At the same time button "ILERI" is used to cut energy from relay "G", in case it was activated, before activating relay "I". This is to prevent a short circuit in the motor circuit. Button "GERI" is also used in the same way. Button "STOP" is used to cut energy from both relays "I" and "G", which cuts energy from the motor.

Section 10a-7 Motor Braking Circuit

Dynamic forced braking



When we cut off energy from the motor, it doesn't stop immediately. It continues rotating while it loses its kinetic energy. To stop the motor immediately, a dynamic braking circuit is used.

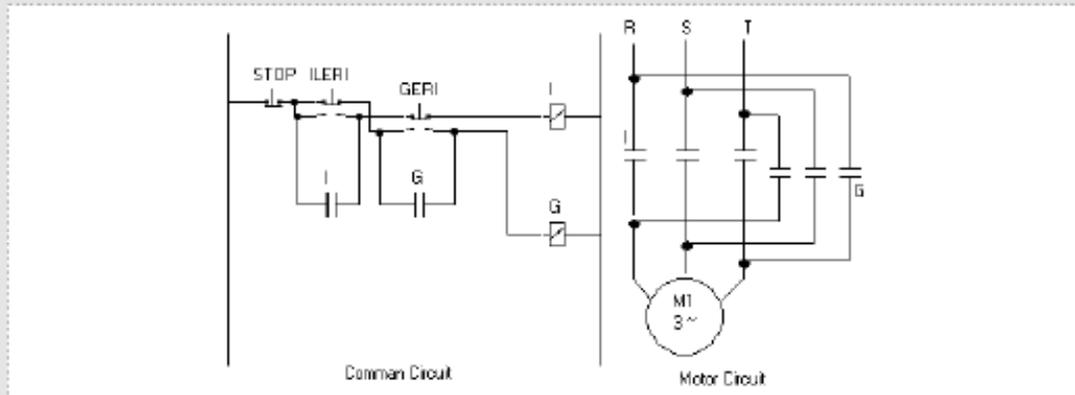
In the command circuit, when we click on the start button, relay "M" and inverse time relay "ZR" are activated. The motor is connected to the power supply by the contacts of relay "M".

When we click the "STOP" button, relay "M" and inverse time relay "ZR" are deactivated. The motor is cut off energy. The counter of the inverse time relay starts to count down. The contact of the inverse time relay "ZR" remains active while the counter counts down. It activates relay "F" which connects the motor to the braking circuit.

When the inverse time relay finishes counting, its contact becomes inactive, that deactivates relay "F". The motor is cut off the braking power supply and remains fully stopped.

Section 10a-6 Reversing Motor Circuit

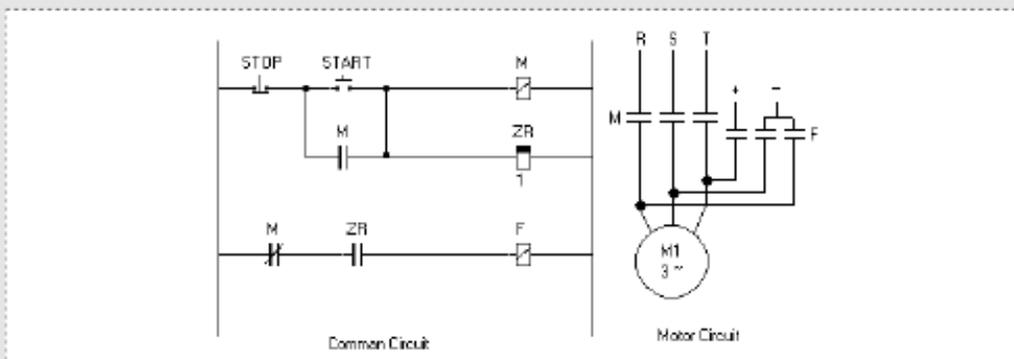
Changing rotation direction with protection lock



We have two buttons to rotate motor in separate directions. "ILERI" button activates Relay "I" which rotates motor in forward direction and "GERI" button activates relay "G" which rotates motor in backward direction. At the same time button "ILERI" is used to cut energy from relay "G", in case it was activated, before activating relay "I". This is preventing from short circuit in motor circuit. Button "GERI" is also used in the same way. Button "STOP" is used to cut energy from both relays "I" and "G", which cuts energy from motor.

Section 10a-7 Motor Braking Circuit

Dynamic forced braking



When we cut off energy from motor it doesn't stop immediately. It continues rotating while it loses its kinetic energy. To stop motor immediately dynamic braking circuit is used.

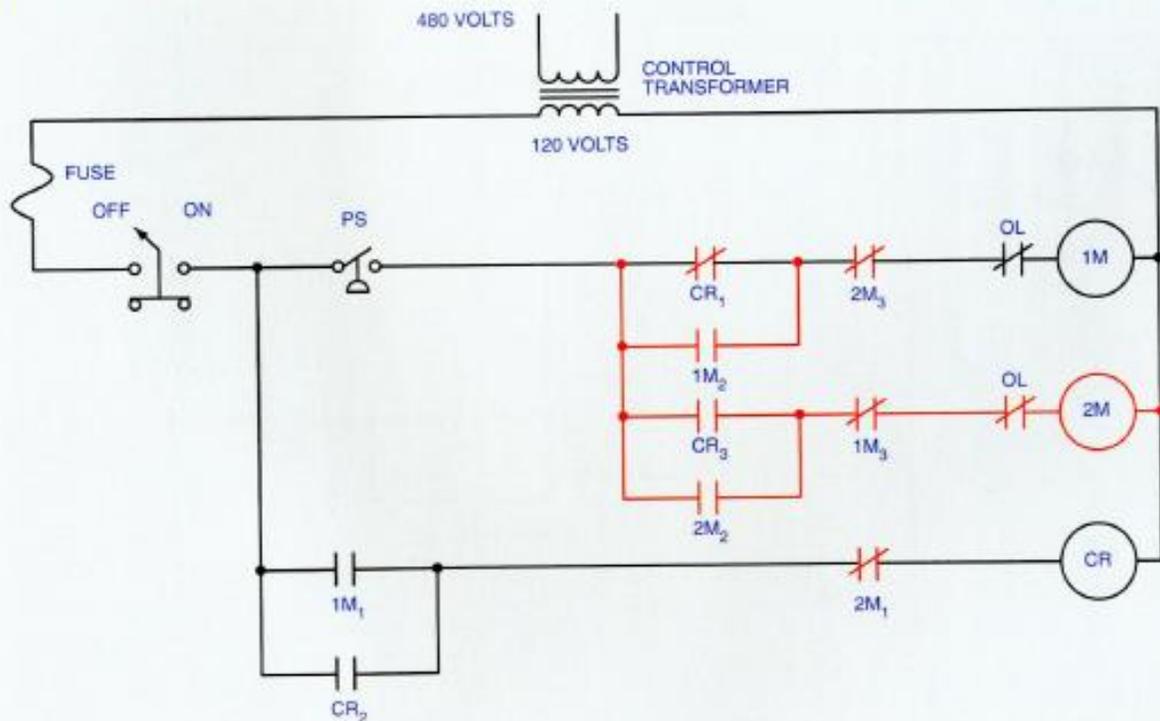
In command circuit when we click on start button relay "M" and inverse time relay "ZR" is activated. Motor connected to power supply by contacts of relay "M".

When we click "STOP" button relay "M" and inverse time relay "ZR" is deactivated. Motor cut off energy. Counter of inverse time relay starts to count down. Contact of inverse time relay "ZR" is remaining active while counter counts down. It activates relay "F" which connects motor to braking circuit.

When inverse time relay finishes counting, its contact became inactive, that deactivates relay "F". Motor is cut off braking power supply and remains fully stopped.

Lesson 10a Problems

- 1) The water for a housing development is supplied by a central tank. The tank is pressurized by the water as it fills. Two separate wells supply water to the tank and each well has a separate pump. Water should be taken from each well equally, but both pumps should not operate at the same time, but rather alternately. Each pump (1M and 2M) operates on a 480V three phase system w/120V control circuit. Each pump motor has an overload protector. If one pump overloads, this will not prevent the second from operating. A manual OFF_ON switch activates the system. A pressure switch mounted on the tank controls operation. When pressure drops to a specific level, one of the pumps will be started. Each time the pressure drops to a low level, the alternative pump will be used.



- How many poles does relay 1M have?
- How many poles does relay 2M have?
- How many poles does relay CR have?
- Immediately after the on-off switch has been turned on and PS is closed, which relays will be activated (energized) - 1M, 2M, CR?
- Why is pump 2 not activated at this time (what switch is open—CR3, 2M2, or 1M3)?
- Motor 1M runs at this time because CR1 is normally closed but the contact immediately opens when CR is energized. Why doesn't 1M immediately shut off?
- What set of contacts 'latches in' 1M?
- After running and then PS opens, is any relay still energized?
- After running, then stopping by PS open, then PS closes again, why isn't 1M energized?
- Which relay 'remembers' which pump ran last?
- Make a truth table for the CR relay.

Motor Control – Jim Pytel

Introduction to Electrically Controlled Systems (Full Lecture)

Jim Pytel 58:14

Build a Motor Control Trainer Board, Plug, and Motor Mount

Jim Pytel 15:07

Contactors (Full Lecture)

Jim Pytel 28:56

Control Relays (Full lecture)

Jim Pytel 26:27

Solenoid Operated Valves (Full Lecture)

Jim Pytel 26:18

Overload Relays (Full Lecture)

Jim Pytel 12:07

Fuses and Circuit Breakers (Full Lecture)

Jim Pytel 20:52

Install a Circuit Breaker

Jim Pytel 5:33

Switches in Electrically Controlled Systems (Full Lecture)

Jim Pytel 48:49

Orientation to the Basic Motor Control Kit

Jim Pytel 19:42

Basic Ladder Logic (Full Lecture)

Jim Pytel 36:05

Ladder Logic Documentation (Full Lecture)

Jim Pytel 38:46

Install a Control Transformer

Jim Pytel 8:34

Alarm Circuit (Full Lecture)

Jim Pytel 10:44

Wiring an Alarm Circuit

Jim Pytel 16:28

Manual and Manual Reversing Motor Starters (Full Lecture)

Jim Pytel 9:08

Install a Manual Motor Starter

Jim Pytel 11:00

Install a Manual Reversing Motor Starter

Jim Pytel 4:48

Troubleshooting a Manual Motor Starter

Jim Pytel 13:32

2 and 3 Wire Magnetic Motor Starters (Full Lecture)

Jim Pytel 19:55

2 and 3 Wire Control Circuits for Fluid Power Systems (Full Lecture)

Jim Pytel 11:29

HAND-OFF-AUTO Circuits (Full Lecture)

Jim Pytel 7:54

Wiring a Two Wire Magnetic Motor Starter HAND-OFF-AUTO Circuit

Jim Pytel 20:27

Troubleshooting a Two Wire Magnetic Motor Starter HAND-OFF-AUTO Circuit

Jim Pytel 13:31

Pumping Circuit Examples (Full Lecture)

Jim Pytel 25:48

Multiple Push Button Stations (Full Lecture)

Jim Pytel 13:23

Wiring a Three Wire Magnetic Motor Starter

Jim Pytel 16:06

Troubleshooting a Three Wire Magnetic Motor Starter

Jim Pytel 14:40

Friction Brakes (Full Lecture)

Jim Pytel 8:39

Motor Starters with Jogging (Full Lecture)

Jim Pytel 21:14

Reversing Motor Starters with Interlocks (Full Lecture)

Jim Pytel 22:29

Reversing Motor Starter Examples (Full Lecture)

Jim Pytel 24:48

Plugging Circuits (Full Lecture)

Jim Pytel 31:06

Wiring a Magnetic Reversing Motor Starter with Interlocks

Jim Pytel 15:53

Troubleshooting a Magnetic Reversing Motor Starter with Interlocks

Jim Pytel 11:29

Reversing Motor Starters with Jogging (Full Lecture)

Jim Pytel 18:21

Megohmmeters (Full Lecture)

Jim Pytel 16:19

Primary Resistors Reduced Voltage Starters (Full Lecture)

Jim Pytel 17:56

Part Winding Reduced Voltage Starters (Full Lecture)

Jim Pytel 16:04

Y Start-Delta Run Reduced Voltage Starters (Full Lecture)

Jim Pytel 21:57

Soft Starters (Full Lecture)

Jim Pytel 22:42

Reduced Voltage Starters with Friction Brakes (Full Lecture)

Jim Pytel 19:32

Timers: On Delay (Full Lecture)

Jim Pytel 40:01

Timers: Off Delay (Full Lecture)

Jim Pytel 34:19

Reduced Voltage Starters with Timers (Full Lecture)

Jim Pytel 22:18

Timers: On and Off Delay (Full Lecture)

Jim Pytel 34:39

Timers: Flash/Repeat/Recycle (Full Lecture)

Jim Pytel 22:38

Timers: One Shots (Full Lecture)

Jim Pytel 42:05

DC Injection Braking (Full Lecture)

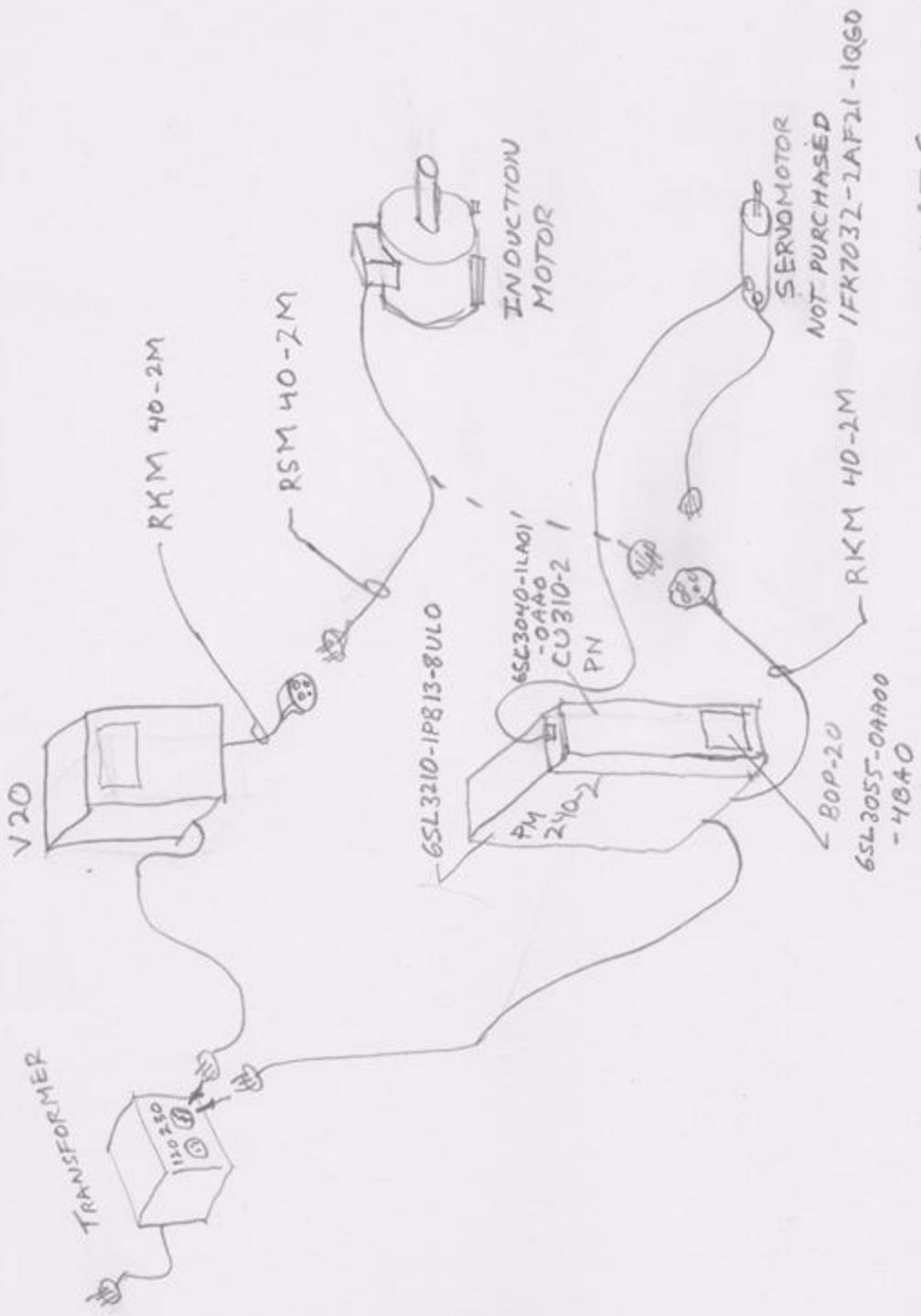
Jim Pytel 12:43

Single Cycle Reciprocation with Dwell Period (Full Lecture)

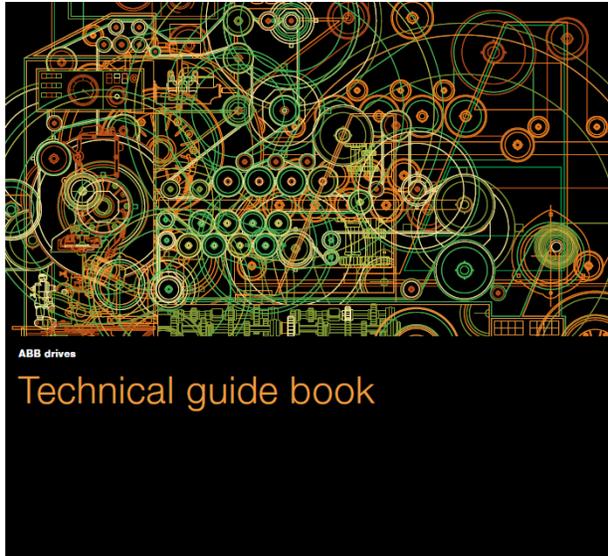
Jim Pytel 16:07

Timers: Cumulative On Delay (Full Lecture)

Jim Pytel 26:56



PROPOSED UofT Siemens
 TRAINER DIAGRAM
 T. WATSON 3/26/19



Contents

1. Direct torque control explains what DTC is; why and how it has evolved; the basic theory behind its success; and the features and benefits of this new technology.	1
2. EU Council Directives and adjustable speed electrical power drive systems is to give a straightforward explanation of how the various EU Council Directives relate to power drive systems.	2
3. EMC compliant installation and configuration for a power drive system assists design and installation personnel when trying to ensure compliance with the requirements of the EMC Directive in the user's systems and installations when using AC drives.	3
4. Guide to variable speed drives describes basics of different variable speed drives (VSD) and how they are used in industrial processes.	4
5. Bearing currents in modern AC drive systems explains how to avoid damages.	5
6. Guide to harmonics with AC drives describes harmonic distortion, its sources and effect, and also distortion calculation and evaluation with special attention to the methods for reducing harmonics with AC drives.	6
7. Dimensioning of a drive system. Making dimensioning correctly is the fastest way of saving money. Biggest savings can be achieved by avoiding very basic mistakes. These dimensioning basics and beyond can be found in this guide.	7
8. Electrical braking describes the practical solutions available in reducing stored energy and transferring stored energy back into electrical energy.	8
9. Guide to motion control drives gives an overview of high performance drives and motion control.	9
10. Functional safety guide introduces the Machinery Directive and the standards that must be taken into account when designing a machine, in order to ensure operational safety.	10

Is direct torque control accepted as superior to flux vector control of ac motors?

ChatGPT said:

Good question — and the short answer is “**it depends on the application.**”

Let’s break down how **Direct Torque Control (DTC)** compares with **Field-Oriented Control (FOC)** (also called **Flux Vector Control**) and when each might be considered “superior.”

⚙️ 1. Basic difference

Feature	Direct Torque Control (DTC)	Field-Oriented Control (FOC / Flux Vector Control)
Control variables	Directly controls torque and flux	Controls stator current components (torque and flux-producing)
Switching	Uses a hysteresis comparator + switching table	Uses PWM with modulation based on reference voltages
Computation	No coordinate transformation or PWM generation needed	Requires transformations (Clarke/Park) and a modulator
Response time	Very fast (microseconds)	Slightly slower (due to modulation stage)
Torque ripple	Higher (in classic DTC)	Lower, smoother torque
Steady-state accuracy	Somewhat lower	Very high
Parameter sensitivity	Less dependent on motor parameters	More dependent on accurate motor model

📊 2. Performance comparison

- **Dynamic performance:**

DTC is generally *faster* — it reacts almost instantly to torque commands. That’s why it’s popular in **high-dynamic drives**, such as traction systems or robotics.

- **Steady-state precision:**
FOC usually provides *smoother torque* and *better low-speed control*, making it preferred in applications like **machine tools** or **positioning systems**.
 - **Implementation complexity:**
DTC is *simpler in concept* (fewer transformations), but its *switching logic* and *torque ripple management* require high-speed DSPs and careful tuning. FOC is *more mathematically complex* but very well understood and widely supported in hardware.
-

3. Industry view (2020s–2025)

- DTC, pioneered by ABB in the 1990s, was promoted as “superior” because of its response time and reduced parameter dependence.
 - However, **modern FOC implementations with fast processors and high-frequency PWM** have closed most of that gap.
 - Today, **DTC is considered superior for dynamic torque response**, but **FOC is still the dominant standard** in general-purpose industrial drives.
-

4. Summary

Criterion	“Winner”
Torque response speed	DTC
Torque smoothness	FOC
Low-speed accuracy	FOC
Computational simplicity	DTC
Parameter robustness	DTC
General industrial use	FOC
High-performance motion / traction	DTC

Conclusion:

DTC isn’t universally accepted as “superior” — it’s **better for certain fast-response applications**,

while **FOC remains preferred** for most precision or low-speed control tasks. The line between them is blurring as modern digital controllers improve.

Which company is leading in the FOC control

ChatGPT said:

There isn't a *single* company universally accepted as the leader in solutions for Field-Oriented Control (FOC), because the ecosystem spans motor drives, microcontrollers/ICs, algorithms and system integration. However, several firms stand out for their strong capabilities and market influence in FOC and vector control. Here are a few of the top names, and what makes them important.

Key companies

STMicroelectronics

- They explicitly promote FOC in their motor-control portfolio for AC / 3-phase motors. [STMicroelectronics](#)
- Their MCUs and motor-control ecosystems (hardware + software) are broadly used in industrial and consumer applications. [Dataintel](#)
- So: ST is a **major player** in the hardware + software enabling FOC.

Infineon Technologies

- They develop motor-control ICs dedicated to FOC / vector control, e.g., their "iMOTION" motor-control family. [EEPower+1](#)
- Their strengths are in power electronics & motor drives (particularly for high-efficiency, high-performance systems). [Dataintel](#)
- Therefore Infineon is another strong leader especially from the drive/IC side.

NXP Semiconductors & Renesas Electronics

- These two also get mentioned as "key players" in the FOC control-chip / motor-control space. [Dataintel+1](#)
 - While perhaps secondary to ST/Infineon in some metrics, they are meaningful in specific application niches (automotive, appliances, industrial).
-

Which one is *leading*?

If I were to pick a “leader” given current publicly-available information, I’d say STMicroelectronics **edges out** as the one with the broadest ecosystem and reference-design support for FOC in many applications (industrial drives, appliances, HVAC, etc.). Infineon is very strong, particularly from the high-power drive side. But “leadership” can depend on your specific domain:

- Are you working in **industrial drives (hundreds of kW)**? Then power electronics + high-performance controllers matter ⇒ Infineon or big drive manufacturers.
- Are you working in **embedded control / MCU + algorithmic** side (small/medium motors, sensors, IoT) ? Then ST has strong position.
- Are you working in **automotive / EV / traction drives**? Then other players or specialised companies may take precedence.

Summary answer

So: yes, FOC is well-established and several companies lead in enabling it. If you want **one** name: **STMicroelectronics** is a strong candidate for the most broadly influential in FOC solutions at this time.

If you tell me *which application* (industrial drives, servo motors, HVAC, EV/traction, etc.) you’re interested in, I can identify **the leader in that specific niche** for FOC

what drive technology does mitsubishi use for ac vfd

ChatGPT said:

Mitsubishi FR-A800 Series VFD is one of the latest variable-frequency drives (VFDs) from Mitsubishi Electric Corporation — here’s a breakdown of the drive technology they use (and how it stacks up).

✅ Key Drive-Technologies Used by Mitsubishi

Here are some of the important control methods and features that Mitsubishi builds into its AC VFDs:

1. **Magnetic Flux Vector Control / Vector Control**

- For example, the FR-A800 series specifies “Advanced magnetic flux vector control” with wide speed-range performance (1:120 in some specs) and high starting torque. [electropower+4MITSUBISHI ELECTRIC UNITED STATES+4Mitsubishi Electric+4](#)
 - Earlier series (like the V/F or vector control drives) mention “digital flux vector controls achieve high response (200 rad/sec) over a wide (1500:1) speed range”. [VFDs](#)
- 2. Sensorless Vector Control (“Real Sensorless Vector (RSV)”)**
- The FR-A800 description mentions “Real sensorless vector control” which allows good performance without the need for a shaft encoder. [Mitsubishi Electric+2MITSUBISHI ELECTRIC UNITED STATES+2](#)
 - With this they claim ability for “0.3 Hz to 60 Hz” driving in some cases. [MITSUBISHI ELECTRIC UNITED STATES+1](#)
- 3. V/f Control (Volts per Hertz)**
- For simpler/less demanding applications, Mitsubishi still supports basic V/f control (voltage to frequency control) for induction motors. [MITSUBISHI ELECTRIC UNITED STATES+2Mercon+2](#)
 - They also mention “square V/f curve” and “linear V/f curve” settings for certain load types (variable torque vs constant torque). [MITSUBISHI ELECTRIC UNITED STATES](#)
- 4. Optimum Excitation Control (OEC) and Flux Optimisation**
- Their product literature mentions “Flux optimisation ensures that the connected motor only gets exactly the amount of magnetic flux required for optimum efficiency.” [Mitsubishi Electric](#)
 - Also “Advanced optimum excitation control ... high starting torque and rapid acceleration ... without the need for complex parameter adjustments.” [MITSUBISHI ELECTRIC UNITED STATES](#)
- 5. Permanent Magnet (PM) Motor Support**
- Some of their VFDs support PM motors (in addition to standard induction motors). For instance the F800 series supports both AC induction and permanent magnet motors. [Automate+1](#)
- 6. Wide Speed/Low Speed Performance**

- The FR-A800 series rates in some cases: starting torque of 200% at low frequencies and speed control ranges of 1 : 120 (or higher) when using vector control. [MITSUBISHI ELECTRIC UNITED STATES+1](#)
-

What this means in plain terms

- Mitsubishi uses **vector control** (flux vector / magnetic flux vector) and **sensorless vector control** in their higher-performance drives. This is aligned with modern best practice for AC drives when you need good torque at low speeds, precision, etc.
 - For less demanding loads (pumps, fans, simple induction motors) they still offer **V/f control**, which is simpler, cheaper, and adequate.
 - The adoption of “optimum excitation” and flux optimisation means they’re trying to improve *efficiency* (less wasted flux, less losses) as well as performance.
 - The support for PM motors as well as induction motors means they’re addressing applications that use permanent-magnet rotor machines (which are more and more common).
 - Because they offer sensorless vector control, you don’t always need a separate encoder, which reduces cost/complexity.
 - For heavy duty / high performance uses (cranes, conveyors, multi-axis) the FR-A800 is rated with high capability, showing that Mitsubishi positions these drives for serious applications, not just simple HVAC/fan loads.
-

Comparison to some other drive-technologies (for context)

- Vector control (flux vector) is generally **higher performance** than basic V/f control (especially for low speed, torque, dynamic response). Mitsubishi’s offering aligns with that.
- Sensorless vector control removes the need for physical feedback but still retains many of the benefits of vector control; Mitsubishi’s “Real sensorless vector” is a sign they’re using that.
- Efficiency improvements: Mitsubishi emphasises flux optimisation and optimum excitation — these help reduce losses compared to older drives that simply ran fixed flux or were tuned more roughly.

- Support for PM motors: Many drives were historically built for induction motors only; Mitsubishi is showing support for PM machines, which require appropriate control (e.g., high dynamic range).
-

✓ Summary

In short: yes — Mitsubishi drives use **advanced vector control technologies** (magnetic flux vector, sensorless vector) as well as simpler V/f control, depending on the model and application. Their high-end drives (e.g., the FR-A800) represent the “top tier” of their technology in terms of response, low-speed torque, flexibility, and efficiency.

Mitsubishi’s detailed control algorithm names and how they compare to competitors’ (e.g., ABB, Siemens, Yaskawa drives) so you can see “how they stack up.” Would you like that?

Here’s a comparison of control-algorithms (or branded control-technologies) used by major drive manufacturers — including Mitsubishi Electric Corporation, ABB Group, and Siemens AG — for AC VFDs (variable-frequency drives). This gives you a sense of how they stack up, what terminology they use, and what features they emphasise.

Mitsubishi Electric

Key algorithm/branding names:

- “Real Sensorless Vector Control” — Mitsubishi’s sensorless vector control mode. [MITSUBISHI ELECTRIC UNITED STATES+2Scribd+2](#)
- “Advanced Magnetic Flux Vector Control” — Mitsubishi’s higher-end vector control variant. [MITSUBISHI ELECTRIC UNITED STATES+1](#)
- “Vector Control” (with encoder) — full vector control when encoder feedback is used. [Scribd+1](#)
- “V/f control” (Volts/Frequency) — the simpler scalar control mode, available on the same drives. [MITSUBISHI ELECTRIC UNITED STATES+1](#)

Highlights / features:

- They emphasise very wide speed/speed-range: e.g., for the FR-A800 series “Real sensorless vector control” allows 0.3 Hz to 60 Hz and starting torque ~200 % at 0.3 Hz. [MITSUBISHI ELECTRIC UNITED STATES+1](#)

- The user manual indicates there are different “control methods” you set via parameters: V/f, Advanced magnetic flux vector, Real sensorless vector, Vector control, PM sensorless vector control. [Manualsnet+1](#)
- They emphasise “Optimum excitation control” (flux optimisation) to improve efficiency. [Mitsubishi Electric+1](#)

In short: Mitsubishi uses a tiered approach: basic scalar (V/f) → sensorless vector → full vector (with encoder) → advanced versions (e.g., for PM motors), with branding that emphasises “Real Sensorless” and “Flux Vector”.

ABB

Key algorithm/branding names:

- “Direct Torque Control (DTC)” — ABB’s premium control technology for high performance. [ABB Group+1](#)

Highlights / features:

- With DTC, ABB says: controls motor flux *and* torque directly, rather than indirectly via current control. [ABB Group](#)
- Claims include ultra-fast dynamic response (torque response time down to ~1 ms in some applications) and very good low-speed performance, often without encoder feedback. [tekcome.com](#)
- The premium drives (e.g., ACS880, ACS580 series) emphasise this advanced control. [tekcome.com](#)

In short: ABB’s offering emphasises the highest performance vector control technology (DTC), focusing on fast response, low speed torque, encoder-less performance.

Siemens

Key algorithm/branding names:

- “Vector control” / “Closed-loop vector control” (with encoder) / “Sensorless vector control” (without encoder) — In their SINAMICS family. [Siemens Industry+2Siemens Industry Support+2](#)

- “V/f control” (Volts/Frequency) and “FCC” (Flux-compensated V/f) for simpler applications. [ManualShelf+1](#)

Highlights / features:

- Their manuals note that in vector control (field-oriented control) the motor model must accurately estimate the rotor field position to allow independent control of flux and torque. [Siemens Industry+1](#)
- They clearly state that vector control improves rise times, better response to set-point changes and load disturbances compared to V/f. [Siemens Industry Support](#)
- They offer both encoder and encoderless vector control, and note when encoderless is appropriate: e.g., for lower speed accuracy, smaller speed range, when torque < certain threshold. [Manuals++1](#)

In short: Siemens uses standard vector control (FOC) terminology, provides both encoder and encoder-less versions, and positions this technology between basic scalar and full servo/position control.

Comparative Summary

Here’s a table summarising how they compare:

Manufacturer	Primary Control Technology	Encoder / Sensorless options	Application Focus
Mitsubishi	Vector control (sensorless & with encoder) + flux optimisation + V/f	Yes (sensorless & encoder)	Wide range: from general industrial to high performance drives (motors & PM motors)
ABB	Direct Torque Control (DTC)	Strong emphasis on encoder-less in premium drives	High performance, dynamic loads, low-speed precision, heavy industrial
Siemens	Field-Oriented/Vector Control (FOC) + V/f	Both sensorless & with encoder	Broad industrial applications: general to high performance, varying speed/torque demands

My take (and what it means for you)

- If you need **highest dynamic performance**, very fast torque response, and perhaps encoder-less control at low speed, ABB's DTC stands out.
- If you need good performance but also wide applicability and a balanced cost / performance budget, Mitsubishi's sensorless vector + full vector options are very solid.
- If you already use Siemens drives (e.g., SINAMICS) or standardise on Siemens infrastructure, their vector control approach is very mature and widely supported.
- Pay attention not just to the algorithm name but to **what the drive can actually do**: starting torque, speed range, encoder vs no encoder, tuning requirements, stability under load changes. The algorithm branding gives a clue but you still need the specs.

Examples of applications using the cam instruction from the S7-1200 PLC from Siemens

ChatGPT said:

Sure — Siemens **S7-1200 “CAM” instruction** (found in **T-technology motion control functions**) is used for **electronic camming**, where a **slave axis** follows a **master axis** according to a pre-defined **cam profile**. It replaces mechanical cams and enables precise motion synchronization in automation systems.

Here are **some practical application examples** using the **CAM instruction** on an **S7-1200 with a technology object (TO)** (for motion control):

1. Packaging Machine – Sealing Jaw Synchronization

Description:

A rotary sealing jaw must close exactly when a film with a product passes beneath it. The sealing jaw (slave axis) must match the film feed (master axis).

How CAM is used:

- The **master axis** is the film feed roller.
 - The **slave axis** is the sealing jaw motor.
 - A **cam table** defines angular synchronization: the jaw rotates fast through the sealing zone, slows outside it.
 - CAM instruction ensures perfect timing even if line speed changes.
-

2. Labeling Machine – Rotary Label Dispenser

Description:

A rotating label applicator needs to align with moving bottles or packages on a conveyor.

How CAM is used:

- Master = conveyor encoder (speed of bottles).
- Slave = servo driving label roll.
- Cam table defines label roll speed vs. conveyor position.
- CAM automatically adjusts label placement as conveyor speed varies.

3. Carton Erector – Folding Arm Motion

Description:

Arms fold and push cartons as the conveyor moves.

The folding arm must move in a non-linear pattern relative to conveyor position.

How CAM is used:

- Master = conveyor axis.
- Slave = folding arm servo.
- Cam profile defines arm position over one carton cycle.
- Smooth motion curve prevents damage or misfolding.

4. Cut-to-Length Machine

Description:

Material runs continuously through rollers, and a flying knife must cut at exact lengths while moving with the material during the cut.

How CAM is used:

- Master = feed roller encoder.
- Slave = flying knife axis.
- CAM profile defines acceleration, match speed, cut, return.
- CAM ensures the knife tracks the material precisely during the cut.

5. Pick-and-Place Unit with Rotary Synchronization

Description:

A pick-and-place arm moves synchronously with a rotating table.

How CAM is used:

- Master = rotary indexing table.
- Slave = pick arm servo.

- Cam table defines pickup and place positions relative to master angle.
 - CAM maintains timing alignment even if the master speed changes dynamically.
-

6. Print Registration / Print Mark Tracking

Description:

In printing or packaging, a roller must adjust rotation to maintain alignment with print marks.

How CAM is used:

- Master = web feed encoder.
 - Slave = print cylinder servo.
 - CAM table defines position correction curve.
 - CAM enables phase correction without stopping the process.
-

Siemens TIA Portal Example

In **TIA Portal (V17 or later)**, a typical implementation for an S7-1200T includes:

- Configure **TO_MA (Master Axis)** and **TO_SA (Slave Axis)**.
 - Define **CAM Table** in Motion > Cam Editor (Position vs. Position pairs).
 - Use instructions:
 - MC_CamTableSelect
 - MC_CamIn
 - MC_CamOut
 - MC_CamPrepare
 - The **CAM instruction** synchronizes the slave axis according to the selected cam table.
-

If you'd like, I can provide a **sample ladder or structured text (ST)** example using a simple **flying shear** or **labeling cam** setup for the S7-1200T — would you like that?

You said:

do you have some more examples

ChatGPT said:

Absolutely — Siemens' **CAM (electronic camming)** function in the S7-1200 and S7-1500 "T" motion CPUs is used across many motion control applications where one motion profile must precisely follow another, often with variable or non-linear relationships.

Here are **additional real-world examples** grouped by industry:

1. Food & Beverage Applications

A. Biscuit or Cookie Cutting Machine

- **Master:** Conveyor or dough feed roller
- **Slave:** Rotary cutter or guillotine knife
- **Use of CAM:**
 - Knife speed and position are synchronized with dough feed.
 - The cam defines acceleration before cut, constant speed during cut, and deceleration after.
 - Allows for consistent shape and spacing at variable conveyor speeds.

B. Bottle Filler or Capper

- **Master:** Star wheel or conveyor transporting bottles
 - **Slave:** Filling nozzle, capping head, or rotary turret
 - **CAM Purpose:**
 - Aligns fill or cap motion to bottle position.
 - Cam defines dwell times (for filling) and retraction profiles (for capping).
-

2. Packaging & Printing

A. Film Feed with Dancer Roller

- **Master:** Film unwind or printing roller

- **Slave:** Dancer or tension control roller
- **CAM Role:**
 - Synchronizes film tension correction to master roller angle.
 - Compensates for speed variation using a sinusoidal cam curve.

B. Shrink Wrapper

- **Master:** Conveyor or infeed screw
 - **Slave:** Sealing bar or film wrapping axis
 - **CAM Role:**
 - Defines precise timing for the bar to descend when a product arrives.
 - Allows dynamic speed change without retuning.
-

3. Building Materials / Metals

A. Wood Saw or Cut-to-Length Line

- **Master:** Feed roller moving wood or metal
- **Slave:** Flying saw carriage
- **CAM Role:**
 - Carriage follows material speed, performs a cut, and returns.
 - Cam curve defines move-in, synchronized cut, and return phases.

B. Tube Bender

- **Master:** Rotary axis controlling bend angle
 - **Slave:** Feed axis pushing the tube
 - **CAM Role:**
 - Feed rate varies non-linearly with bend angle.
 - Cam ensures uniform bend radius and prevents wrinkling.
-

4. Automotive & Assembly

A. Robotic Assembly Press

- **Master:** Conveyor carrying components
- **Slave:** Press or torque driver
- **CAM Role:**
 - Synchronizes press engagement with part position.
 - Defines position-dependent speed profile for gentle contact.

B. Paint Line Conveyor

- **Master:** Main conveyor
 - **Slave:** Oscillating spray gun arm
 - **CAM Role:**
 - The cam defines arm oscillation amplitude tied to conveyor position, ensuring uniform coverage.
-

5. Printing, Textile, and Paper

A. Yarn Winder

- **Master:** Drum rotation
- **Slave:** Traverse guide motor
- **CAM Role:**
 - The traverse follows a sine-wave cam to distribute yarn evenly on bobbin.
 - Cam defines reversing motion at end points.

B. Paper Folder

- **Master:** Roller pulling paper
- **Slave:** Folding blade
- **CAM Role:**
 - Blade movement synced to roller rotation for accurate fold timing.
 - Cam curve defines smooth approach, fold, and retract motion.

⚙️ 6. Custom Machinery / Robotics

🏠 A. Battery Assembly Line

- **Master:** Indexing conveyor
- **Slave:** Pick-and-place gantry
- **CAM Role:**
 - Gantry accelerates and decelerates in sync with conveyor index cycle.
 - Cam table ensures coordinated movement without collisions.

🧠 B. Servo Press Profile Control

- **Master:** Encoder simulating crankshaft
- **Slave:** Servo actuator controlling slide
- **CAM Role:**
 - Cam defines mechanical linkage simulation curve.
 - Enables programmable motion without physical cam mechanism.

🔗 Key Siemens Function Blocks in TIA Portal:

Typical instruction flow for all these examples:

1. MC_CamTableSelect – Choose a predefined cam table.
2. MC_CamPrepare – Load and prepare the cam for synchronization.
3. MC_CamIn – Engage slave axis with master axis via the CAM table.
4. MC_CamOut – Disengage camming (return to independent control).

Would you like me to show **one or two full examples in Structured Text (ST)** — for instance a **flying saw** or **label applicator** — with actual Siemens S7-1200 CAM instructions (MC_CamTableSelect, MC_CamIn, etc.) in code form?

You said:

yes

ChatGPT said:

Perfect 👍 — let's go through **two complete examples** of how to use the Siemens **CAM instruction** on an **S7-1200T** (T-CPU with motion control).

I'll show both the **concept** and a **Structured Text (ST)** implementation using Siemens motion blocks.

⚙️ Example 1: Flying Saw Application

🎯 Goal

Cut material into fixed lengths **while it's moving**, by synchronizing a saw carriage (slave) with the feed roller (master).

🔧 Hardware Setup

- **Master Axis:** TO_Master (feed roller motor with encoder)
- **Slave Axis:** TO_Slave (flying saw carriage motor)
- **Control Type:** Electronic cam via a cam table

📄 Cam Table Concept

Master Pos (mm) Slave Pos (mm)

0	0
200	180
400	400
600	180
800	0

This creates a motion profile where:

- The slave accelerates to match feed speed,
- Moves synchronously while cutting (midsection),
- Then decelerates and returns.

Structured Text Example

VAR

```
camSelect   : MC_CamTableSelect;
camPrepare  : MC_CamPrepare;
camIn       : MC_CamIn;
camOut      : MC_CamOut;
camTableID  : INT := 1;      // ID of the cam table from TIA
masterAxis  : Axis_Ref;     // Reference to master axis
slaveAxis   : Axis_Ref;     // Reference to slave axis
startCam    : BOOL;        // Command to start camming
stopCam     : BOOL;        // Command to stop camming
```

END_VAR

// Step 1: Select the cam table

```
camSelect(Axis := slaveAxis,
          CamTableID := camTableID,
          Execute := TRUE);
```

// Step 2: Prepare the cam (must complete before engaging)

```
camPrepare(Slave := slaveAxis,
           Master := masterAxis,
           CamTableID := camTableID,
           Execute := camSelect.Done);
```

```
// Step 3: Engage the cam when ready
camIn(Slave := slaveAxis,
      Master := masterAxis,
      CamTableID := camTableID,
      Execute := startCam,
      BufferMode := 0,      // immediate
      MasterScaling := 1.0,
      SlaveScaling := 1.0);
```

```
// Step 4: Disengage cam (return slave to independent control)
camOut(Slave := slaveAxis,
      Execute := stopCam);
```

Logic sequence:

1. Load cam table (created in *Motion & Technology* → *Cams*).
2. Prepare and engage when master speed reaches target.
3. Slave follows cam curve automatically.
4. CamOut disengages control.

Example 2: Label Applicator

Goal

Apply labels to bottles on a conveyor; the label roller (slave) must synchronize with conveyor motion (master).

Setup

- **Master Axis:** Conveyor motor (with encoder feedback)
- **Slave Axis:** Label roll servo
- **Cam Table:** Defines speed ramp for label application zone

Master Pos (mm) Slave Pos (mm)

0	0
100	90
200	200
300	210
400	400

Structured Text Example

VAR

```
camSelect2 : MC_CamTableSelect;  
camPrepare2 : MC_CamPrepare;  
camIn2    : MC_CamIn;  
camOut2   : MC_CamOut;  
masterAxis2 : Axis_Ref;  
slaveAxis2 : Axis_Ref;  
camID2    : INT := 2; // Label cam table  
camStart2 : BOOL;  
camStop2  : BOOL;
```

END_VAR

// Select cam

```
camSelect2(Axis := slaveAxis2,  
           CamTableID := camID2,  
           Execute := TRUE);
```

```
// Prepare cam  
camPrepare2(Slave := slaveAxis2,  
            Master := masterAxis2,  
            CamTableID := camID2,  
            Execute := camSelect2.Done);
```

```
// Engage cam when bottle detected
```

```
camIn2(Slave := slaveAxis2,  
       Master := masterAxis2,  
       CamTableID := camID2,  
       Execute := camStart2,  
       BufferMode := 0,  
       MasterScaling := 1.0,  
       SlaveScaling := 1.0);
```

```
// Disengage after label applied
```

```
camOut2(Slave := slaveAxis2,  
        Execute := camStop2);
```

Process flow:

- Bottle sensor triggers camStart2.
- Label roll servo follows conveyor position using cam profile.
- When label applied, camStop2 disengages servo control.

 **Tips for TIA Portal Implementation**

- Define **Cam Tables** under:
Technology Objects → *Cams* → *New Cam Table*
 - You can view or edit cam shape graphically (position vs. position).
 - Make sure to assign **TO_Master** and **TO_Slave** in PLC tags.
 - Test each cam motion offline using *Trace* in TIA to ensure smooth motion.
-

Would you like me to show the **same flying saw example in Ladder (LAD)** format next, or keep it in **Structured Text** with added motion interlocks (e.g., speed threshold, homing check)?

You said:

lad

ChatGPT said:

Excellent 👍 — here's the **Flying Saw** example again, this time in **LADDER (LAD)** form for a Siemens **S7-1200T** using the **CAM instruction set** (MC_CamTableSelect, MC_CamPrepare, MC_CamIn, MC_CamOut).

⚙️ Example: Flying Saw – Ladder Logic Implementation

🎯 Objective

A flying saw (slave axis) follows a moving feed roller (master axis) during cutting. When the feed is running at production speed and a cut command is received, the saw axis engages a **cam profile** that synchronizes motion for one full cut cycle.

🧩 Required Elements

Technology Objects:

- TO_Master → Feed roller motor (master)
- TO_Slave → Saw carriage motor (slave)
- Cam table → e.g. Cam_FlySaw (created in *Technology Objects* → *Cams*)

PLC Tags (BOOLS):

Tag	Description
StartCam	Operator “Start Cut” signal
StopCam	Disengage / return home
CamSelected	Internal status bit
CamPrepared	Internal status bit
MasterAtSpeed	Interlock that master is at proper speed
CutDone	Signal that cut is complete

Ladder Network Breakdown

Network 1 – Select Cam Table

Instruction: MC_CamTableSelect

Input	Value
Axis	TO_Slave
CamTableID 1 (Cam_FlySaw)	
Execute	TRUE (rising edge at machine start)

Outputs:

Output Tag

Done CamSelected

 *Purpose:* Loads the correct cam table into memory. Usually done once after startup.

Network 2 – Prepare Cam

Instruction: MC_CamPrepare

Input	Value
Slave	TO_Slave
Master	TO_Master
CamTableID	1
Execute	CamSelected AND MasterAtSpeed

Outputs:

Output Tag

Done CamPrepared

 *Purpose:* Prepares synchronization parameters between master and slave before engaging.

Network 3 – Engage CAM (Run Cut)

Instruction: MC_CamIn

Input	Value
Slave	TO_Slave
Master	TO_Master
CamTableID	1
Execute	StartCam AND CamPrepared
BufferMode	0 (immediate)
MasterScaling	1.0
SlaveScaling	1.0

Outputs:

Output Tag

Active CamEngaged

Output Tag

Done CutDone

 *Purpose:* Starts the cammed motion; slave now follows master according to the cam curve. When motion completes, CutDone can be used to trigger the return or next cycle.

Network 4 – Disengage CAM

Instruction: MC_CamOut

Input Value

Slave TO_Slave

Execute StopCam OR CutDone

 *Purpose:* Returns the slave axis to independent control after the cut.

Network 5 – Home and Reset (optional)

You may use MC_Home or MC_MoveAbsolute to return the saw to a defined starting position once the cam disengages.

Logic Flow Summary

1. After power-up → select cam table (MC_CamTableSelect).
 2. When master is running → prepare cam (MC_CamPrepare).
 3. Operator presses “Start Cut” → MC_CamIn engages.
 4. Slave follows master via cam curve.
 5. When cycle done → MC_CamOut disengages.
 6. Return to home or await next trigger.
-

Tips in TIA Portal

- You can see these blocks under **“Instructions → Motion Control → Camming”**.

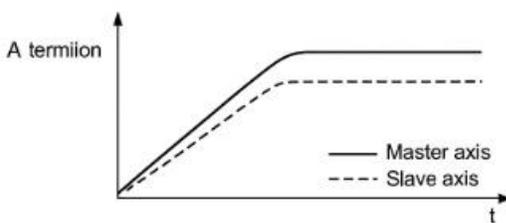
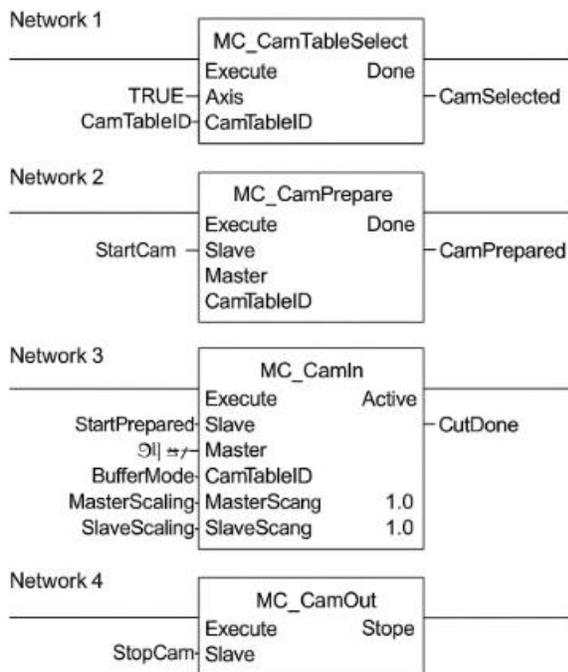
- Use the **Cam Editor** to graph position-to-position curve visually.
- Use the **Technology Trace** tool to verify synchronization timing.
- Make sure both axes are **homed and enabled** before running cam functions.

Would you like me to add a **diagram showing how the master and slave motion curves look (cam shape)** for this flying saw example? It helps visualize how the slave follows the master during the synchronized portion.

You said:

yes

ChatGPT said:



P. Cammina orcu a cammina profile

what are some good programs for choosing and sizing of pumps and valves

ChatGPT said:

Great question. The “right” program for pump and valve sizing depends a lot on your use case (hydronic HVAC, process plant, water systems, safety relief sizing, etc.). Here are some solid software tools — plus their strengths — that engineers commonly use:

Recommended Programs for Pump & Valve Sizing

1. AFT Fathom

- Very popular for incompressible fluid (liquid) systems. [Empowering Pumps and Equipment+2docs.aft.com+2](#)
- Supports pump sizing and selection, control valve sizing, system simulation, scenario management. [Datacor+1](#)
- Can import manufacturer pump curves, and it has a “Suggest Pump” feature to match a real pump. [docs.aft.com](#)
- Handles variable speed, viscosity, NPSH, and more. [Empowering Pumps and Equipment](#)

2. FluidFlow (by ADL / CASPEO)

- Very flexible “pipe-flow” simulation software for liquids, gases, two-phase, and slurries. [adltechnologies.com+1](#)
- Has auto-sizing functionality: can automatically size pipes, centrifugal or PD pumps, control valves, relief valves, etc. [knowledge.fluidflowinfo.com+1](#)
- Comes with a built-in database of fluids, pumps, valves, and fittings—plus you can add your own. [knowledge.fluidflowinfo.com](#)
- Supports heat transfer, so you can model temperature effects, insulation, exchangers, etc. [adltechnologies.com](#)
- Import of piping layout via PCF (Piping Component Files) from CAD. [CASPEO+1](#)

3. Nelprof (Valve Sizing & Selection)

- A web-based tool for valve sizing, especially control valves, on-off valves, and actuators. [Valmet](#)

- Includes performance analysis: cavitation, flashing, erosion, gas noise, plus actuator sizing. [Everest Automation](#)
- SIL (safety/integrity) module for final element PFD (probability of failure) calculations. [Valmet](#)
- Free-to-use via browser. [Everest Automation](#)

4. myValve® (ARI-Armaturen)

- Online or offline software for sizing various valve types: control, shut-off, safety, etc. [ARI-Armaturen](#)
- Computes Kv, flow, pressure drop, sound level, torque of actuator, etc. [ARI-Armaturen](#)
- Project management: tracking of valve tags, spare parts, drawings. [ARI-Armaturen](#)

5. PRV²SIZE (Emerson)

- Specialized for sizing over-pressure protection devices (safety/relief valves) and vents. [Valve Sizing](#)
- Supports common design standards (API 520, API 521, ISO, etc.) for gas, liquid, steam, fire sizing. [Valve Sizing](#)

6. SizeMaster (Curtiss-Wright / Farris)

- Web-based tool for designing relief system / pressure relief valves. [Valvesgroup+1](#)
- Lets you define overpressure scenarios (fire, thermal expansion, tube rupture, etc.) per API 521. [Valvesgroup](#)
- Allows customization of valve material, accessories, and other parameters. [Curtiss Wright Valves](#)

7. PumpLinx

- A 3D CFD (*Computational Fluid Dynamics*) software: good for detailed pump and valve flow/pressure analysis, especially for rotating parts, cavitation, transient effects. [Wikipedia](#)
- Useful if you need to validate very detailed hydraulic behavior, not just sizing.

8. DWSIM

- Open-source chemical process simulator. [Wikipedia](#)
- Good for process plants where you need to simulate not just hydraulics, but thermodynamics, reactions, streams, etc.
- *Not* as specialized for detailed pump curve matching as the dedicated hydraulic tools, but valuable in process engineering.

9. EPANET

- Public-domain software by the EPA for modeling water distribution networks. [Wikipedia](#)
- Models extended-period hydraulics in pressurized pipe networks, including valves and pump placement.
- More suited for water distribution systems than process or industrial pumping plants.

How to Choose the Right Tool

Here are some criteria and tips to help you pick the right program:

- **Purpose / Scale:**
 - For *detailed system engineering* (pipes, pumps, valves): go for AFT Fathom or FluidFlow.
 - For *relief system/safety valve sizing*: PRV²SIZE or SizeMaster.
 - For *process simulations* (chemistry, thermodynamics): DWSIM.
 - For *detailed CFD work*: PumpLinX.
- **Complexity:**
 - If you need multi-phase flows, heat transfer, non-Newtonian fluids => FluidFlow is very strong.
 - For simpler incompressible networks, Fathom is very efficient.
- **Cost:**
 - CFD tools (like PumpLinX) are likely more expensive, and require more expertise.
 - Some valve sizing tools/web tools are free or low cost (e.g. Nelprof).

- **Standards / Compliance:**
 - Make sure your tool supports relevant standards (e.g., API, ISO) if you need to meet regulatory or vendor requirements.
- **Data Integration:**
 - Do you need to pull in manufacturer pump curves or valve data? Tools like Fathom support “Suggest Pump” from manufacturer libraries. docs.aft.com
 - If you're using CAD, FluidFlow supports PCF import. adltechnologies.com
- **Reporting / Documentation:**
 - Tools like FluidFlow let you generate reports in Excel, Word, PDF. adltechnologies.com+1
 - For audit or safety-critical systems, you might need a sizing report — ensure the tool supports that.

If you like, I can put together a **comparison table** (cost, pros/cons, ideal use cases) of the top 5 tools as of now — do you want me to do that?

You said:

yes

ChatGPT said:

Good plan. Here’s a comparison table (5-tools) + breakdown: cost, pros/cons, and ideal use cases. Some of the pricing is approximate / depends on licensing, so treat as guidance.

Comparison Table: Top 5 Pump & Valve Sizing / Hydraulic Tools

Software	Rough Cost / Licensing	Strengths / Pros	Limitations / Cons	Best Use Cases
AFT Fathom	~ \$5,100 for a standalone license. TrustRadius	Very mature, great for incompressible fluid networks, pump sizing, NPSH, system simulation, “suggest	Cost can climb with modules or network licenses; steep learning	Hydronic systems, water networks, process plants where you need reliable pump +

Software	Rough Cost / Licensing	Strengths / Pros	Limitations / Cons	Best Use Cases
		pump” features. AFT Docs+1	curve for very complex systems.	valve analysis + cost comparison.
FluidFlow	<p>- Permanent license: liquid \$2,500; gas \$3,000; two-phase \$7,500; full (all) ~ \$10,000. Fluid Flow Info</p> <p>- Annual lease: liquid \$1,000; gas \$1,200; two-phase \$3,000; full \$4,000. Fluid Flow Info</p>	<p>Very flexible; can do auto-sizing of pumps, pipes, valves; supports multi-phase, heat transfer; good for process simulation. Fluid Flow Info+1</p>	<p>Significant cost for full modules; maintenance / support cost after first year (if perpetual) ~20% of list price. Fluid Flow Info</p>	<p>Process engineering, chemical plants, power plants, systems with mixed-phase flow, or where sizing optimization is needed.</p>
Nelprof (Valve Sizing)	<p>Free, web-based. Valin</p>	<p>Easy to access; strong for control valve sizing; handles cavitation, flashing, noise, actuator sizing, SIL. Valin</p>	<p>Limited to Valmet valves / components (or at least heavily oriented), not a full “system simulation” tool.</p>	<p>Control valve selection, preliminary valve work, SIL analysis, actuator/trip sizing.</p>
AioFlo	<p>Commercial, but relatively modest cost (license is paid; they offer a fully-functional trial). Katmar Software</p>	<p>Very lightweight, fast, good for quick pipe sizing / pressure drop / flow rate calculations, for both liquids and gases. Katmar Software</p>	<p>Not designed for complex system-level simulation, two-phase, or detailed pump curve matching.</p>	<p>Preliminary design, pipe-sizing checks, “back-of-the-envelope” hydraulics, simple systems.</p>
PRV²SIZE / SizeMaster (Safety /	<p>(Note: pricing varies and often requires a quote)</p>	<p>Designed for relief valve sizing per API / ISO, supports</p>	<p>Very specialized: not useful for general pump</p>	<p>Safety system design, overpressure</p>

Software	Rough Cost / Licensing	Strengths / Pros	Limitations / Cons	Best Use Cases
Relief Valves)		pressure-relief scenarios, allows scenario management, fire-case, etc.	network sizing; requires domain knowledge.	protection, relief valve selection and sizing in process plants.

My Take: Recommendations Based on Use Case

- If you need **general system hydraulics** (pumps + valves + pipes), **FluidFlow** or **AFT Fathom** are top picks.
- If you are mostly focused on **valve sizing/control valves**, **Nelprof** is excellent and low cost (free).
- For **safety relief**, go with a specialized tool (PRV²SIZE or SizeMaster).
- For quick, lightweight checks (single pipe or simple network), **AioFlo** is very efficient.

If you like, I can also include **open-source** or **free** alternatives (or lower-cost tools) in this comparison (in case budget is a concern). Do you want me to add those?

You said:

yes

ChatGPT said:

Good — here are some **free / open-source** (or very low-cost) tools you can use for pump, valve, and hydraulic / fluid-system sizing / simulation, plus trade-offs:

Open-Source / Free Tools for Pump & Valve / Hydraulic Sizing

Software	What It Is / Does Well	Limitations / When It Might Not Be a Great Fit
EPANET	Very commonly used for water distribution networks. Supports pumps, valves (pressure-reducing,	Focused on water distribution . Not designed for chemical-process pumps or highly dynamic/mechatronic systems. GUI is

Software What It Is / Does Well

Limitations / When It Might Not Be a Great Fit

	flow control, etc.), tanks, reservoirs, and simulates pressure, flow, and water quality over time. EPA+2EPA+2	somewhat dated. Doesn't handle two-phase flow or detailed valve actuator modeling.
DWSIM	Open-source process simulator (CAPE-OPEN compatible). Has unit operations like pumps, valves, pipes, pipe segments, and dynamic / steady-state simulation. DWSIM+1	More geared to <i>process engineering</i> (chemistry, thermodynamics) than pure hydraulic network design. Less intuitive for detailed pump-curve matching unless you build that into your flowsheet.
Hopsan	Free, open-source simulation environment for mechatronic and hydraulic systems , developed by Linköping University. Supports hydraulic components, mechatronics, control, etc. Wikipedia	More "simulation" than "sizing": good for dynamic / transient behavior, control, system interaction. Not super focused on optimizing valve sizing for industrial control valves in the same way as a dedicated sizing tool. Requires some modeling expertise.
Elmer FEM	A multi-physics finite-element solver. Can model fluid flow, heat transfer, structural, etc. Wikipedia	Very general-purpose. If you're just doing pump / valve sizing, this might be overkill (and more work) because it's not specifically tailored for hydraulic network simulation or sizing of control valves.
SimPy	A Python discrete-event simulation framework. Wikipedia	Not a hydraulic solver per se. Useful for process-level simulation (e.g., modeling events, queuing), but not for physical fluid flow or detailed pressure / valve sizing.

My Recommendations (for Free / Open-Source Tools)

- Use **EPANET** if your primary interest is **water distribution networks** — pumps + valves + tanks + diurnal demand, etc.
- Use **DWSIM** if you need to do **process-plant simulation**, where hydraulics is just one part of the flowsheet.

- Use **Hopsan** if you're modeling **hydraulic-mechatronic systems** (actuators, feedback, dynamic valves) and need good simulation fidelity.
- Use **Elmer** only if you need to couple hydraulics with **structural, thermal, or other physics**.
- Use **SimPy** if you're modeling **system-level behavior** (e.g., “when does this pump turn on?”, event-driven modeling), but not for detailed flow calculations.